

Turnaround in Cyclic Cosmology

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It is speculated how dark energy in a brane world can help reconcile an infinitely cyclic cosmology with the second law of thermodynamics. A cyclic model is described, in which dark energy with $w < -1$ equation of state leads to a turnaround at a time, extremely shortly before the would-be big rip, at which both volume and entropy of our Universe decrease by a gigantic factor, while very many independent similarly small contracting universes are spawned. The entropy of our model decreases almost to zero at turnaround but increases for the remainder of the cycle by a vanishingly small amount during contraction, empty of matter, then by a large factor during inflationary expansion.

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One of the oldest questions in theoretical cosmology is whether an infinitely oscillatory universe which avoids an initial singularity can be consistently constructed. As realized by Friedmann [1] and especially by Tolman [2,3] one principal obstacle is the second law of thermodynamics which dictates that the entropy increases from cycle to cycle. If the cycles thereby become longer, extrapolation into the past will lead back to an initial singularity again, thus removing the motivation to consider an oscillatory universe in the first place. This led to the abandonment of the oscillatory universe by the majority of workers.

Nevertheless, an infinitely oscillatory universe is a very attractive alternative to the big bang. One new ingredient in the cosmic makeup is the dark energy discovered only in 1998, and so it is natural to ask whether this can avoid the difficulties with entropy.

Some work has been started to exploit the dark energy in allowing cyclicity possibly without the need for inflation in [4–8]. Another new ingredient is the use of branes and a fourth spatial dimension as in [9–12], which examined consequences for cosmology. The big rip and replacement of dark energy by modified gravity were explored in [13,14].

If the dark energy has a supernegative equation of state, $\omega_\Lambda = p_\Lambda/\rho_\Lambda < -1$, it leads to a big rip [15] at a finite future time where there exist extraordinary conditions with regard to density and causality as one approaches the rip. In the present Letter, we explore whether these exceptional conditions can assist in providing an infinitely cyclic model.

We consider a model where, as we approach the rip, expansion stops due to a brane contribution just short of the big rip and there is a turnaround at time $t = t_T$ when the scale factor is deflated to a very tiny fraction (f) of itself and only one causal patch is retained, while the other $1/f^3$ patches contract independently to separate universes. Turnaround takes place an extremely short time ($< 10^{-27}$ s) before the big rip would have occurred, at a time when the Universe is fractionated into many independent causal patches [14].

We discuss contraction which occurs with a very much smaller universe than in expansion and with almost vanishing entropy because it is assumed empty of dust, matter, and black holes.

A bounce takes place a short time before a would-be big bang. After the bounce, entropy is injected by inflation [16], where is assumed that an inflation field is excited. Inflation is thus to be a part of the present model which is one distinction from the work of [5–8].

A possible shortcoming of the proposal could have been the persistence of spacetime singularities in cyclic cosmologies [17], but to our understanding, for the model we outline, this problem is avoided, provided that the time average of the Hubble parameter during expansion is equal in magnitude and opposite in sign to its average during contraction.

This model is published because it gives renewed hope for the infinitely oscillatory universe sought in [1–3]. Time will tell whether the present model is consistent, but at present we see no fatal flaw.

Friedmann equation for expansion phase.—Let the period of the Universe be designated by τ and the bounce take place at $t = 0$ and turnaround at $t = t_T$. Thus the expansion phase is for times $0 < t < t_T$ and the contraction phase corresponds to times $t_T < t < \tau$. We employ the following Friedmann equation for the *expansion* period $0 < t < t_T$:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3} \left[\left(\frac{(\rho_\Lambda)_0}{a(t)^{3(\omega_\Lambda+1)}} + \frac{(\rho_m)_0}{a(t)^3} + \frac{(\rho_r)_0}{a(t)^4} \right) - \frac{\rho_{\text{tot}}(t)^2}{\rho_c} \right], \quad (1)$$

where the scale factor is normalized to $a(t_0) = 1$ at the present time $t = t_0 \approx 14$ Gyr. To explain the notation, $(\rho_i)_0$ denotes the value of the density ρ_i at time $t = t_0$. The first two terms are the dark energy and total matter (dark plus luminous) satisfying

