Noise Squeezing in a Nanomechanical Duffing Resonator

R. Almog, S. Zaitsev, O. Shtempluck, and E. Buks

Department of Electrical Engineering, Technion, Haifa 32000, Israel

(Received 5 July 2006; published 16 February 2007)

We study mechanical amplification and noise squeezing in a nonlinear nanomechanical resonator driven by an intense pump near its dynamical bifurcation point, namely, the onset of Duffing bistability. Phase sensitive amplification is achieved by a homodyne detection scheme, where the displacement detector's output, which has a correlated spectrum around the pump frequency, is down-converted by mixing with a local oscillator operating at the pump frequency with an adjustable phase. The down-converted signal at the mixer's output could be either amplified or deamplified, yielding noise squeezing, depending on the local oscillator phase.

DOI: 10.1103/PhysRevLett.98.078103 PACS numbers: 87.80.Mj, 05.45.−a

Micro- or nanoelectromechanical resonators are widely employed for ultrasensitive force/mass measurements [1–3]. A possible technique to improve the signal to noise ratio in such devices is to implement an on-chip mechanical amplification. Such an amplification, as well as thermomechanical noise squeezing in microresonators, has been achieved before using parametric excitation [4,5]. In the present work, we use a different mechanical amplification scheme based on a bifurcating dynamical system, exploiting its high sensitivity to fluctuations near its bifurcation point [6–10]. This amplification scheme has been used lately for quantum measurements of superconducting qubits [11]. In our case, we use the onset of bistability in a nanomechanical Duffing resonator as the bifurcation point. In a Duffing resonator, above some critical driving amplitude, the response becomes a multivalued function of the frequency in some finite frequency range, and the system becomes bistable with jump points in the frequency response [12,13]. Previously, we exploited this property to demonstrate high intermodulation gain [14]. This was achieved by employing an intense pump signal to drive the resonator near the onset of bistability, thus enabling amplification of a small signal in a narrow bandwidth. Here we employ this mechanism for the first time in nanomechanical resonators to demonstrate experimentally phase sensitive amplification and noise squeezing. The system under study, coined a nanomechanical bifurcation amplifier, consists of a nonlinear doubly clamped nanomechanical PdAu beam, excited capacitively by an adjacent gate electrode.

The experimental setup is shown in Fig. 1. The resonator is excited by two sources (a pump and a small test signal or noise), and its vibrations are detected optically using a knife-edge technique [15]. The device is located at the focal point of a lensed fiber which is used to focus laser light at the beam and to collect the reflected light back to the fiber and to a photodetector (PD). The PD signal is amplified, mixed with a local oscillator (LO), low pass filtered, and measured by a spectrum analyzer. The measurement is done in vacuum (10^{-5} torr) and at room temperature. The resonator has length $l = 100$ μm, width $w = 600$ nm, and thickness $t = 250$ nm. The gap separating the doubly clamped beam and the stationary side electrode is $d = 4$ μm wide. The device is fabricated using a bulk nanomachining process together with electron beam lithography [16].

The nonlinear dynamics of the fundamental mode of a doubly clamped beam driven by an external force per unit mass $F(t)$ can be described by a Duffing oscillator equation for a single degree of freedom x:

$$\ddot{x} + 2\mu(1 + \beta x^2)\dot{x} + \omega_0^2(1 + \kappa x^2)x = F(t), \qquad (1)$$

where μ and β are the linear and nonlinear damping constants, respectively, $\omega_0/2\pi$ is the resonance frequency of the fundamental mode, and κ is the cubic nonlinear constant. Our resonator has a quality factor $Q = \omega_0/2\mu \simeq 2000$ (at 10^{-5} torr), and its fundamental mode resonance

