Coherent Decay of Bose-Einstein Condensates

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Atomic Bose-Einstein condensates are singular forms of matter with the coherence between constituent atoms as a defining characteristic. Although this viewpoint is increasingly validated through experimental findings, the mechanisms behind the observed losses are still understood with classical recombinant collision arguments between particles within the condensate itself. By incorporating a general interparticle interaction into the Hamiltonian, a coherent decay rate can be obtained, thus providing a direct link between the observed losses and the microscopic two-body parameters. Appearing in the lifetime, the interaction strength, λ, is expressed as $\lambda = 8\pi a/(1-\delta)$, where the small parameter δ is obtained from a fit to experimental loss data. Most importantly, the lowest order rate exhibits a novel density dependence ($\rho^{3/2}$) that can be identified in low temperature tests.

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As the coldest form of matter known to exist, atomic Bose-Einstein condensates (BECs) are ideal superfluids in which the constituent atoms lose their individual identities, becoming absorbed into a single macroscopic quantum cloud that inherits its properties directly from the quantum world. One important quantum property arises from the microscopic propensity for molecule formation, recognized as the driving force behind the decay of the condensate. Despite the wealth of experiments that accentuate coherence as a fundamental aspect of the BEC [1], its decay mechanisms continue to be primarily understood through semiclassical collision kinetics. At finite temperature, the presence of quasiparticle excitations would induce interactions that can manifest in a manner consistent with a classical three-body interaction scheme. In addition to the presence of these excitations, we find that the BEC itself has an instability resulting in a decay of its coherent state. Initially discovered as a property of condensates having attractive interparticle interactions [2], this coherent loss can arise as a general feature of the many-body physics.

Although molecule formation had been long recognized as the fundamental reason for atom loss, the exact kinetic mechanism through which it manifested was at first poorly understood. To fit the experimental observations of cold hydrogen samples, a three-body process was eventually identified as being primarily responsible for the loss [3]. Specifically, a three-body recombinant event occurs when an atom causes a molecule to relax into a deeply bound state thereby releasing energy resulting in the expulsion of both atom and molecule from the trap. Although these initial gas samples were cold, they were relatively far from quantum degeneracy which allowed them to be described classically. As such, the proposed recombinant model was completely consistent with the theoretical framework underpinning the description of these gases.

To understand decay as a coherent mechanism, we first examine the order parameter, $\psi(\mathbf{r}, t)$, which is sometimes identified with the single-particle state into which all particles coalesce, but is normalized to the total number of condensate atoms, N_0. Denoting μ as the phase of the stationary, single-particle wave function, φ, we have

$$\psi(\mathbf{r}, t) = \sqrt{N_0}\varphi(\mathbf{r})e^{-i\mu t/\hbar}. \tag{1}$$

When substituted into the many-body Schrödinger equation, $i\hbar\partial_t\psi = \hat{H}\psi$, the time-independent Gross-Pitaevskii equation emerges, where μ is identified as the chemical potential in the usual way [4]. Therefore, this definition of the order parameter indicates that coherent decay arises for a complex valued chemical potential, with the imaginary part quantifying the decay rate. Through this argument it is seen that condensate loss can be incorporated as a fundamental aspect of the underlying physics, provided that the model Hamiltonian contains the necessary flexibility to yield complex values for μ in an equation of state.

Because no such complex solution exists in the mean field Gross-Pitaevskii treatment, our starting point is a more general Hamiltonian that is consistent with the low-energy scattering physics. In most treatments, a delta function interaction is specified where the strength is in proportion to the scattering length, $V(\mathbf{r}) = 8\pi a\delta(\mathbf{r})$. Although this model has been successful in many applications, it is inconsistent with the fact that, in the absence of a molecular state, the scattering length must vanish at zero range. In addition, it implies that attraction or repulsion is dictated entirely by the sign of a, thus precluding the possibility of having an attractive interaction with a positive scattering length. Hence, we remedy these deficiencies with the more general separable form given by the two-body matrix element,

$$\langle \mathbf{k}\mathbf{k}'|\hat{V}|\mathbf{k}''\mathbf{k}'''\rangle = \lambda\delta(\mathbf{P}-\mathbf{P}')f(\mathbf{q})f(\mathbf{q}'). \tag{2}$$

