N-to-Δ Axial Transition Form Factors from Lattice QCD

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We evaluate the N to Δ axial transition form factors in lattice QCD with no dynamical sea quarks, with two degenerate flavors of dynamical Wilson quarks, and using domain wall valence fermions with three flavors of staggered sea quarks. We predict the ratio $C_5^A(q^2)/C_3^V(q^2)$ relevant for parity violating asymmetry experiments and verify the off-diagonal Goldberger-Treiman relation.

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The electromagnetic structure of the nucleon including the transition form factors for electroproduction of the Δ has been the subject of recent experimental [1] and theoretical studies [2,3]. The N to Δ transition has the advantages that the $\Delta(1232)$ is the dominant, clearly accessible nucleon resonance, and the isovector spin-flip transition provides selective information on hadron structure. The weak structure functions provide valuable input often complimentary to that obtained from electromagnetic probes. In the nucleon, for example, measurements of the elastic parity violating asymmetry yield information on strange quark contributions. The N to Δ transition filters out the isoscalar $s\bar{s}$ contributions, so the parity violating asymmetry in weak neutral and charge changing N to Δ transitions probes isovector structure [4,5] not accessible in the study of strange isoscalar quark currents. Furthermore its isovector nature could be used to detect physics beyond the standard model since it is sensitive to additional heavy particles not appearing in the standard model. However, in order to compete with other low-energy semileptonic measurements, the N to Δ transition requires a determination of the parity violating asymmetry to significantly better than 1% precision [4]. Hence, a lattice prediction for the axial form factors provides valuable input for ongoing experiments [5]. In this Letter, we evaluate the dominant contribution to the parity violating asymmetry, determined by the ratio C_5^A/C_3^V. This is the off-diagonal analogue of the g_A/g_V ratio extracted from neutron β-decay and therefore tests low-energy consequences of chiral symmetry, such as the off-diagonal Goldberger-Treiman relation. In addition, the ratio of axial form factors C_6^A/C_5^A provides a measure of axial current conservation.

Since this is the first lattice computation of the N to Δ axial transition form factors, the starting point is an evaluation in the quenched theory using the standard Wilson action. A quenched calculation allows us to use a large lattice in order to minimize finite volume effects and obtain accurate results at small momentum transfers reaching pion mass, m_π, down to about 410 MeV. To study the role of the pion cloud, which is expected to provide an important ingredient in the description of the properties of the nucleon system, one requires dynamical configurations with light quarks on large volumes. In this Letter, the light-quark regime is studied in two ways. First, we use configurations with the lightest available dynamical two flavor Wilson fermions spanning approximately the same pion mass range as in the quenched calculation [6,7]. Second, we use a hybrid combination of domain wall valence quarks, which have chiral symmetry on the lattice, and configurations generated with three flavors of staggered sea quarks using the Asqtad improved action (MILC configurations) [8]. The effectiveness of this hybrid combination has recently been demonstrated in the successful precision calculation of the axial charge, g_A [9]. Since Wilson fermions have discretization errors in the lattice spacing, a, of $O(a)$ and break chiral symmetry whereas the hybrid action has discretization errors of $O(a^2)$ and chirally symmetric valence fermions, agreement between calculations using these two lattice actions provides a nontrivial check of consistency of the lattice results. The hybrid calculation is the most computationally demanding, since it requires propagators on a five-dimensional lattice. The bare quark mass for the domain wall fermions, the size of the fifth dimension and the renormalization factors, Z_A, for the four-dimensional axial current are taken from Ref. [9]. As in the case of Wilson fermions, we consider three values of light-quark mass with the strange sea quark mass fixed to approximately its physical value [8]. In all cases we use Wuppertal smeared [10] interpolating fields at the source and sink. In the unquenched Wilson case, to minimize fluctuations [11] we use hypercubic (HYP) smearing [12] on the spatial links entering in the Wuppertal smearing function at the source and sink whereas for the hybrid case all gauge links in the fermion action are HYP smeared. We list the parameters used in our computations in Table I. The value of the lattice spacing is determined from the nucleon mass at the physical limit for the case of Wilson fermions and for the staggered sea quark configurations, we take the value determined from heavy quark spectroscopy [13].