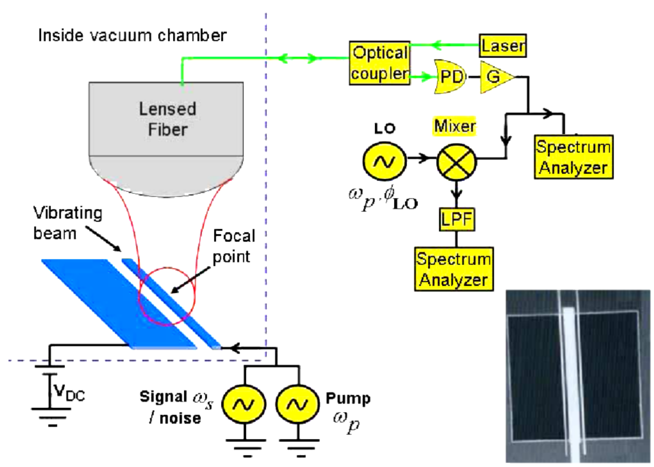

FIG. 1 (color online). The experimental setup. The device consists of a suspended doubly clamped nanomechanical resonator. The resonator is excited by two phase locked sources (one source is used as a pump, and the other one as a small test signal or as a noise source). The resonator's vibrations are detected optically. The inset shows an electron micrograph of the device.

frequency is $\omega_0/2\pi \simeq 500$ kHz. Generally, for resonators driven using a bias voltage applied to a side electrode, Eq. (1) should contain additional parametric terms [4,17]. In our case, however, the effect of these parametric terms on the dynamics is negligibly small [18].

The relative importance of nonlinear damping can be characterized by the dimensionless parameter $p = 2\sqrt{3}\mu\beta/\kappa\omega_0$ [18]. The relatively low value of $p \simeq 0.05$, obtained from the measured values of ω_0, μ, and the frequency of the bifurcation point [18], indicates that the effect of nonlinear damping in the present case is relatively weak. To investigate nonlinear amplification of a small test signal, the resonator is driven by an applied force $F(t) = f_p \cos(\omega_p t) + f_s \cos(\omega_s t + \varphi)$, composed of an intense *pump* with frequency $\omega_p = \omega_0 + \sigma$, amplitude f_p, and a small force (called *signal*) with frequency $\omega_s = \omega_p + \delta$, relative phase φ, and amplitude f_s, where $f_s \ll f_p$ and $\sigma, \delta \ll \omega_0$. This is achieved by applying a voltage of the form $V = V_{\mathrm{dc}} + V_p \cos(\omega_p t) + V_s \cos(\omega_s t + \varphi)$, where V_{dc} is a dc bias (employed for tuning the resonance frequency) and $V_s \ll V_p \ll V_{\mathrm{dc}}$. The resonator's displacement has spectral components at ω_p, at ω_s, and at the intermodulations $\omega_p \pm k\delta$, where k is an integer. The one at frequency $\omega_i = \omega_p - \delta$ is called the *idler* component.

Strong correlation between the signal and the idler, occurring near the edge of the bistability region, could be exploited for phase sensitive amplification and noise squeezing [7,19]. This is achieved by a homodyne detection scheme, where the displacement detector's output is down-converted by mixing with a LO operating at frequency ω_p with an adjustable phase ϕ_{LO} and phase locked to the pump. The mixer's output (IF port) has a spectral component at frequency δ, which is proportional to the phasor sum of the signal and the idler, yielding phase sensitive amplification, controlled by ϕ_{LO}. In the notation of Ref. [14], the displacement $x(t)$ is given by

$$x(t) = \tfrac{1}{2}A(t)e^{i\omega_p t} + \text{c.c.}, \tag{2}$$

where $A(t) = a_p + a_s e^{i\delta t} + a_i e^{-i\delta t}$ is a slowly varying function (relative to the time scale $1/\omega_p$), and the complex numbers a_p, a_s, and a_i are the pump, signal, and idler spectral components of $A(t)$, respectively. Suppose that the LO voltage is given by $V^{\mathrm{LO}}(t) = V_0^{\mathrm{LO}} \cos(\omega_p t + \phi_{\mathrm{LO}})$, and the mixer's output is given by $V_{\mathrm{MO}} = Mx(t)V^{\mathrm{LO}}(t)$, where M is a constant term depending on the optical detector's sensitivity, amplification, and the mixing factor. After passing through a low pass filter (LPF), the output signal is

$$\tfrac{1}{4}MV_0^{\mathrm{LO}}[A(t)e^{-i\phi_{\mathrm{LO}}} + \text{c.c.}].$$

The measured quantity is the amplitude $R(\delta)$ of the spectral component of the output signal at frequency δ. $R(\delta)$ depends on the LO phase ϕ_{LO} and is given by

$$R(\delta) = \tfrac{1}{2}MV_0^{\mathrm{LO}}|a_s e^{-i\phi_{\mathrm{LO}}} + a_i^* e^{i\phi_{\mathrm{LO}}}|. \tag{3}$$