Taking all energies on the scale of $\hbar^2/2m$, λ is identified with the strength whereas f is the dimensionless form factor. In addition, the arguments are simplified with new momenta defined by $\mathbf{q} = (\mathbf{k} + \mathbf{k}')/2$, $\mathbf{q}' = (\mathbf{k}'' + \mathbf{k}''')/2$, $\mathbf{P} = \mathbf{k} - \mathbf{k}'$, and $\mathbf{P}' = \mathbf{k}'' - \mathbf{k}'''$. By solving the Lippmann-Schwinger equation, $T = V + VGT$, the strength is related to the scattering length through $(8\pi a)^{-1} = \lambda^{-1} + b^{-1}$, where the inverse range is given by $b^{-1} = (4\pi^2)^{-1} \int f(\mathbf{k})^2 dk$.

Since molecule formation is responsible for the decay, the model must include coupling to the molecular state which effectively changes the scattering length. This change is accounted for by a shift in the strength, $\lambda - g^2/\epsilon$, where ϵ is the energy difference between the atoms and the molecules and g is the coupling strength between the two components [5]. Using this substitution, the full scattering length becomes

$$\frac{1}{8\pi a} = \frac{1}{\lambda - g^2/\epsilon} + \frac{1}{b}. \tag{3}$$

For positive scattering lengths, the background ($g = 0$) strength must be greater than or equal to $8\pi a$, but inclusion of the bound state allows for an arbitrary value of λ since g^2 and ϵ are unknowns.

With the separable form in (2), we consider the zero temperature thermodynamic potential, $E - \mu N$, which is the expectation value of the operator

$$\begin{aligned}\hat{H} - \mu\hat{N} = &\int \hat{\psi}^\dagger(\mathbf{k})(k^2 - \mu)\hat{\psi}(\mathbf{k}) \\ &+ (\epsilon - 2\mu)\int \hat{\phi}^\dagger(\mathbf{k})\hat{\phi}(\mathbf{k}) \\ &+ \frac{\lambda}{2}\int \hat{\psi}^\dagger(\mathbf{q}_+)\hat{\psi}^\dagger(\mathbf{q}_-)f(\mathbf{q})f(\mathbf{q}')\hat{\psi}(\mathbf{q}'_-)\hat{\psi}(\mathbf{q}'_+) \\ &+ \frac{g}{\sqrt{2}}\int \hat{\phi}^\dagger(\mathbf{P})f(\mathbf{q})\hat{\psi}(\mathbf{q}_+)\hat{\psi}(\mathbf{q}_-) + \text{H.c.},\end{aligned} \tag{4}$$

where $\hat{\psi}$ and $\hat{\phi}$ are the respective atomic and molecular field operators coupled through the form factor $gf(\mathbf{q})/\sqrt{2}$ [6]. In order of appearance, the terms on the right-hand side comprise a kinetic part, the molecular contribution, the atom-atom potential, and the coupling term. Finally, the last two terms have been simplified using the same variables in (2), but with $\mathbf{q}_\pm = \mathbf{q} \pm \mathbf{P}/2$ and $\mathbf{q}'_\pm = \mathbf{q}' \pm \mathbf{P}/2$.

In a uniform system, the momentum space order parameters [see Eq. (1)] are simply constants multiplied by delta functions which appear in the field operators as $\hat{\psi}(\mathbf{k}) = u(\mathbf{k})\hat{a}(\mathbf{k}) + v(\mathbf{k})\hat{a}^\dagger(\mathbf{k}) + \frac{1}{\sqrt{2}}\psi\delta(\mathbf{k})$ and $\hat{\phi}(\mathbf{k}) = \frac{1}{\sqrt{2}}\phi\delta(\mathbf{k})$, where the $\hat{\psi}$ has been separated into a mean field part, denoted by $\psi\delta(\mathbf{k})$, and a fluctuation defined through the usual canonical Bogoliubov transformation [7]. Because of the absence of any $\hat{\phi}\,\hat{\phi}$ terms in (4), the variational analysis causes any molecular fluctuations to vanish, thereby reducing the molecular field to its mean. To calculate the expectation value of (4), we take a vacuum state, $|0\rangle$, such that $\hat{a}|0\rangle = 0$, then use Wick's theorem [8]. Furthermore, a step function form factor, $f(\mathbf{k}) = \theta(4\pi^2/b - |\mathbf{k}|)$, renders all integrals independent, where the cutoff is $4\pi^2/b$. Although this integral decoupling emerges in the variational treatment, we do not prove it here due to space constraints. Because of the system's uniformity, it is convenient to normalize the thermodynamic potential by the volume of the space, $\mathcal{V}$, which obtains the pressure as