The invariant N to Δ weak matrix element, expressed in terms of four transition form factors [14,15] can be written as [5]

$$\langle\Delta(p',s')|A^3_\mu|N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{M_\Delta M_N}{E_\Delta(\mathbf{p}')E_N(\mathbf{p})}\right)^{1/2}\bar{u}^\lambda(p',s')\left[\left(\frac{C_3^A(q^2)}{M_N}\gamma^\nu + \frac{C_4^A(q^2)}{M_N^2}p'^\nu\right)(g_{\lambda\mu}g_{\rho\nu} - g_{\lambda\rho}g_{\mu\nu})q^\rho + C_5^A(q^2)g_{\lambda\mu} + \frac{C_6^A(q^2)}{M_N^2}q_\lambda q_\mu\right]u(p,s) \tag{1}$$

where $q_\mu = p'_\mu - p_\mu$ is the momentum transfer, $A^3_\mu(x) = \bar{\psi}(x)\gamma_\mu\gamma_5\frac{\tau^3}{2}\psi(x)$ is the isovector part of the axial current, and τ^3 is the third Pauli matrix. We evaluate this matrix element on the lattice by computing the nucleon two-point function $\langle G^N(t;\mathbf{p};\Gamma_4)\rangle$, the Δ two-point function $\langle G^\Delta_{ii}(t;\mathbf{p};\Gamma_4)\rangle$, and the three point function $\langle G^{\Delta A^\mu N}_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle$ and forming the ratio [2]

$$R_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma;\mu) = \frac{\langle G^{\Delta A^\mu N}_\sigma(t_2,t_1;\mathbf{p}',\mathbf{p};\Gamma)\rangle}{\langle G^\Delta_{ii}(t_2;\mathbf{p}';\Gamma_4)\rangle} \times\left[\frac{\langle G^N(t_2-t_1;\mathbf{p};\Gamma_4)\rangle\langle G^\Delta_{ii}(t_1;\mathbf{p}';\Gamma_4)\rangle\langle G^\Delta_{ii}(t_2;\mathbf{p}';\Gamma_4)\rangle}{\langle G^\Delta_{ii}(t_2-t_1;\mathbf{p}';\Gamma_4)\rangle\langle G^N(t_1;\mathbf{p};\Gamma_4)\rangle\langle G^N(t_2;\mathbf{p};\Gamma_4)\rangle}\right]^{1/2} \stackrel{t_2-t_1\gg 1, t_1\gg 1}{\Rightarrow} \Pi_\sigma(\mathbf{p}',\mathbf{p};\Gamma;\mu), \tag{2}$$

where the indices i are summed, $\Gamma_4 = \frac{1}{2}\begin{pmatrix} I & 0 \\ 0 & 0\end{pmatrix}$, and $\Gamma_j = \frac{1}{2}\begin{pmatrix} \sigma_j & 0 \\ 0 & 0\end{pmatrix}$. For large time separations between t_1, the time when a photon interacts with a quark, and t_2, the time when the Δ is annihilated, the ratio of Eq. (2) becomes time independent and yields the transition matrix element of Eq. (1). The source-sink time separation is optimized as in Ref. [11] so that a plateau is clearly identified when varying t_1. We use kinematics where the Δ is produced at rest and $Q^2 = -q^2$ is the Euclidean momentum transfer squared. There are various choices for the Rarita-Schwinger spinor index σ and projection matrices Γ that can be used in the computation of the three point function, each requiring a sequential inversion. We use this freedom to construct optimized Δ sources that maximize the number of lattice momentum vectors contributing to a given value of Q^2 in analogy with the evaluation of the electromagnetic N to Δ transition form factors [2].

In Fig. 1 we show quenched and unquenched results obtained with Wilson fermions. In all cases the errors are determined using a jackknife analysis. We observe that C_3^A is consistent with zero and that unquenching effects are small for the dominant form factors, C_5^A and C_6^A. In contrast, for the form factor C_4^A, dynamical fermions produce a dramatic increase at low momentum transfer relative to the quenched results. Such large deviations between quenched

TABLE I. The number of configurations, the hopping parameter, κ, for Wilson fermions or the light-quark mass, m_l, for staggered quarks, and the pion, nucleon and Δ masses in lattice units.

