Stochastic Background of Gravitational Waves from Hybrid Preheating

Juan García-Bellido and Daniel G. Figueroa

Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

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The process of reheating the Universe after hybrid inflation is extremely violent. It proceeds through the nucleation and subsequent collision of large concentrations of energy density in bubblelike structures, which generate a significant fraction of energy in the form of gravitational waves. We study the power spectrum of the stochastic background of gravitational waves produced at reheating after hybrid inflation. We find that the amplitude could be significant for high-scale models, although the typical frequencies are well beyond what could be reached by planned gravitational wave observatories. On the other hand, low-scale models could still produce a detectable stochastic background at frequencies accessible to those detectors. The discovery of such a background would open a new window into the very early Universe.

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According to general relativity, the present Universe should be permeated by a diffuse gravitational wave background (GWB) with a variety of origins, from unresolved point sources (gravitational collapse of supernovae, neutron star, and black hole coalescence, etc.) to relic stochastic backgrounds from early Universe phase transitions, inflation, turbulent plasmas, cosmic strings, etc. [1]. These backgrounds have very different spectral shapes and amplitudes that may, in the future, allow gravitational wave observatories like the Laser Interferometer Gravitational Wave Observatory (LIGO), Laser Interferometer Space Antenna (LISA), Big Bang Observer (BBO), or Decihertz Interferometer Gravitational Wave Observatory (DECIGO) [1] to disentangle their origin. There are already a series of constraints on some of these backgrounds, the most stringent one coming from the large-scale polarization anisotropies in the cosmic microwave background (CMB), which may soon be measured by Planck, if the scale of inflation is sufficiently high [2]. There are also constraints coming from big bang nucleosynthesis [3] and from millisecond pulsar timing [4], while it has recently been proposed a new constraint on primordial GWB's coming from CMB anisotropies [5]. However, most of these constraints come at very low frequencies (typically from 10^{-18} to 10^{-8} Hz), while present and planned observatories range from 10^{-3} Hz of LISA to 10^3 Hz of Advanced-LIGO [1], which could detect GW associated with early Universe phenomena like first-order phase transitions [6,7], or cosmic turbulence [8], if these occur around the electroweak scale.

In this Letter we want to describe a stochastic GWB that may open a new window into the early Universe. Recent observations of the CMB anisotropies seem to suggest that something like inflation must have occurred very early in the evolution of the Universe. The process by which the energy density driving inflation was converted into all the radiation and matter we observe today is called reheating. The first stage of conversion, preheating [9], is known to be explosive, and generates in less than a Hubble time the huge entropy measured today. In chaotic inflation, the coherent oscillations of the inflaton during preheating generates, via parametric resonance, a population of highly occupied modes that behave like waves of matter, which collide among themselves and whose scattering leads to homogenization and local thermal equilibrium. These collisions occur in a highly relativistic and very asymmetric way, being responsible for the generation of a stochastic background of gravitational waves [10,11] with a typical frequency today of the order of 10^7–10^9 Hz, corresponding to the present size of the causal horizon at the end of high-scale inflation. There is at present no chance to detect such a background.

However, in hybrid inflation models the end of inflation is sudden [12], and the conversion into radiation occurs almost instantaneously. Indeed, we know that hybrid models preheat in an even more violent way than chaotic models, thanks to the spinodal instability of the symmetry breaking field that triggers the end of inflation, irrespective of the couplings that this field may have to the rest of matter. Such a process is known as tachyonic preheating [13,14] and could be responsible for copious production of dark matter particles [15], lepto and baryogenesis [16], topological defects [13], primordial magnetic fields [17], etc. Moreover, it was speculated in Ref. [18] that in (low-scale) models of hybrid inflation it might be possible to generate a stochastic GWB in the LIGO frequency range, if the scale of inflation is as low as $H_{\rm inf} \sim 1$ TeV. However, the amplitude was estimated using the parametric resonance formalism of chaotic preheating, which may not be applicable in this case. In Ref. [14] it was shown that the process of symmetry breaking proceeds via the nucleation of dense bubblelike structures moving at the speed of light, which collide and break up into smaller structures (see Figs. 7 and 8 of Ref. [14]). We conjectured at that time that such collisions would be a very strong source of gravitational waves, analogous to the gravity wave production associated with strongly first-order phase transitions [6].

