Induced Time-Reversal Symmetry Breaking Observed in Microwave Billiards

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Using reciprocity, we investigate the breaking of time-reversal (T) symmetry due to a ferrite embedded in a flat microwave billiard. Transmission spectra of isolated single resonances are not sensitive to T violation, whereas those of pairs of nearly degenerate resonances do depend on the direction of time. For their theoretical description a scattering matrix model from nuclear physics is used. The T-violating matrix elements of the effective Hamiltonian for the microwave billiard with the embedded ferrite are determined experimentally as functions of the magnetization of the ferrite.

We investigate time-reversal symmetry breaking (TRSB) in resonant scattering through a flat microwave cavity. Such cavities, also known as "microwave billiards," have long been used to simulate quantum billiards [1–3], and interest has mainly been focused on statistical properties of the eigenvalues [4,5], which have also been studied in microwave networks [6], and eigenfunctions [7], and on dissipative transport [8]. We report on experiments where time-reversal (T) symmetry is broken by the magnetic and absorptive properties of a piece of ferrite. Here, the two-state system is the simplest one to show effects of TRSB. Thus, our investigations focus on pairs of nearly degenerate resonances (doublets) and, as a test, also on isolated resonances (singlets). We determine all relevant parameters of the system as functions of the magnetization of the ferrite and thereby the properties of both, the two-state system and the ferrite. Needless to say, our work is not related directly to studies of TRSB in fundamental interactions. Indeed, we recall that in the absence of dissipation all classical physics is T invariant, and that among the four fundamental interactions, only the weak force is known to cause TRSB [9]. Rather, the microwave billiard with the ferrite inside serves as a paradigmatic example of how to study and analyze TRSB in resonance scattering experiments.

The experiments were performed with a flat, cylindrical microwave resonator made of copper. The resonator has a circular shape with a diameter of 250 mm and a height of 5 mm (see Fig. 1). An HP 8510C vector network analyzer (VNA) coupled power into and out of the system through two antennas, metal pins of 0.5 mm in diameter reaching about 2.5 mm into the resonator. The microwave resonator with the antennas as entrance and exit channels defines a scattering system [10]. The VNA measures the relative phase and amplitude of the input and output signals, thereby providing the complex elements of the scattering matrix. Effects of the connecting coaxial cables have been eliminated by a careful calibration of the VNA. The TRSB is caused by a ferrite inside the resonator [4,5,7,8]. The ferrite is a calcium vanadium garnet and has the shape of a

cylinder, 5 mm high and 4 mm in diameter [11]. The material has a resonance line width of $\Delta H = 17.5$ Oe and a saturation magnetization of $4\pi M_S = 1859$ Oe. At the position of the ferrite a static magnetic field was created with a NdFeB magnet, which was placed outside the resonator and connected to a screw thread. Thus, magnetic field strengths of up to 90 mT (with an uncertainty below 0.5 mT) were obtained at the ferrite.

The spins within the ferrite precess with their Larmor frequency about the static magnetic field; this introduces a chirality into the system. A circularly polarized rf magnetic field interacts with the precessing spins if it has the same rotational direction and frequency. The magnetic field component of the rf field of the resonator can be split into two circularly polarized fields rotating in opposite directions. Only one of these is attenuated by the ferrite, this causes TRSB. In contradistinction to Ref. [12], T invariance is experimentally tested by interchanging input and output at the resonator. Thereby, the attenuated direction of polarization is interchanged with the unaffected one. The TRSB effect is strongest at the ferromagnetic resonance where the Larmor frequency of the ferrite matches the eigenfrequency of the cavity.
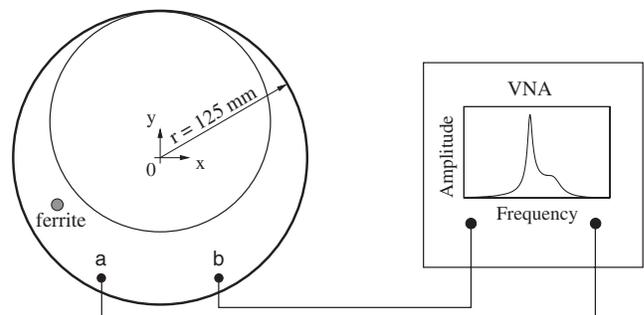

FIG. 1. Scheme of the experimental setup (not to scale). The antennas connected to the vector network analyzer (VNA) were located at the positions a and b. The inner circle is the copper disk introduced into the resonator to transform the circular into an annular billiard.