No. confs	κ or am_l	am_π	aM_N	aM_Δ
	Quenched $32^3\times 64$ $a^{-1} = 2.14(6)$ GeV			
200	0.1554	0.263(2)	0.592(5)	0.687(7)
200	0.1558	0.229(2)	0.556(6)	0.666(8)
200	0.1562	0.192(2)	0.518(6)	0.646(9)
	$\kappa_c = 0.1571$	0.	0.439(4)	0.598(6)
	Unquenched Wilson $24^3\times 40$ [6] $a^{-1} = 2.56(10)$ GeV			
185	0.1575	0.270(3)	0.580(7)	0.645(5)
157	0.1580	0.199(3)	0.500(10)	0.581(14)
	Unquenched Wilson $24^3\times 32$ [7] $a^{-1} = 2.56(10)$ GeV			
200	0.158 25	0.150(3) [16]	0.423(7)	0.533(8)
	$\kappa_c = 0.1585$	0.	0.366(13)	0.486(14)
	MILC $20^3\times 64$ $a^{-1} = 1.58$ GeV			
150	0.03	0.373(3)	0.886(7)	1.057(14)
150	0.02	0.306(3)	0.800(10)	0.992(16)
	MILC $28^3\times 64$ $a^{-1} = 1.58$ GeV			
118	0.01	0.230(3)	0.751(7)	0.988(26)

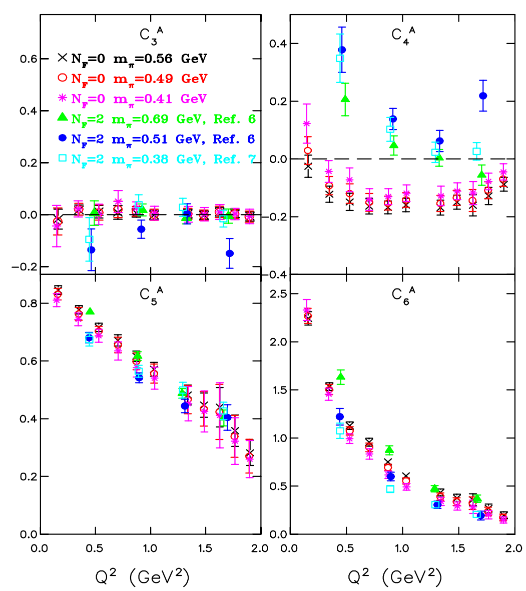

FIG. 1 (color online). The axial form factors C_3^A, C_4^A, C_5^A, and C_6^A as a function of Q^2. We show quenched lattice results at $\kappa = 0.1554$ (crosses), at $\kappa = 0.1558$ (open circles), and at $\kappa = 0.1562$ (asterisks) and unquenched Wilson results at $\kappa = 0.1575$ (solid triangles), $\kappa = 1580$ (solid circles), and $\kappa = 0.15825$ (open squares). We use $Z_A = 0.8$ [17,18].

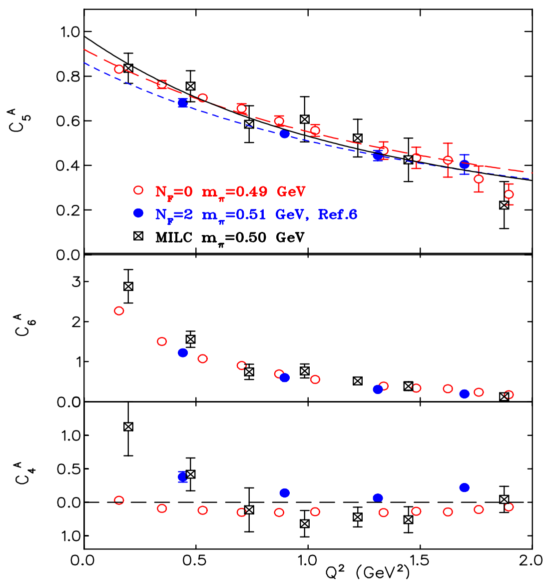

FIG. 2 (color online). The dominant axial form factors C_5^A (top) and C_6^A (middle) and C_4^A as a function of Q^2, for similar pion mass, for quenched at $\kappa = 0.1558$ (open circles) and dynamical Wilson fermions at $\kappa = 0.1580$ (solid circles) and in the hybrid approach at $am_l = 0.01$ (squares). The curves are fits to the dipole form $C_5^A(0)/(1 + Q^2/M_A^2)^2$.