Hybrid inflation models [12] arise in theories with symmetry breaking fields ("Higgs fields") coupled to flat directions, and are present in many extensions of the standard model, both in string theory and in supersymmetric theories [19]. These models do not require small couplings in order to generate the observed CMB anisotropies; e.g., a working model with grand unified theory (GUT) scale symmetry breaking, $v = 10^{-3} M_P$, with a Higgs self-coupling λ and a Higgs-inflaton coupling g given by $g = \sqrt{2\lambda} = 0.05$, satisfies all CMB constraints [20], and predicts a tiny tensor contribution to the CMB polarization. The main advantage of hybrid models is that, while most chaotic inflation models can only occur at high scales, with Planck scale values for the inflaton, and $V_{\rm inf}^{1/4} \sim 10^{16}$ GeV, one can choose the scale of inflation in hybrid models to range from GUT scales all the way down to TeV scales, while the Higgs vacuum expectation value (VEV) can range from the Planck scale to the Electroweak scale, see Refs. [12,16].

Reheating in hybrid inflation goes through four well-defined regimes: first, the exponential growth of long wave modes of the Higgs field via spinodal instability, which drives the explosive growth of all particles coupled to it, from scalars [13] to gauge fields [16] and fermions [15]; second, the nucleation and collision of high density contrast and highly relativistic bubblelike structures associated with the peaks of a Gaussian random field like the Higgs field [14]; third, the turbulent regime that ensues after all these "bubbles" have collided and the energy density in all fields cascades towards high momentum modes; finally, thermalization of all modes when local thermal and chemical equilibrium induces equipartition. The first three stages can be studied in detailed lattice simulations [14,16] thanks to the semiclassical character of the process of preheating [21], while the last stage is intrinsically quantum and has never been studied in the lattice.

In this Letter we use lattice simulations to study the generation of a GWB during preheating in hybrid inflation and analyze the dependence of the shape and amplitude of the GWB spectrum on the scale of hybrid inflation, and more specifically on the VEV of the Higgs field. GW's are represented by a tensor metric perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ whose equation of motion in the (traceless) radiation gauge is $\Box h_{\mu\nu} = 16\pi G T_{\mu\nu}$, with the harmonic gauge condition $\partial^\mu h_{\mu\nu} = 0$ ensured by conservation of the energy-momentum tensor. Moreover, we can fix $h_{00} = 0$, and the resulting field is the usual tensor gauge-invariant metric perturbation h_{ij}, which satisfies the evolution equation $\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G \Pi_{ij}$, with Π_{ij} the anisotropic (traceless) stress tensor, sourced by both the inflaton and Higgs fields, $\Pi_{ij} = \nabla_i \phi^a \nabla_j \phi^a + \nabla_i \chi \nabla_j \chi - 1/3 \delta_{ij} [(\nabla \phi^a)^2 + (\nabla \chi)^2]$. We solve the evolution equations of the gravity waves h_{ij} together with those of the other coupled scalar fields in a discretized lattice, assuming initial quantum fluctuations for all fields and only a zero mode for the inflaton, following the prescription adopted in Ref. [14]. We also included the GW backreaction on the scalar fields' evolution via the gradient terms $h^{ij} \nabla_i \phi \nabla_j \phi$, although for all practical purposes these are negligible throughout GW production. We then evaluate the mean field values, as well as the different energy components; see Fig. 1. For the energy in gravitational waves we use the expression $(32\pi G) t_{\mu\nu} = \langle \partial_\mu h_{ij}^{\rm TT} \partial_\nu h_{\rm TT}^{ij} \rangle = \frac{2}{5} \langle \partial_\mu h_{ij} \partial_\nu h^{ij} \rangle$, where the expectation value is over a region sufficiently large to encompass enough physical curvature to have a gauge-invariant measure of the GW energy [22], and we have expressed the average over the transverse traceless tensor $h_{ij}^{\rm TT}$ in terms of the average over h_{ij}, the solution of the (traceless) tensor evolution equation. The fractional energy density in gravitational waves is then $\rho_{\rm gw}/\rho_0 = 4 t_{00}/v^2 m^2$, which can be used to compute the corresponding density parameter today (with $\Omega_{\rm rad} h^2 \simeq 3.5 \times 10^{-5}$),