To interpret our experiments, we use the scattering formalism for microwave billiards of Ref. [10]. The element S_{ab} of the scattering matrix S is the amplitude for the transition from an initial state (in the connecting coaxial cables) $|b\rangle$ to a final state $|a\rangle$. For a resonator with two attached, pointlike antennas S has dimension two. T invariance implies that S is symmetric [13], i.e.,

$$S_{ab} = S_{ba}. \qquad (1)$$

Property (1) is referred to as *reciprocity*. Experiments in nuclear physics [14] have tested the weaker principle of detailed balance [13] which implies $|S_{ab}|^2 = |S_{ba}|^2$. In the present experiment we test the stronger statement of reciprocity.

How does the absorption of the circularly polarized rf field lead to a violation of Eq. (1)? We write S in the form

$$S_{ab}(\omega) = \delta_{ab} - 2\pi i \langle a|W^\dagger(\omega - H^{\mathrm{eff}})^{-1}W|b\rangle, \qquad (2)$$

originally derived in the context of nuclear physics in Refs. [15,16]. Here $\omega/(2\pi)$ is the frequency of the rf field. The matrix W describes the coupling of the waves in the coaxial cables ($|a\rangle$, $|b\rangle$) with the resonator state(s). Since that coupling is T invariant, W can be chosen real. For a single resonance with resonator state $|1\rangle$, we deal with the two matrix elements $\langle 1|W|a\rangle$ and $\langle 1|W|b\rangle$. For a doublet with the two resonator states $|1\rangle$ and $|2\rangle$, W can be written as

$$W|a\rangle = N_a \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \qquad W|b\rangle = N_b \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}. \qquad (3)$$

The resonator with the ferrite is represented by the effective Hamiltonian H^{eff}. For a doublet of resonances, H^{eff} has dimension two. The frequencies ω_μ with $\mu = 1, 2$ of the resonances appear in the diagonal elements. In addition, H^{eff} contains the matrix (see Eq. (4.2.20b) of Ref. [16])

$$F_{\mu\nu}(\bar\omega) = \sum_{i=a,b,f} \int_{-\infty}^{\infty} d\omega' \frac{W_{\mu i}(\omega')W_{\nu i}^*(\omega')}{\bar\omega^+ - \omega'}, \qquad (4)$$

where $\bar\omega^+ = \bar\omega + i\varepsilon$ is the angular frequency $\bar\omega$ shifted infinitesimally to complex values with $\varepsilon > 0$. In the sum on the right-hand side of Eq. (4), the terms with $i = a, b$ stand for the couplings to the antennas with matrix elements $W_{\mu i}$ as in Eq. (3). As these are real, the integration over $W_{\mu i}(\omega')W_{\nu i}^*(\omega')$ yields for $i = a, b$ a complex symmetric matrix. However, because of TRSB the matrix elements $W_{\mu f}$ that couple the resonator states $|\nu\rangle$, $\nu = 1, 2$, to the ferromagnetic resonance in the ferrite, cannot be chosen real. Hence the integration over $W_{\mu f}(\omega')W_{\nu f}^*(\omega')$ leads to both, a complex symmetric matrix and a complex antisymmetric matrix H^a with

$$H^a = \begin{pmatrix} 0 & H_{12}^a \\ -H_{12}^a & 0 \end{pmatrix}. \qquad (5)$$

The matrix H^a is the hallmark of TRSB in H^{eff}. Since H^a is antisymmetric, TRSB cannot be observed for a single isolated resonance. In the case of a pair of resonances, it is useful to choose the doublet spacing as small as possible to enhance TRSB. We observe that H^a does not depend on the choice of the resonator basis (it is invariant under orthogonal transformations).