and full QCD results for these relatively heavy quark masses are unusual, making this an interesting quantity with which to study effects of unquenching. In Fig. 2, we compare the Wilson and hybrid results for the two dominant form factors for $m_\pi \sim 500$ MeV. The hybrid results thus corroborate the small unquenching effects, and similar behavior is observed at the heavier and lighter quark masses. For C_4^A, also shown in Fig. 2, we observe the same large unquenching effects observed for dynamical Wilsons. In addition, although they are not shown, hybrid calculations yield $C_3^A \sim 0$. A dipole Ansatz $C_5^A(0)/(1 + Q^2/M_A^2)^2$ describes the Q^2 dependence of C_5^A well, as shown by the curves in Fig. 2, yielding, at this pion mass, an axial mass $M_A \sim 1.8(1)$ GeV. In the range of pion masses considered in this work, we observe a weak quark mass dependence for M_A that, however, needs to be checked for lighter quarks before comparing to the experimental result of 1.28 ± 0.10 GeV [19].

In the chiral limit, axial current conservation leads to the relation $C_6^A(Q^2) = M_N^2 C_5^A(Q^2)/Q^2$. In Fig. 3, we show the ratio $(Q^2/M_N^2)C_6^A(Q^2)/C_5^A(Q^2)$ for Wilson and domain wall fermions at the lightest quark mass available in each case. The expected value in the chiral limit for this ratio is one. For finite quark mass, the axial current is not conserved and for Wilson fermions chiral symmetry is broken, so that deviations from one are expected. We observe that this ratio differs from unity at low Q^2 but approaches unity at higher values of Q^2. For the quenched case, where we have accurate results, a linear extrapolation in m_π^2 yields values that are consistent with unity for $Q^2 \gtrsim 0.5$ GeV^2. It is reassuring that this chiral restoration is seen on the lattice

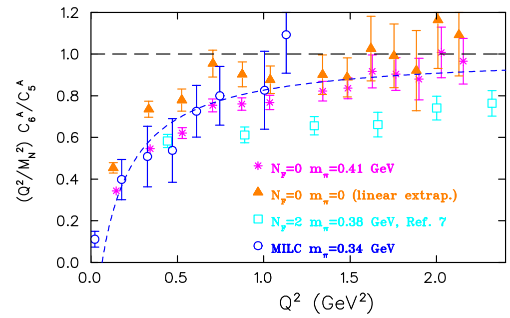

FIG. 3 (color online). The ratio $(Q^2/M_N^2)C_6^A/C_5^A$ is shown versus Q^2 in the quenched theory at $\kappa = 0.1562$ (asterisks) and in the physical limit (solid triangles), for dynamical Wilson fermions at $\kappa = 0.15825$ (open squares) and in the hybrid approach at $am_l = 0.01$ (open circles). The short dashed line denotes the soft pion fit described in the text.

even for Wilson fermions, confirming that the discrete theory is correctly representing the continuum physics. For finite pion mass, deviations from unity are expected to be proportional to $m_\pi^2/(Q^2 + m_\pi^2)$ for chiral lattice fermions. As can be seen by the dashed line in the figure, this form describes well the Q^2 dependence of the results obtained with domain wall fermions.

For finite mass pions, partial conservation of the axial current $[\partial_\mu A_\mu^a(x) = f_\pi m_\pi^2 \pi^a(x)]$ leads to the off-diagonal Goldberger-Treiman relation $C_5^A(Q^2) = f_\pi g_{\pi N\Delta}(Q^2)/2M_N$, where $g_{\pi N\Delta}(Q^2)$ is determined from the matrix element of the pseudoscalar density $\langle\Delta^+|\bar{\psi}(x)\gamma_5 \frac{\tau^3}{2}\psi(x)|p\rangle$ [20] and the pion decay constant f_π is calculated from the two-point function $\langle 0|A_4(x)|\pi\rangle$. To relate the lattice pion matrix element to its physical value, we need the pseudoscalar renormalization constant Z_p, and we use for quenched [17] and dynamical Wilson fermions [18] the value $Z_p(\mu^2 a^2 \sim 1) = 0.5$. In Fig. 4 we show the ratio $f_\pi g_{\pi N\Delta}/(2M_N C_5^A)$ for Wilson fermions, which is almost independent of Q^2, indicating similar Q^2 dependence for $g_{\pi N\Delta}$ and C_5^A. As the quark mass decreases, the ratio becomes consistent with unity, in agreement with the off-diagonal Goldberger-Treiman relation. We find a similar behavior for hybrid results albeit with larger errors.