$$\Omega_{\rm gw} h^2 = \Omega_{\rm rad} h^2 \frac{1}{8\pi G v^2 m^2} \langle \partial_0 h_{ij}^{\rm TT} \partial_0 h_{\rm TT}^{ij} \rangle,$$

where we have assumed that all the vacuum energy ρ_0 gets converted into radiation, an approximation which is always valid in generic hybrid inflation models with $v \ll M_P$, and thus $H \ll m = \sqrt{\lambda} v$. We have shown in Fig. 1 the evolution in time of the fraction of energy density in GW. The first (tachyonic) stage is clearly visible, with a slope twice that of the anisotropic tensor Π_{ij}. Then there is a small plateau corresponding to the production of GW from bubble collisions, and finally there is the linear growth due to turbulence. Note that in the case that $H \ll m$, the maximal production of GW occurs in less than a Hubble time, soon after symmetry breaking, while turbulence lasts several decades in time units of m^{-1}. Therefore, we can safely ignore the dilution due to the Hubble expansion, until the Universe finally reheats and the energy in gravitational waves redshifts like radiation thereafter.

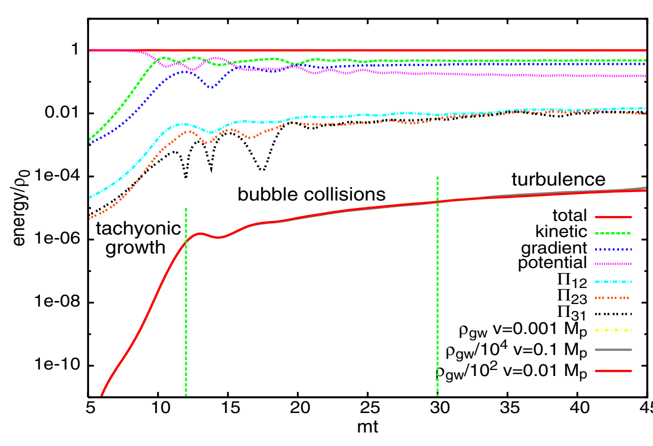

FIG. 1 (color online). The time evolution of the different types of energy, normalized to the initial vacuum energy, for a model with $v = 10^{-3} M_P$. There are three well characterized stages: tachyonic growth, bubble collisions, and turbulence.

We then compute the power spectrum per logarithmic interval in GW by performing a Fourier transform of the energy density $\Omega_{\rm gw} = \int df/f \Omega_{\rm gw}(f)$ as a function of the frequency f, where $\Omega_{\rm gw}(k) = k^3 \rho_{\rm gw}(k)/2\pi^2 \rho_c$, with ρ_c the critical density today. Since gravitational waves below Planck scale remain decoupled from the plasma immediately after production, we can evaluate the power spectrum today from that obtained at preheating by simply converting the wave number k into frequency [10],

$$f = 6 \times 10^{10}\ {\rm Hz} \frac{k}{\sqrt{HM_P}} = 5 \times 10^{10}\ {\rm Hz} \frac{k}{m} \lambda^{1/4}.$$

We have shown in Fig. 2 the power spectrum of gravitational waves as a function of wave number k/m. We have used different lattices in order to have lattice artifacts under control, especially at late times and high wave numbers. We have checked that the power spectrum of GW follows (turbulent) scaling after $mt \sim 40$, and we can thus estimate the subsequent growth in energy density beyond our simulations [23].