$$\begin{aligned}-P = \frac{1}{\mathcal{V}}\langle\hat{H} - \mu\hat{N}\rangle = &\int (k^2 - \mu)v(\mathbf{k})^2 - \frac{1}{2}\mu\psi^2 \\ &+ \frac{\lambda}{2}\left[\int u(\mathbf{k})v(\mathbf{k})\right]^2 + \frac{1}{2}\lambda\psi^2\int u(\mathbf{k})v(\mathbf{k}) \\ &+ \lambda\left[\int v(\mathbf{k})^2\right]^2 + \lambda\psi^2\int v(\mathbf{k})^2 + \frac{\lambda}{8}\psi^4 \\ &+ \left(\frac{1}{2}\epsilon - \mu\right)\phi^2 + g\phi\left[\int u(\mathbf{k})v(\mathbf{k}) + \frac{1}{2}\psi^2\right].\end{aligned} \tag{5}$$

Arriving at this expression also requires the change of variables $\mathbf{P} \to -\mathbf{P}$, such that $\langle\hat{\psi}^\dagger(\mathbf{q}_\pm)\hat{\psi}(\mathbf{q}'_\mp)\rangle \to \langle\hat{\psi}^\dagger(\mathbf{q}_\pm)\hat{\psi}(\mathbf{q}'_\pm)\rangle$.

Under the usual constraint $u(\mathbf{k})^2 - v(\mathbf{k})^2 = 1$, the pressure is extremized by $\delta P/\delta v(\mathbf{k}) = \delta P/\delta\phi = \delta P/\delta\psi = 0$. Using the definitions

$$\gamma^2 = -\mu + 2\lambda\int v(\mathbf{k})^2 + \lambda\psi^2, \tag{6a}$$

$$\xi = \lambda\int u(\mathbf{k})v(\mathbf{k}) + \frac{1}{2}\lambda\psi^2 + g\phi, \tag{6b}$$

the variations reduce to the following set of equations

$$v(\mathbf{k})^2 = \frac{1}{2}\left[\frac{k^2 + \gamma^2}{\sqrt{(k^2 + \gamma^2)^2 - \xi^2}} - 1\right], \tag{7a}$$

$$u(\mathbf{k})v(\mathbf{k}) = -\frac{1}{2}\frac{\xi}{\sqrt{(k^2 + \gamma^2)^2 - \xi^2}}, \tag{7b}$$

$$\phi = -\frac{g}{\epsilon - 2\mu}\left[\int u(\mathbf{k})v(\mathbf{k}) + \frac{1}{2}\psi^2\right], \tag{7c}$$

$$\lambda\psi^2 = \xi + \gamma^2. \tag{7d}$$

Comprising a complete set, Eqs. (6) and (7) allow all quantities to be expressed in terms of a single parameter. Along with (6a), (7c), and (7d), the integrals of $v(\mathbf{k})^2$ and $u(\mathbf{k})v(\mathbf{k})$ are substituted into (6b), resulting in an expression relating ξ to γ^2. Further progress is achieved by defining the parameter δ through $\lambda = 8\pi a/(1 - \delta)$, which is substituted into (3) to give

$$\delta = \frac{(8\pi a - b)g^2 - (8\pi a)^2\epsilon}{(8\pi a - b)g^2 - 8\pi a b\epsilon}. \tag{8}$$

Assuming this to be a small quantity allows corresponding expansions to be made. Confirming the validity of this assumption, δ is determined from a fit of the coherent decay rate to the observed loss data [9].