$$\Omega_\Lambda = \frac{8\pi G(\rho_\Lambda)_0}{3H_0^2} = 0.72 \quad (2)$$

and $\Omega_m = \frac{8\pi G(\rho_m)_0}{3H_0^2} = 0.28,$

where $H_0 = \dot{a}(t_0)/a(t_0)$. The third term in the Friedmann equation is the radiation density which is now $\Omega_r = 1.3 \times 10^{-4}$. The final term $\sim \rho_{\text{tot}}(t)^2$ is derivable from a brane setup [9,10,12]; we use a negative sign arising from negative brane tension (a negative sign can arise also from a second timelike dimension but that gives difficulties with closed timelike paths); $\rho_{\text{tot}} = \sum_{i=\Lambda,m,r} \rho_i$. As the turnaround is approached, the only significant terms in Eq. (1) are the first (where $\omega_\Lambda < -1$) and the last. As the bounce is approached, the only important terms in Eq. (1) are the third and the last. (We shall later argue that the second term, for matter, is absent during contraction.) In particular, the final term of Eq. (1), $\sim \rho_{\text{tot}}(t)^2$, arising from the brane setup is insignificant for almost the entire cycle but becomes dominant as one approaches $t \rightarrow t_T$ for the turnaround and again for $t \rightarrow \tau$ approaching the bounce.

Turnaround.—Let us assume for algebraic simplicity $\omega_\Lambda = -4/3 = \text{const}$. This value is already almost excluded by WMAP3 [18], but to begin we are aiming only at consistency of infinite cyclicity. More realistic values may be discussed elsewhere. With the value $\omega_\Lambda = -4/3$ we learn from [13] that the time to the big rip is $(t_{\text{rip}} - t_0) = 11 \text{ Gyr}(-\omega_\Lambda - 1)^{-1} = 33 \text{ Gyr}$, which is, as we shall discuss, within 10^{-27} s or less, when turnaround occurs at $t = t_T$. So if we adopt $t_0 = 14 \text{ Gyr}$, then $t_T = t_0 + (t_{\text{rip}} - t_0) \sim (14 + 33) \text{ Gyr} = 47 \text{ Gyr}$. From the analysis in [13–15] the time when a system becomes gravitationally unbound corresponds approximately to the time when the dark energy density matches the mean density of the bound system. For an object like the Earth or a hydrogen atom water density $\rho_{\text{H}_2\text{O}}$ is a practical unit.

With this in mind, for the simple case of $\omega = -4/3$ we see from Eq. (1) that the dark energy density grows proportional to the scale factor $\rho_\Lambda(t) \propto a(t)$, and so given that the dark energy at present is $\rho_\Lambda \sim 10^{-29} \text{ g/cm}^3$, it follows that $\rho_\Lambda(t_{\text{H}_2\text{O}}) = \rho_{\text{H}_2\text{O}}$ when $a(t_{\text{H}_2\text{O}}) \sim 10^{29}$. We can estimate the time $t_{\text{H}_2\text{O}}$ by taking on the right-hand side of the Friedmann equation only dark energy $(\frac{\dot{a}}{a})^2 = H_0^2 \Omega_\Lambda a^{-\beta}$ with $\beta = 3(1 + \omega)$. When we specialize to $\omega = -4/3$, it follows that

$$\frac{a(t_{\text{H}_2\text{O}})}{[a(t_0) = 1]} = \left(\frac{(t_{\text{rip}} - t_0)}{(t_{\text{rip}} - t_{\text{H}_2\text{O}})} \right)^2 \quad (3)$$

so that $(t_{\text{rip}} - t_{\text{H}_2\text{O}}) = 33 \text{ Gyr} \times 10^{-14.5} \simeq 10^{3.5} \text{ s} \sim 1 \text{ h}$. (The value is sensitive to ω .) It is instructive to consider approach to the rip for a more general critical density $\rho_c = \eta \rho_{\text{H}_2\text{O}}$ and to compute the time $(t_{\text{rip}} - t_\eta)$ such that $\rho_\Lambda(t_\eta) = \rho_c = \eta \rho_{\text{H}_2\text{O}}$. We then find, using $a(t_\eta) =$

$10^{29} \eta$, that

$$(t_{\text{rip}} - t_\eta) = (t_{\text{rip}} - t_0) 10^{-14.5} \eta^{-1} \simeq \eta^{-1} \text{ h}, \quad (4)$$

which is the required result. We shall see $\eta > 10^{31}$, so the time in (4) is $< 10^{-27}$ s.