As ϕ_{LO} is varied, the term $|a_s e^{-i\phi_{\mathrm{LO}}} + a_i^* e^{i\phi_{\mathrm{LO}}}|$ oscillates between the minimum value $||a_s| - |a_i||$ and the maximum one $|a_s| + |a_i|$. When $\delta \to 0$, and a_p is tuned to the bifurcation point, we have $|a_s| = |a_i| \simeq f_s/2\omega_0\delta$ [14]; hence, $R(\delta)_{\max} = MV_0^{\mathrm{LO}} f_s/2\omega_0\delta$ and $R(\delta)_{\min}/R(\delta)_{\max} \to 0$. The factor $\Delta \equiv R(\delta)_{\max} - R(\delta)_{\min}$ characterizes the phase dependence of the amplification. An example of a measurement of $R(\delta)$ vs ϕ_{LO} is shown in Fig. 3(a).

To study the response to injected noise, the resonator is excited by a fixed pump near the bifurcation point, together with white noise. In this case, Eq. (1) is a Langevin equation with $F(t) = f_p \cos(\omega_p t) + F_n(t)$, where $F_n(t)$ is a white noise having a vanishing mean $\langle F_n(t)\rangle = 0$ and spectral density $S_{F_n} = 4\omega_0 k_B T_{\mathrm{eq}}/mQ$ [3]. Here T_{eq} is the equivalent temperature of the applied voltage noise; m is the effective mass of the fundamental mode. In this case, the displacement spectral density measured at the mixer's output will consist of two contributions, namely, the pump response (δ function peaked at $\delta = 0$) and a continuous part $S_x(\delta)$ due to noise. In the limit $\delta \to 0$, the spectral density S_x, which was calculated in Ref. [20], is given by

$$S_x = \frac{1 + 2\zeta\cos(\phi_{\mathrm{LO}} - \phi_0) + \zeta^2}{(1 - \zeta^2)^2} S_{x0}, \tag{4}$$

where

$$S_{x0} = S_{F_n}/\{4\omega_0^2[\mu^2 + (\omega_p - \omega_0 - \tfrac{3}{2}\omega_0\kappa|a_p|^2)^2]\},$$

and ϕ_0 and ζ are real parameters. While ζ vanishes in the linear region, its largest value $\zeta = 1$ is obtained along the edge of the bistability region. Equation (4) implies that when $\delta \to 0$, the output noise will oscillate between a maximum value, corresponding to the amplified quadrature, and a minimum one, corresponding to the deamplified (or squeezed) quadrature, as ϕ_{LO} is varied [20]:

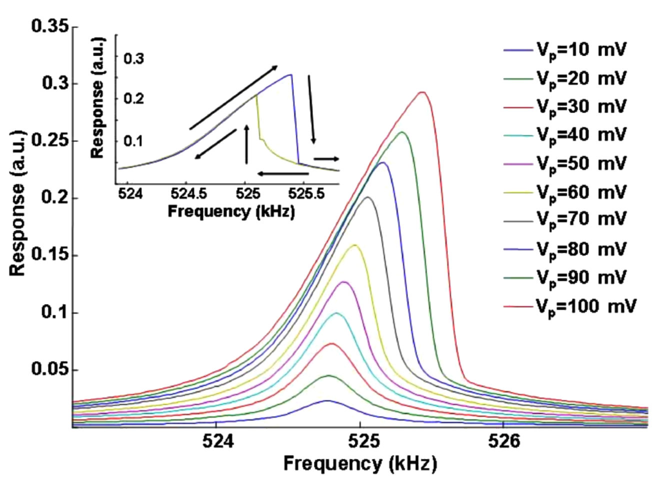

FIG. 2 (color online). Typical frequency response curves for various excitation voltages V_p and upward frequency scan. The inset shows hysteresis response for $V_p = 90$ mV.

$$[S_x]_{\max} = \frac{S_{x0}}{(1-\zeta)^2}; \qquad [S_x]_{\min} = \frac{S_{x0}}{(1+\zeta)^2}.$$

Thus, the largest amplification obtained by this model diverges at the bifurcation point, whereas noise squeezing is limited to a factor of 4.