We will now compare our numerical results with analytical estimates. The tachyonic growth is dominated by the faster than exponential growth of the Higgs modes towards the true vacuum [14]. The (traceless) anisotropic stress tensor Π_{ij} grows rapidly to a value of order $k^2|\phi|^2 \sim 10^{-3} m^2 v^2$, which gives a tensor perturbation $|h_{ij}^{\rm TT} h_{\rm TT}^{ij}|^{1/2} \sim 16\pi G v^2 (m\Delta t)^2 10^{-3}$ and an energy density in GW, $\rho_{\rm gw}/\rho_0 \sim 64\pi G v^2 (m\Delta t)^2 10^{-6} \sim G v^2$, for $m\Delta t \sim 16$. In the case at hand, with $v = 10^{-3} M_P$, we find $\rho_{\rm gw}/\rho_0 \sim 10^{-6}$ at symmetry breaking, which coincides with the numerical simulations at that time; see Fig. 1. The production of gravitational waves in the next stage proceeds through bubble collisions. Assuming the bubble walls contain most of the energy density, and since they travel close to the speed of light [14], it is expected that the asymmetric collisions will copiously produce GW, like those of a strongly first-order phase transition. In that case, a quick estimate suggests that the fraction of energy density is given by [6,24] $\rho_{\rm gw}/\rho_0 \sim 1/20(RH)^2 \sim 8\pi/60(Rm)^2 G v^2 \sim 2 G v^2$, of the same order or slightly larger than the previous stage, for the typical size of bubbles, $R \sim 3m^{-1}$, upon collision [14], which again corresponds to what is observed in the numerical simulations, see Fig. 1. The subsequent turbulent stage [17,25] is expected to further produce GW with a spectrum that scales with time in a well-defined manner; see also [23] for a detailed analysis,

$$\frac{k^3}{2\pi^2} \frac{\rho_{gw}(k)}{\rho_0} = 0.2 G v^2 \tau^{1.0} k^2 \exp(-0.25 k^2 \tau^{-2p}),$$

where $\tau = mt$ and $p = \frac{1}{7}$ is the corresponding turbulent exponent [17,25]. This spectrum has a maximum at $k/m \sim 1$, and falls as k^2 for small k until it reaches the maximum wavelength $k \sim H$, corresponding to the minimum frequency today, $f_{\rm min} \sim 5 \times 10^{10}\ {\rm Hz} \lambda^{1/4} v/M_P$. For the case we were considering in our numerical simulations, with $v = 10^{-3} M_P$ and $\lambda \sim g^2 \sim 0.1$, we find the power spectrum of Fig. 2.

We have plotted in Fig. 3 the sensitivity of planned GW interferometers like LIGO, LISA, and BBO, together with the present bounds from CMB anisotropies (GUT inflation), from big bang nucleosynthesis (BBN) and from millisecond pulsars (ms pulsar). Also shown are the expected stochastic backgrounds of chaotic inflation models such as $\lambda\phi^4$ [10,11], as well as the predicted background from two different hybrid inflation models, a high-scale model, with $v = 10^{-2} M_P$ and $\lambda \sim g^2 \sim 0.05$, and a low-scale model, with $v = 10^{-5} M_P$ and $\lambda \sim g^2 \sim 10^{-14}$, corresponding to a rate of expansion $H \sim 100$ GeV. The high-scale hybrid model produces typically as much gravita-