To discuss the turnaround analytically we keep only the first and last terms, the only significant ones, on the right-hand side of Eq. (1), which becomes for the special case $\omega = 4/3$:

$$\left(\frac{\dot{a}}{a} \right)^2 = \alpha_1 a - \alpha_2 a^2, \quad (5)$$

in which

$$\alpha_1 = \frac{8\pi G}{3} (\rho_\Lambda)_0 \alpha_2 = \frac{8\pi G}{3} \frac{(\rho_\Lambda)_0^2}{\rho_c}. \quad (6)$$

Writing $a = z^2$ and $z = (\alpha_1/\alpha_2)^{1/2} \sin\theta$ gives

$$dt = \frac{2\sqrt{\alpha_2}}{\alpha_1} \frac{d\theta}{\sin^2\theta} = \frac{2\sqrt{\alpha_2}}{\alpha_1} d(-\cot\theta). \quad (7)$$

Integration then gives for the scale factor

$$a(t) = \left(\frac{\alpha_1}{\alpha_2} \right) \sin^2\theta = \frac{\rho_c}{(\rho_\Lambda)_0} \left[\frac{1}{1 + \left(\frac{t - t_T}{C} \right)^2} \right], \quad (8)$$

where $C = -(3/2\pi G \rho_c)^{1/2}$. At turnaround $t = t_T$, $a(t_T) = [\rho_c/(\rho_\Lambda)_0] = [a(t)]_{\text{max}}$. At the present time $t = t_0$, $a(t_0) = 1$ and $\sin^2\theta_0 = [(\rho_\Lambda)_0/\rho_c] \ll 1$, increasing during subsequent expansion to $\theta_T = \pi/4$.

A key ingredient in our model is that at turnaround $t = t_T$ our universe deflates dramatically with effective scale factor $a(t_T)$ shrinking before contraction to $\hat{a}(t_T) = f a(t_T)$ where $f < 10^{-28}$. This jettisoning of almost all, a fraction $(1 - f)^3$, of the accumulated entropy is permitted by the exceptional causal structure of the Universe. We shall see later that the parameter η at turnaround lies in the range $\eta = 10^{31}$ to $\eta = 10^{87}$ which implies a dark energy density at turnaround (Planckian density of $\rho_\Lambda \sim 10^{104} \rho_{\text{H}_2\text{O}}$ can be avoided) such that, according to the big rip analysis of [13,14], all known, and yet unknown smaller, bound systems have become unbound and the constituents causally disconnected. Recall that the density of a hydrogen atom is approximately $\rho_{\text{H}_2\text{O}}$ and we are reaching a dark energy density of from 31 to 87 orders of magnitude higher.

According to these estimates, at $t = t_T$ the universe has already fragmented into an astronomical number $(1/f^3)$ of causal patches, each of which independently contracts as a separate universe leading to an infinite multiverse. The entropy at $t = t_T$ is thus divided between these new contracting universes and our universe retains only a fraction f^3 . Since our model universe has cycled an infinite number of times, the number of parallel universes is infinite.

Deflation.—A central assumption in our cyclic model is that almost all of the entropy is jettisoned at turnaround by the retention of one causal patch. We cannot justify this

step rigorously but hope to convince the reader by the following physical argument. Let us take a bounce temperature $T_B = 10^p$ GeV with $p > 3$. This gives (see below) $\eta = 10^{(19+4p)}$ and hence, from Eq. (4), $(t_{\text{rip}} - t_T) \sim 10^{-(19+4p)}$ h. The dark energy density $\rho_c \sim 10^{(19+4p)}$ g/cm³ at turnaround implies the prior disintegration of all bound systems with mean density $\rho < \rho_c$, which for $p > 3$ includes atoms, nuclei, nucleons (10^{15} g/cm³), and even smaller bound systems, if any. As shown in [13,15], at a similar time, actually somewhat but not too much later, these constituents become causally disconnected. For such a density, a generic causal patch contains no quarks or leptons, only dark energy together with a small number of highly-infrared photons. Black holes are also absent, having been torn apart by the approach to the would-be big rip, e.g., [12]. The entropy of such a patch is essentially zero, by which we mean $S = O(10^1)$ compared to the earlier $S > 10^{88}$. This dramatic decrease in entropy is called deflation for obvious reasons. During contraction, as we shall describe, the entropy remains constant at essentially zero because dark energy has zero entropy and radiation contracts adiabatically. Although these heuristic arguments about deflation seem clear, a more rigorous justification would be desirable.