We now turn to describe the experimental steps. As a first step, we find the onset of bistability and characterize the bistability region. This is achieved by sweeping the pump frequency upward and back downward for different constant excitation amplitudes, without an additional small signal or noise. Typical response curves are shown in Fig. 2. The bistability region and the bifurcation point B_p (marked with a circle) are shown in Fig. 3(b). In the next step, we characterize the small signal amplification by exciting the resonator with a pump and a small test signal where $V_p/V_s = 25$ and $\delta = 30$ Hz. Measurements of Δ vs frequency are shown in Fig. 3(c) for four pump amplitudes [related to lines 1–4 in Fig. 3(b)]. The response of the frequency upward (downward) sweep is depicted with a black (green) line. For $V_p = 50$ mV [Fig. 3(c)(1)], the frequency sweep is contained within the monostable region, and, consequently, the value of Δ is relatively small. For $V_p = 70$ mV [Fig. 3(c)(2)], $V_p = 90$ mV [Fig. 3(c)(3)], and $V_p = 110$ mV [Fig. 3(c)(4)], on the other hand, the frequency sweeps cross the bistability region and two peaks are seen for Δ, corresponding to the upward and downward frequency sweeps. These peaks originate from the high signal amplification in the jump points of the pump response. Note that in this case the width of the hysteresis loop (which is the distance between the peaks) is smaller relative to the case when the pump is the only excitation.

We now turn to investigate the resonator response to pump and noise. First, the bifurcation point (B_p) is located. A frequency response of the beam, excited by the pump (without noise) in the vicinity of B_p is shown in Fig. 4(a).

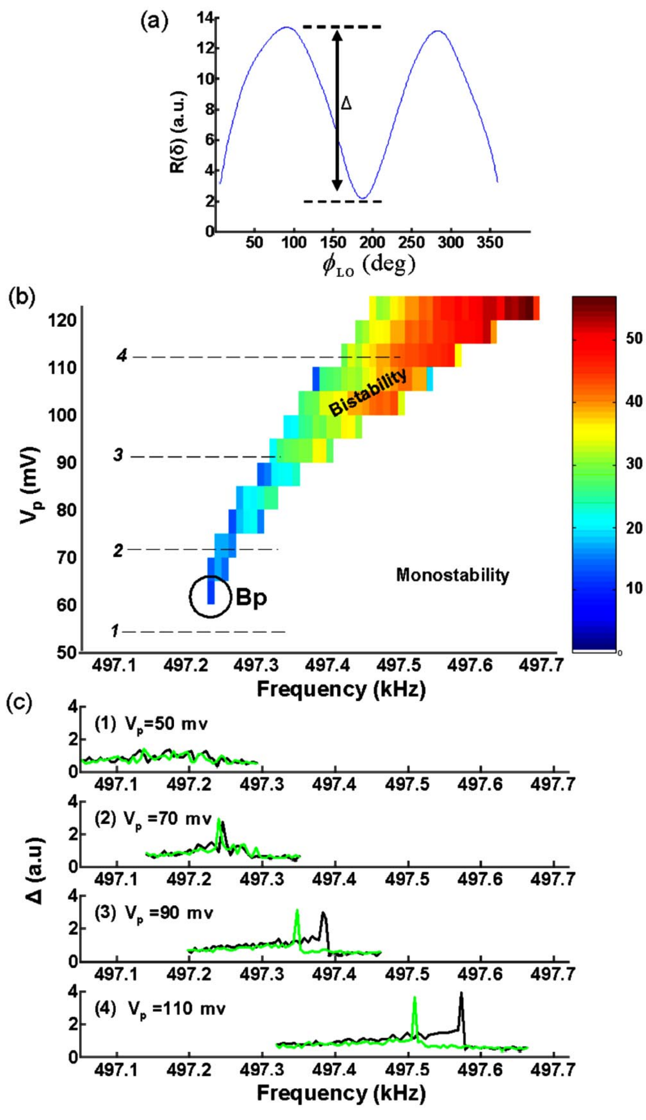

FIG. 3 (color online). (a) Measured $R(\delta)$ vs LO phase $\phi_{\rm LO}$. (b) Measurement of the bistability (hysteresis) region. The bifurcation point B_p is marked with a circle. (c) The parameter Δ vs frequency for four different V_p values [related to lines 1–4 in (b)].