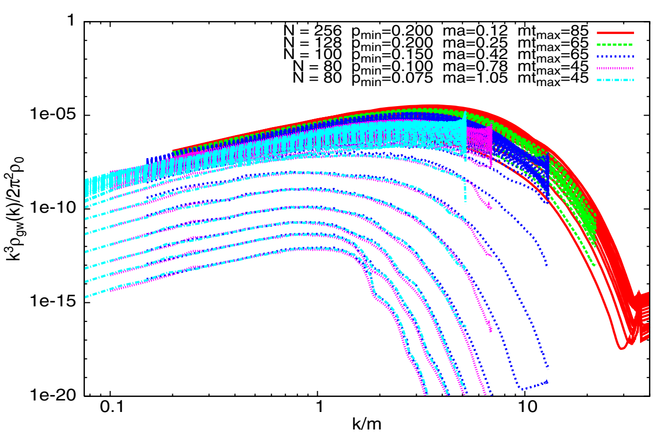

FIG. 2 (color online). The comparison between the GW power spectrum obtained with increasing lattice resolution, to prove the robustness of our method. The different realizations are characterized by the number of lattice points (N), the minimum lattice momentum ($p_{\rm min}$), and the lattice spacing (ma). The growth is shown in steps of $m\Delta t = 1$ for the lower spectra and $m\Delta t = 5$ for the rest.

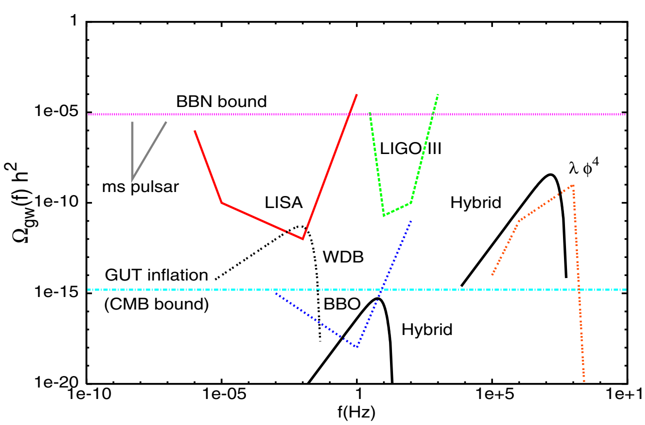

FIG. 3 (color online). The sensitivity of the different gravitational wave experiments, present and future, compared with the possible stochastic backgrounds; we include the white dwarf binaries (WDB) [26] and chaotic preheating ($\lambda\phi^4$) [10] for comparison.

tional waves from preheating as the chaotic inflation models. The advantage of low-scale hybrid models of inflation is that the background produced is within reach of future GW detectors such as BBO [27].

To summarize, we have shown that hybrid models are very efficient generators of gravity waves at preheating, in three well-defined stages, first via the tachyonic growth of Higgs modes, which act as sources of gravity waves, then via the collisions of highly relativistic bubblelike structures with large energy density contrast, and finally via the turbulent regime that drives the system towards thermalization. These waves remain decoupled since the moment of their production, and thus the predicted amplitude and shape of the gravitational wave spectrum today can be used as a probe of the widely unknown reheating period just after inflation. The characteristic spectrum can be used to distinguish between this stochastic background and others, like those arising from neutron star and black hole pairs' coalescence, which are decreasing with frequency, or those arising from inflation, that are flat [28].

For a high-scale model of inflation, we may never see the predicted GW background coming from preheating, in spite of its large amplitude, because it appears at very high frequencies, much beyond present experiments' sensitivities, where no detector has yet shown to be sensitive. On the other hand, if inflation occurred at low scales, even though we will never have a chance to detect the GW produced during inflation in the polarization anisotropies of the CMB, we do expect gravitational waves from preheating to contribute with an important background in sensitive detectors like BBO. The detection and characterization of such a GW background, coming from the complicated and mostly unknown epoch of reheating of the Universe, may open a new window into the very early Universe, while providing a new test on inflation.

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