Friedmann equation for contraction phase.—The contraction phase for our universe occurs for the period $t_T < t < \tau$. The scale factor for the contraction phase will be denoted by $\hat{a}(t)$, while we use always the same linear time t subject to the periodicity $t + \tau \equiv t$. At the turnaround we retain a fraction f^3 of the entropy with $\hat{a}(t_T) = fa(t_T)$ and for the contraction phase the Friedmann equation is

$$\left(\frac{\dot{\hat{a}}(t)}{\hat{a}(t)}\right)^2 = \frac{8\pi G}{3} \left[\left(\frac{\hat{\rho}_\Lambda}{\hat{a}(t)^{3(\omega_\Lambda+1)}}\right) + \left(\frac{\hat{\rho}_r}{\hat{a}(t)^4}\right) - \frac{\hat{\rho}_{\text{tot}}(t)^2}{\hat{\rho}_c} \right], \quad (9)$$

where we have defined

$$\hat{\rho}_i(t) = \frac{(\rho_i)_0 f^{3(\omega_i+1)}}{\hat{a}(t)^{3(\omega_i+1)}} = \frac{(\hat{\rho}_i)_0}{\hat{a}(t)^{3(\omega_i+1)}}, \quad (10)$$

but in contrast to Eq. (1) we have set $\hat{\rho}_m = 0$ because our hypothesis is that the causal patch retained in the model contains only dark energy and radiation but no matter including no black holes. This is necessary because during a contracting phase dust or matter would clump even more readily than during expansion, and inevitably interfere with cyclicity. Perhaps more importantly, the presence of dust or matter would require that our universe go in reverse through several phase transitions (recombination, QCD, and electroweak to name a few), which would violate the second law of thermodynamics. We thus require that our Universe comes back empty. Any tiny entropy associated with radiation is constant during adiabatic contraction.

The contraction of our universe will proceed from one of the $1/f^3$ causal patches following Eq. (9) until the radiation balances the brane tension at the bounce.

Bounce.—At the bounce, the contraction scale is given, using $\rho_c = \eta\rho_{\text{H}_2\text{O}}$, from Eq. (1) as

$$a(\tau)^4 = \left(\frac{(\rho_r)_0}{\eta\rho_{\text{H}_2\text{O}}}\right). \quad (11)$$

Now the model's bounce at $t = \tau$ must be before the electroweak transition at $t_{\text{EW}} = 10^{-10}$ s when $a(t_{\text{EW}}) = 10^{-15}$ and after the Planck scale when $a(t_{\text{Planck}}) = 10^{-32}$ in order to accommodate the well established weak transition and to avoid uncertainties associated with quantum gravity. With this in mind, here are three illustrative values [(A), (B), and (C)] for the bounce temperature T_B : (A) At a grand unified theory scale $T_B = 10^{17}$ GeV, $a(t_B) = 10^{-30}$. (B) At an intermediate scale $T_B = 10^{10}$ GeV, $a(t_B) = 10^{-23}$. (C) At a weak scale $T_B = 10^3$ GeV, $a(t_B) = 10^{-16}$. From Eqs. (4) and (11) for these three cases one finds (A) $\eta = 10^{87}$ and $(t_{\text{rip}} - t_T) = 10^{-87}$ h; (B) $\eta = 10^{59}$ and $(t_{\text{rip}} - t_T) = 10^{-59}$ h; (C) $\eta = 10^{31}$ and $(t_{\text{rip}} - t_T) = 10^{-31}$ h.

Immediately after the bounce, we assume that an inflation field is excited and there is conventional inflation with enhancement $E = a(\tau + \delta)/\hat{a}(\tau)$. Successful inflation requires $E > 10^{28}$. Consistency therefore requires that $f < E^{-1}$ to allow for the entropy accrued during expansion after inflation. The fraction of entropy jettisoned from our universe at deflation is thus extremely close to 1, being less than 1 and more than $(1 - 10^{-28})^3$.

Entropy.—Consider first the present epoch $t = t_0$. The contributions of the radiation to the entropy density s follows the relation

$$s = \frac{2\pi^2}{45} g_* T^3. \quad (12)$$

Photons contribute $g_* = 2$. The present cosmic microwave background temperature is $T = 2.73$ K $\equiv 0.235$ meV ~ 1.191 (mm)⁻¹. Substitution in Eq. (12) gives a present radiation entropy density $s_\gamma(t_0) = 1.48$ (mm)⁻³. Using a volume estimate $V = (4\pi/3)R^3$ with $R = 10$ Gly $\approx 10^{29}$ mm gives a total radiation entropy $S_\gamma \sim 6.3 \times 10^{87}$. Including neutrinos increase g_* in Eq. (12) from $g_* = 2$ to $g_* = 3.36 = 2 + 6 \times (7/8) \times (4/11)^{4/3}$. This increases $S_\gamma = 6.3 \times 10^{87}$ to $S_{\gamma+\nu} \sim \times 10^{88}$.