Before a low-density expansion of μ can be made, it is first necessary to determine the magnitude of $|\xi|/|\gamma^2|$,

consistent with $|\delta| \ll 1$. Ignoring the quantum fluctuation terms will give the lowest order results which, by Eqs. (6a) and (7d), imply $\xi \sim \mu$. Because $\lambda \sim 8\pi a$, we have $\lambda\psi^2 \sim 16\pi a\rho \sim \mu + \gamma^2$, from Eq. (7d). Finally, the usual low-density chemical potential, $\mu \sim 8\pi a\rho$, gives $\xi \sim \gamma^2$.

With this approximation, the density $\rho = \psi^2/2 + \int v(\mathbf{k})^2 + \phi^2$, has the small range ($b\gamma \ll 1$) expansion given by [10]

$$\rho = \frac{1}{\lambda(1+\delta)}\gamma^2 + \frac{\sqrt{2}}{12\pi^2}\frac{1}{(1+\delta)^{5/2}} \times \left[4 - 5\delta + \frac{15}{8}\delta^2 \ln(\delta) + \ldots\right]\gamma^3 + \ldots. \quad (9)$$

Realized in terms of the small quantity δ, the expression in square braces is an expansion for $\xi \sim \gamma^2$ [10]. Inversion of the density equation results in an expression of γ in powers of $\sqrt{\rho}$ which, when used in (6a), obtains a chemical potential of the form

$$\mu = 8\pi a\rho + \mu_1\rho^{3/2} + \mu_2\rho^2 + \ldots. \quad (10)$$

To linear order in density, μ has the expected dependence, but the coefficients μ_1 and μ_2 are in general complex. Note that the expansion (10) includes a ρ^2 term which can be interpreted as a three-body effect, perhaps becoming dominant at higher temperature [11]. However, at low density the $\rho^{3/2}$ term dominates, having a coefficient dependent only on the scattering length and the two-body interaction strength, λ [12].

Since μ is also the phase of the order parameter, the imaginary part of μ_1 determines the dominant, lowest order coherent decay rate coefficient as

$$\Gamma_0 = \frac{\hbar}{m}\frac{5\sqrt{2}}{32\pi}(\delta^2 + \cdots)\lambda^{5/2}. \quad (11)$$

Obtaining a nonzero rate requires $\lambda \lesssim 8\pi a$ which, as demonstrated by (3), is a condition made possible only by inclusion of a molecular state. In addition, we see that a sufficiently small range ($b \ll a$) implies the two-body relationship $g^2/\epsilon \sim 8\pi a$, which can be used as a test for the applicability of this description. Given that λ is proportional to the scattering length, the rate coefficient is roughly seen to have an $a^{5/2}$ dependence that is reminiscent of the earlier result [2].

Although we have shown that a complex chemical potential can arise in a realistic Hamiltonian, the predicted density dependence must be consistent with experimental findings. Extension of the uniform medium result to the experiments' trapping geometries requires the application of the local density approximation, where it is assumed that each point inside the trap admits a locally uniform value for μ. Proceeding in the same manner as the treatment of the three-body collisional loss [13], the coherent rate is described by $-\Gamma_0 \int \rho(\mathbf{r})^{5/2} d^3r = -\Gamma_0\langle\rho^{3/2}\rangle N$, with N being the total number of atoms in the trap. Note that the averaging is weighted by density which, in the Thomas-Fermi limit, assumes a parabolic profile, $\rho(\mathbf{r}) = [m^2\bar{\omega}^2/(8\pi\hbar^2 a)](r_0^2 - r^2)$, where $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$, and r_0 is the BEC radius found from normalization to N. With this profile, the integral is calculated to be $\langle\rho^{3/2}\rangle = c_{5/2}N^{3/5}$, where $c_{5/2} = (75\pi/4096)(7c_2)^{3/2}$ and $c_2 = [15^{2/5}/(14\pi)][m\bar{\omega}/(\hbar\sqrt{a})]^{6/5}$. Inside the chamber, background atoms will impinge on the BEC resulting in quasiparticle excitations which manifest as the loss of individual particles from the collective. Taking τ_0 as the lifetime from this background effect, the averaged coherent rate equation is given by

$$\frac{1}{N}\frac{dN}{dt} = -\Gamma_0 c_{5/2}N^{3/5} - \frac{1}{\tau_0}. \quad (12)$$

To test its consistency with experiment, we choose an appropriate value of δ to fit the time evolution of N [Eq. (12)] to the loss data [14]. Demonstrated in the inset of Fig. 1, these data provide no appreciable distinction between the loss as a coherent phenomenon and a purely three-body effect. Because of a lack of detailed loss measurements, the value of λ in other atomic species must be found by a coarse one-point lifetime observation. A summary of some different cases is displayed in Table I.