As a test of these arguments we first discuss isolated resonances (singlets). It is well known that in circular billiards resonances with vanishing angular momentum are already singlets [2]. However, our billiard provides only two truly isolated singlets. Therefore, we suppressed the existing degeneracies by breaking the rotational symmetry. We have introduced a copper disk (187.5 mm diameter, 5 mm height) into the resonator, see Fig. 1. The resulting "annular billiard" is fully chaotic [17]. In the presence of the ferrite, its spectrum contains several well isolated resonances (widths ≈14 MHz, spacings ≈300 MHz). We have studied eight singlets. For the singlet at frequency $f = 2.84$ GHz, we show in Fig. 2 both S_{ab} and S_{ba}. Within the errors due to noise the complex function S_{ab} agrees with S_{ba}; i.e., reciprocity holds despite the presence of the ferrite. The same statement applies to the other seven singlets. This shows that the coupling to the leads [see Eq. (2)] is indeed T invariant.

We used the circular billiard (see Fig. 1) with its numerous degeneracies to study isolated doublets of resonances. The ferrite destroys the rotational symmetry and, thus, partly lifts the degeneracies. We found four isolated doublets at 2.43 GHz, 2.67 GHz, 2.89 GHz, and 3.2 GHz. The influence of the magnetized ferrite on the resonance shape of the second doublet is illustrated in Fig. 3. In contrast to the case of singlets, the direction of transmission is now important; reciprocity is violated. This was also observed for the first and the third doublet, but not for the fourth. A simulation of the electromagnetic field patterns inside the resonator [18] showed that for one of the two states of the fourth doublet, the magnetic field vanished at the location of the ferrite, so that the ferrite interacted with just one
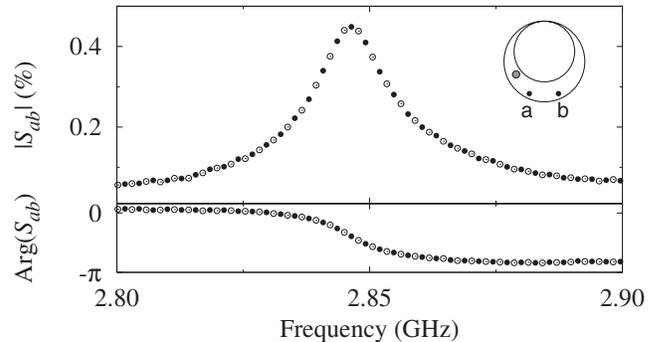

FIG. 2.   A singlet in the annular billiard. For an external field of 119.3 mT we show S_{ab} (open circles) and S_{ba} (solid circles). Both amplitudes and phases coincide and reciprocity holds. The statistical errors of the data are smaller than the symbols.
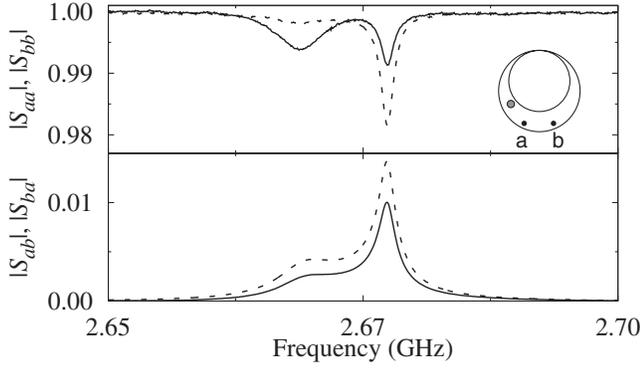
FIG. 3.   The doublet at 2.67 GHz in the circular billiard with an external magnetic field of 36.0 mT. The upper part shows the absolute values of S_{aa} (solid) and S_{bb} (dashed), the lower one those of S_{ab} (solid) and S_{ba} (dashed). The uncertainties of S_{ab} and S_{ba} are about 5×10^{-4}. Reciprocity is clearly violated.

state and $H^a = 0$. Moving the ferrite to a position where it interacted with both states, resulted in a violation of reciprocity. This confirms the importance of coupling a pair of states for observing TRSB.