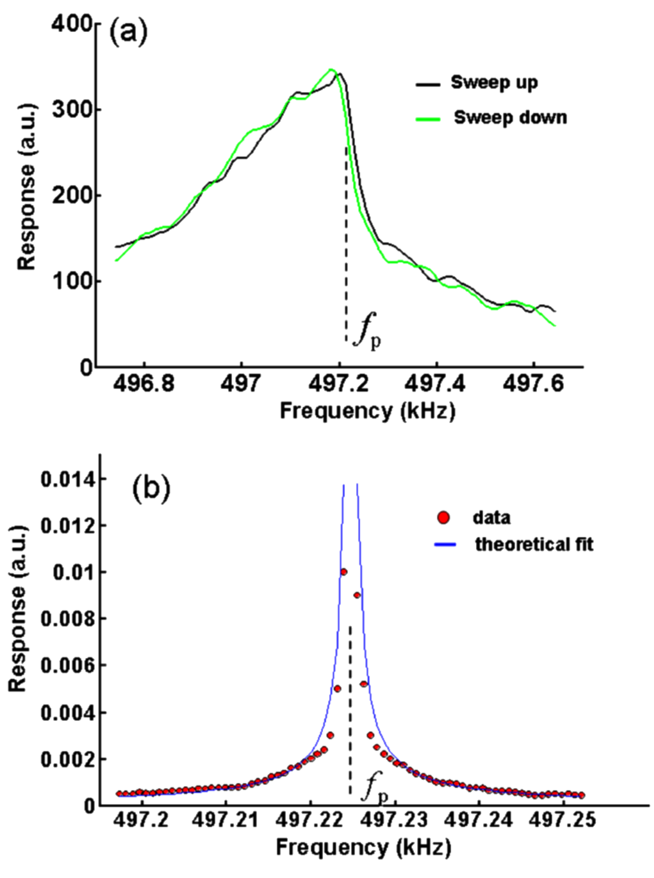

FIG. 4 (color online). (a) Pump response near B_p. Upward and downward sweeps are seen in black and green, respectively. (b) Averaged spectrum response for pump and noise excitation. The input noise spectral density is 1 mV/$\sqrt{\rm Hz}$. Circles indicate the experimental data, whereas a theoretical fit is seen as a blue line.

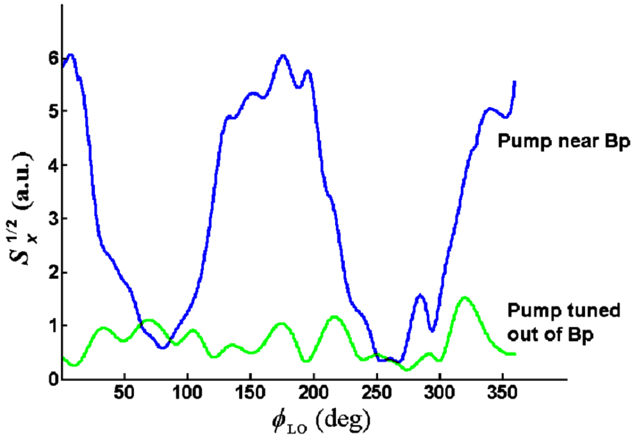

FIG. 5 (color online). Noise squeezing. The spectral component $S_x^{1/2}$ vs $\phi_{\rm LO}$ for $\delta = 10$ Hz. The resonator is excited by pump and noise. Blue line—pump near B_p; green line—pump tuned out of B_p (200 Hz higher).

In the next step, the pump frequency is fixed to the bifurcation point, and we add white noise to the excitation having spectral density $S_{V\rm noise}^{1/2} = 1$ mV$/\sqrt{\rm Hz}$ (since the thermomechanical fluctuations are relatively weak, we employ externally injected noise).

The measured spectrum taken around the pump frequency [see Fig. 4(b)] demonstrates strong amplification occurring in this region, a manifestation of the noise rise phenomenon [21]. There is good agreement between the theoretical fit (δ^{-1} dependence) [7] to the experimental data for $\delta > 50$ Hz. For smaller values of δ, the model breaks down due to high order terms.