This total entropy is interpretable as $\exp(10^{88})$ degrees of freedom, or in information theory [19] to a number I of qubits where $2^I = e^S$ so that $I = S/(\ln 2 = 0.693) \sim 10^{88}$. This is well below the holographic bound which is dictated by the area in terms of Planck units 10^{-64} mm² which gives $S_{\text{holog}}(t_0) = 4\pi(10^{29} \text{ mm})^2/(10^{-32} \text{ mm})^2 \sim 10^{123}$ about 10^{35} times bigger. In [19] it is suggested that at least some of this difference may come from supermassive black holes. The entropy contribution from the baryons is smaller than S_γ by some 10 orders of magnitude, so like that of the dark matter, it is negligible.

What is the entropy of the dark energy? If it is perfectly homogeneous and noninteracting, it has zero temperature and entropy. Finally, the fourth term in Eq. (1) corresponding to the brane term is negligible, as we have already estimated. The conclusion is that at present $S_{\text{tot}}(t_0) \sim 10^{88}$.

Now consider the entropy approaching turnaround at $t = t_T$. We have estimated that $a(t_T) = 10^{29} \eta$ and that representative values for $\eta = \rho_c / \rho_{\text{H}_2\text{O}}$ are 10^{31} , 10^{59} , and 10^{87} . The temperature T_γ of the radiation scales as $T_\gamma \propto a(t)^{-1}$ so using the entropy density of Eq. (12) a comoving 3-volume $\propto a(t)^3$ will contain the same total radiation entropy $S_\gamma(t_T) = S_\gamma(t_0)$ as at present; this is simply the usual adiabatic expansion. The expansion from $t = 0$ to t_T is not purely adiabatic because irreversible processes take place. The first is inflation, which increases entropy by $>10^{84}$. There are phase transitions such as the electroweak transition at $t_{\text{EW}} \sim 100$ ps, the QCD phase transition at $t_{\text{QCD}} \sim 100 \mu\text{s}$, and recombination at $t_{\text{rec}} \sim 10^{13}$ s. Further irreversible processes occur during stellar evolution. Although the expansion of the radiation, the dominant contributor to the entropy, is adiabatic, the entropy of matter increases in accord with the second law of thermodynamics. In our model, the entropy of the matter increases between $t = 0$ and $t_T \sim 47$ Gyr. Setting the entropy of the dark energy to zero and the radiation as adiabatic, the matter part represented by ρ_m will cause the entropy to rise from $S(t = 0)$ to $S(t_T) = S(t = 0) + \Delta S$, where ΔS causes the contradiction plaguing previous oscillatory model universes [1–3].

Our main point is that in order for entropy to be cyclic, the entropy which was enhanced by a huge factor $E^3 > 10^{84}$ at inflation must be reduced dramatically at some point during the cycle so that $S(t) = S(t + \tau)$ becomes possible. Since it increases during expansion and contraction, the only logical possibility is the decrease at turnaround as accomplished by our causal patch idea. The second law of thermodynamics continues to obtain for other causal patches, each with practically vanishing entropy at turnaround, but these are permanently removed from our universe contracting instead into separate universes.

For contraction $t_T < t < \tau$, we are assuming that the Universe during contraction is empty of matter until the bounce, so its entropy is vanishingly small. Immediately after the bounce inflation arises from an inflation field, assumed to be excited. We find the counterpoise of inflation at the bounce and deflation at turnaround an appealing aspect of the present model.

Conclusion.—The standard cosmology based on a big bang augmented by an inflationary era is impressively consistent with the detailed data from WMAP3 [18] when dark energy, most conservatively a cosmological constant, is included. Our objections to this standard model are more aesthetic than motivated directly by observations. The first objection is the nature of the initial singularity and

the initial conditions. A second objection, not of concern to all colleagues, is that the predicted fate of the Universe is an infinitely long expansion. We have outlined here a cyclic cosmology resting on phantom dark energy where these objections are ameliorated: the classical density and temperature never become infinite and future expansion is truncated. Also, our proposal of deflation naturally leads to a multiverse picture, somewhat reminiscent of that predicted in eternal inflation, though here the new proliferation of universes must be infinite and originates at the opposite end of a cyclic cosmology, at its maximum rather than at its minimum size.

We publish our infinitely cyclic model mainly in the hope that it will stimulate a more detailed and compelling formulation.

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