We measured all four S-matrix elements (S_{aa}, S_{ab}, S_{ba}, S_{bb}) of the system, each at 15 settings of the external magnetic field. From these data, it is possible to determine H^a_{12}. Indeed, Eq. (2) has 12 real parameters: α, β, N_a, N_b and the four complex matrix elements of H^{eff}. We assume that α, β, N_a, N_b, $W_{\mu a}$, $W_{\mu b}$ are independent of the external magnetic field and, within the range of frequencies of the doublet, of ω.

For the second and third doublet, the values deduced for H^a_{12} are shown as dots in Fig. 4. Care was taken to rule out any TRSB effects feigned by the VNA. The data points show a clear resonancelike structure: As a function of the external magnetic field, H^a_{12} goes through a maximum; at the same time the phase of H^a_{12} decreases by about π. We now show that this behavior is intimately connected with the ferromagnetic resonance.

We use Eq. (4) to express H^a_{12} in terms of the matrix elements $W_{\nu f}(\omega')$. As explained above, the ferrite is coupled to only one of the two circular polarizations of the rf magnetic field in the resonator. The unitary matrix

$$U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} \tag{6}$$

induces a transformation from the real resonator basis states $\{|1\rangle, |2\rangle\}$ with matrix elements W to circular basis states with matrix elements $\tilde{W}$ so that $W_{\nu f}(\omega') = \sum_{\xi=1}^2 U^*_{\xi\nu}\tilde{W}_{\xi f}(\omega')$. By definition, one of the $\tilde{W}_{\xi f}$ ($\tilde{W}_{2f}$, say) vanishes so that $W_{1f}(\omega)W^*_{2f}(\omega) = |\tilde{W}_{1f}(\omega)|^2/(2i)$. Moreover, $\tilde{W}_{1f}$ is proportional [19] to the magnetic susceptibility

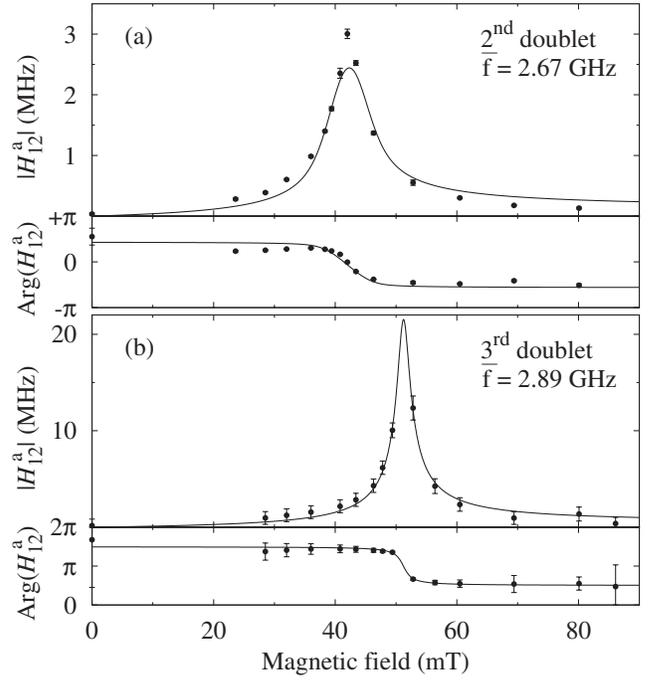$$\tilde{W}_{1f}(\omega') \propto \chi(\omega') = \frac{\omega_M}{\omega_0(B) - \omega' - i/T}, \tag{7}$$

FIG. 4.   The T violating matrix element H^a_{12} for (a) the second doublet at 2.67 GHz (upper panel) and (b) the third doublet at 2.9 GHz (lower panel). The error bars include the systematic, statistical, and numerical errors. The line shape of the ferromagnetic resonance for the second doublet (upper panel) has been convoluted with a Gaussian distribution of magnetization with width 15.5 Oe, see Eq. (10).