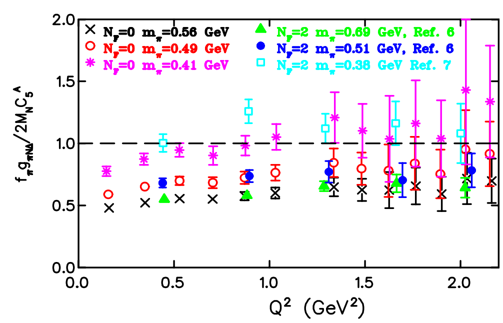

FIG. 4 (color online). The ratio $f_\pi g_{\pi N\Delta}/(2M_N C_5^A)$ as a function of Q^2. The notation is the same as in Fig. 1.

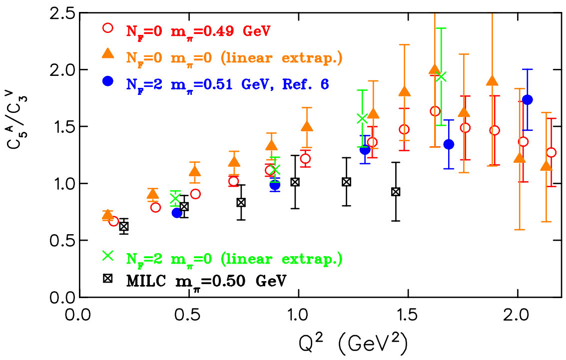

FIG. 5 (color online). The ratio C_5^A/C_3^V as a function of Q^2 for quenched QCD at $\kappa = 0.1558$ (open circles) and at the physical pion mass (solid triangles), for dynamical Wilson fermions at $\kappa = 0.1580$ (solid circles) and at the physical pion mass (crosses) and for the hybrid action at $am_l = 0.02$ (squares with cross).

The presently unmeasured ratio C_5^A/C_3^V is an interesting prediction of our lattice calculation. The form factor C_3^V can be obtained from the electromagnetic N to Δ transition. Using our lattice results for the dipole and electric quadrupole Sachs factors, $\mathcal{G}_{M1}$ and $\mathcal{G}_{E2}$ [21], we extract C_3^V using the relation

$$C_3^V = \frac{3}{2}\frac{M_\Delta(M_N + M_\Delta)}{(M_N + M_\Delta)^2 + Q^2}(\mathcal{G}_{M1} - \mathcal{G}_{E2}) \qquad (3)$$

and evaluate the ratio C_5^A/C_3^V shown in Fig. 5. We observe that quenched and unquenched results at $m_\pi \sim 500$ MeV agree within errors, and that the quenched data have a sufficiently weak mass dependence that when extrapolated to the physical pion mass, the results nearly coincide with the 500 MeV data within errors. Hence, it is reasonable to use the extrapolated quenched results as a first estimate of this ratio. This ratio, besides being the off-diagonal analogue of g_A/g_V, provides the basis for a physical estimate of the parity violating asymmetry. Under the assumptions that $C_3^A \sim 0$ and C_4^A is suppressed as compared to C_5^A, both of which are supported by our lattice results, the parity violating asymmetry can be shown to be proportional to this ratio [4]. Thus, our lattice results show that this ratio and, to a first approximation, the parity violating asymmetry is nonzero at $Q^2 = 0$ and increases for Q^2 values up to about 1.5 GeV^2.

In summary, we have provided a first lattice calculation of the axial transition form factors, which are to be measured at Jefferson Lab [5]. The first conclusion is that C_3^A is consistent with zero, whereas C_4^A is small but shows unusually high sensitivity to unquenching effects. The two dominant form factors are C_5^A and C_6^A, which are related in the chiral limit by axial current conservation. The ratio $(Q^2/M_N^2)C_6^A/C_5^A$ is shown to approach unity as the quark mass decreases, as expected from chiral symmetry. For any quark mass, the strong coupling $g_{\pi N\Delta}$ and the axial form factor C_5^A show similar Q^2 dependence, and the off-diagonal Goldberger-Treiman relation is reproduced as the quark mass decreases. The ratio of C_5^A/C_3^V, which provides a first approximation to the parity violating asymmetry, is predicted to be nonzero at $Q^2 = 0$ with a twofold increase when $Q^2 \sim 1.5$ GeV.

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