The imaginary part of μ gives a decay rate consistent with experiment, but it does not indicate how atom loss occurs. Addressing this first requires recognition of a lower energy solution corresponding to the two-body bound state [9]. A small oscillation expansion reveals that the present solution lies inside a continuum of excitations of that lower state. Hence, the decay represents a transition into collective modes which can induce quasiparticle ejection from the condensate [20].

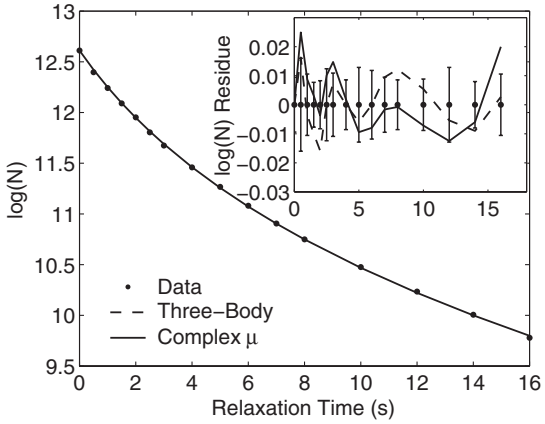

FIG. 1. Fit of the decay rate given in Eq. (12) to the experimental loss data of ^{87}Rb [13] with an inset showing a close-up view comparing the difference between the coherent decay and the semiclassical three-body recombinant model. In a least squares sense, the best fit is obtained for a decay rate coefficient of $\Gamma_0 = 2.7732 \times 10^{-22}$ cm$^{9/2}$ s^{-1} which, by Eq. (11), corresponds to an interaction strength $\lambda = (0.974)8\pi a$. Also, the background lifetime is found to be $\tau_0 = 38.8$ s, implying it is not the dominant effect.

TABLE I. Table showing the numerical values of the interaction strength, λ, as determined from a fit of Eq. (11) to the observed lifetime, τ. The ^{87}Rb strength was found from the loss curve in Fig. 1 whereas ^{23}Na, ^{133}Cs, and ^{4}He were determined by matching a single observed lifetime measurement (obtained from [15–17], respectively) to the exponential rate $-\Gamma_0\bar{\rho}^{3/2}$. In the latter cases, an average trap density was used in finding the $1/e$ lifetime. For the ^{133}Cs condensate produced using a Feshbach resonance, we show the magnetic field value, B, corresponding to the scattering length, a, at which the lifetime was observed. Depicted in the table are experiments in which the loss was primarily due to the internal physics of the BEC itself, but not its interaction with the trapping potential. The only exception is the case for the ^{4}He where the causes of the loss were unclear. Not shown are the cases of ^{7}Li [18] and ^{41}K [19] which involve additional physics thus prohibiting a clean comparison from being made.

Isotope	a (nm)	B (G)	$\bar{\rho}$ (10^{14} cm^{-3})	τ (s)	$\frac{\lambda}{8\pi a}$
^{87}Rb	5.8	⋯	1	16	0.974
^{23}Na	4.5	⋯	1	1	0.967
^{133}Cs	15.9	23	0.1	15	0.976
^{4}He	20	⋯	0.1	∼3	0.993

In conclusion, the chemical potential's imaginary part quantifies the coherent decay rate through the phase of the condensate wave function. In a variational analysis of a general Hamiltonian, a complex valued chemical potential arises for suitable parameter values. From this perspective, observed condensate losses can be regarded as a signature of the two-body interaction present in the model Hamiltonian. In particular, this result establishes the two-body condition $g^2/\epsilon \sim 8\pi a$, which can be checked by molecular theorists. Most importantly, the decoherence mechanism has a lowest order $\rho^{3/2}$ dependence that may be distinguished by experiments at lower densities.

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