where $T(\Delta H) = (\pi\gamma\Delta H)^{-1}$ is the relaxation time, $\omega_M = 2\pi\gamma 4\pi M_S$ with the gyromagnetic ratio $\gamma = 2.8$ MHz/Oe and

$$\omega_0(B) = 2\pi(1.7 \text{ GHz} + 0.0237 \text{ GHz/mT } B). \tag{8}$$

The relation between the strength of the external magnetic field B and the frequency of the ferromagnetic resonance $\omega_0(B)$ has been established by prior measurements with the ferrite in a waveguide.

Collecting Eqs. (4)–(8), we find

$$H^a_{12}(B) = \frac{i\pi}{2}\lambda BT \frac{\omega^2_M}{\omega_0(B) - \bar{\omega} - i/T}. \tag{9}$$

The proportionality stated in Eq. (7) has here been expressed in terms of a linear dependence on B and on a coupling strength parameter λ. Equation (9) depends on two free parameters, $\bar{\omega}$ and λ. The parameter $\bar{\omega}$ averages the TRSB effect of the ferrite over a doublet and is expected to be close to the resonance frequencies of the doublet. Equations (8) and (9) imply that the phase of H^a_{12} decreases with increasing B, independently of the choice of the phase of the matrix element (7). This is in agreement with the data.

In Fig. 4(b), we show for the third doublet the measured values of H^a_{12} and the function (9). The agreement is very

TABLE I. Parameters of Eq. (10) for the first three doublets. Doublet No. 3 is not convoluted; hence, σ and α are not defined in this case.

No.	σ (Oe)	$\bar{\omega}/2\pi$ (GHz)	λ (mT^{-1})	α (deg)
1	42.1 ± 9.3	2.427 ± 0.037	35.7 ± 4.6	-5 ± 8
2	15.5 ± 3.3	2.696 ± 0.011	10.8 ± 1.0	168 ± 7
3	$\cdots$	2.914 ± 0.003	37.3 ± 1.6	$\cdots$

good. We cannot obtain data points at the maximum of the ferromagnetic resonance since there the susceptibility (7) varies appreciably over the frequency range of the doublet. The theory of Eqs. (4)–(9) is not valid in that regime. For the second doublet shown in Fig. 4(a), the resonance is too broad to be explicable by a ferromagnetic resonance width of $\Delta H = 17.5$ Oe. This is because Eqs. (7) and (8) apply to a completely saturated ferrite only. Saturation is achievable only with a strong magnetic field, however. To account for the incomplete saturation we assume that the magnetization has a Gaussian distribution with rms value σ and phase α, $\Psi(\omega) = e^{i\alpha} e^{-(1/2)(\omega/\sigma)^2}/(\sigma\sqrt{2\pi})$. The convolution with H_{12}^a of Eq. (9),

$$\tilde{H}_{12}^a(B) = \int_{-\infty}^{\infty} d\omega' H_{12}^a(\omega')\Psi[\omega_0(B) - \omega'], \qquad (10)$$

yields agreement with the experimental data [see Fig. 4(a)]. The numerical values of the parameters in Eq. (9) are given in Table I. We note that λ depends on the properties of the ferrite as well as on the values of the wave functions of the doublet states at the position of the ferrite.

In summary, we have investigated TRSB in singlets and doublets in ferrite-loaded cavities. To the best of our knowledge this is the first time that the principle of reciprocity has been investigated in microwave resonators with broken time-reversal symmetry. While TRSB cannot be detected at isolated resonances, we have shown that reciprocity is violated for resonance doublets. We extracted the T violating matrix element of the effective Hamiltonian which describes the resonator plus the ferrite. That matrix element can be modeled in terms of a ferromagnetic resonance due to the coupling of the rf magnetic field with the ferrite. We plan to use these insights to study intensity fluctuations at high excitation energies ("Ericson fluctuations" [14]) as well as exceptional points [20,21] in a TRSB regime.

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