Noise squeezing is demonstrated in Fig. 5, where $\delta = 10$ Hz, $S_{V\rm noise}^{1/2} = 1$ mV$/\sqrt{\rm Hz}$, and $S_x^{1/2}$ is plotted vs the LO phase $\phi_{\rm LO}$. Here the sweep time is 6 s, and the resolution bandwidth is 2 Hz. The blue line demonstrates the case where the pump is in the vicinity of the bifurcation point, whereas the green line demonstrates the case where the pump is far from the bifurcation point. The noise amplitude amplification is about 6. The deamplified (squeezed) quadrature is below the measurement noise floor; hence, it cannot be measured. Using the measured room temperature thermomechanical fluctuations, we estimate the noise floor (of the measurement system) to be 3.7×10^{-13} m$/\sqrt{\rm Hz}$ [16].

To summarize, in this experiment we have demonstrated experimentally phase sensitive amplification and noise squeezing in a nanomechanical resonator using the onset of Duffing bistability and a homodyne detection scheme. This could be exploited for both signal amplification and noise reduction, which could be useful for detection of weak forces. Moreover, our noise squeezing scheme can be exploited for enhancing the sensitivity of resonant detectors based on nanomechanical resonators (e.g., mass detector) [20]. On the other hand, such enhancement is typically accompanied by an undesirable slowing down of the response of the detector.

This work is supported by the Israeli Ministry of Science, Intel Corporation, the Israel–U.S. Binational Science Foundation, and the Henry Gutwirth Foundation. The authors thank Bernard Yurke for helpful discussions.

[1] M. L. Roukes, Phys. World **14**, No. 2, 25 (2001).
[2] H. G. Craighead, Science **290**, 1532 (2000).
[3] A. N. Cleland, *Foundation of Nanomechanics* (Springer, New York, 2003).
[4] D. Rugar and P. Grütter, Phys. Rev. Lett. **67**, 699 (1991).
[5] D. W. Carr, S. Evoy, L. Sekaric, H. G. Craighead, and J. M. Parpia, Appl. Phys. Lett. **77**, 1545 (2000).
[6] K. Wiesenfeld and B. McNamara, Phys. Rev. A **33**, 629 (1986).
[7] B. Yurke and E. Buks, quant-ph/0505018 [J. Lighwave Tech. (to be published)].
[8] H. Krömmer, A. Erbe, A. Tilke, S. Manus, and R. H. Blick, Europhys. Lett. **50**, 101 (2000).
[9] S. Savel'ev, A. L. Rakhmanov, and F. Nori, Phys. Rev. E **72**, 056136 (2005).
[10] M. I. Dykman, D. G. Luchinsky, R. Mannella, P. V. E. McClintock, N. D. Stein, and N. G. Stocks, Phys. Rev. E **49**, 1198 (1994).
[11] I. Siddiqi, R. Vijay, F. Pierre, C. M. Wilson, M. Metcalfe, C. Rigetti, L. Frunzio, and M. H. Devoret, Phys. Rev. Lett. **93**, 207002 (2004).
[12] A. H. Nayfeh and D. T. Mook, *Nonlinear Oscillations* (Wiley, New York, 1979).
[13] L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon, Oxford, 1976).
[14] R. Almog, S. Zaitsev, O. Shtempluck, and E. Buks, Appl. Phys. Lett. **88**, 213509 (2006).
[15] D. Karabacak, T. Kouh, C. C. Huang, and K. L. Ekinci, Appl. Phys. Lett. **88**, 193122 (2006).
[16] E. Buks and M. L. Roukes, Phys. Rev. B **63**, 033402 (2001).
[17] M. P. Blencowe and M. N. Wybourne, cond-mat/9811232.
[18] Stav Zaitsev, Ronen Almog, Oleg Shtempluck, and Eyal Buks, cond-mat/0503130.
[19] R. Movshovich, B. Yurke, P. G. Kaminsky, A. D. Smith, A. H. Silver, R. W. Simon, and M. V. Schneider, Phys. Rev. Lett. **65**, 1419 (1990).
[20] E. Buks and B. Yurke, Phys. Rev. E **74**, 046619 (2006).
[21] P. Bryant, K. Wiesenfeld, and B. McNamara, J. Appl. Phys. **62**, 2898 (1987).