Periodic-Orbit Theory of Level Correlations

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We present a semiclassical explanation of the so-called Bohigas-Giannoni-Schmit conjecture which asserts universality of spectral fluctuations in chaotic dynamics. We work with a generating function whose semiclassical limit is determined by quadruplets of sets of periodic orbits. The asymptotic expansions of both the nonoscillatory and the oscillatory part of the universal spectral correlator are obtained. Borel summation of the series reproduces the exact correlator of random-matrix theory.

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Quantum spectra of individual chaotic systems can be phenomenologically described in terms of random-matrix theory (RMT) [1,2]. This universality, asserted by the celebrated Bohigas-Giannoni-Schmit conjecture (BGS) [3], is an empirical fact, supported by a huge body of experimental and numerical data. Proving its origin remains a challenge in quantum or wave chaos.

Spectral fluctuations are conveniently characterized in terms of the two-point correlation function, $R(\epsilon) \equiv \pi\delta(\epsilon) - Y_2(\epsilon) = \Delta^2\langle\rho(E+\frac{\epsilon\Delta}{2\pi})\rho(E-\frac{\epsilon\Delta}{2\pi})\rangle - 1$, where $\rho(E)$ is the energy-dependent density of states, $\Delta \equiv 1/\langle\rho\rangle$ the mean level spacing, and $\langle\cdot\rangle$ denotes averaging over the energy E. Predictions made by RMT for this correlation function are universal in that they depend only on the parameter ϵ, and the fundamental symmetries of the system considered, in particular, on whether it is time-reversal invariant (orthogonal case) or not (unitary case). Specifically, the complex representation $R(\epsilon) = \lim_{\gamma\to 0}\mathrm{Re}C(\epsilon^+)$ where $\epsilon^\pm = \epsilon \pm i\gamma$ and $C(\epsilon^+) = \frac{\Delta^2}{2\pi^2}\langle\mathrm{Tr}G(E+\epsilon^+\Delta/2\pi)\mathrm{Tr}G(E-\epsilon^+\Delta/2\pi)\rangle - \frac{1}{2}$ is employed, with $G(x) = (x-H)^{-1}$ and H the Hamiltonian. The Wigner-Dyson unitary (u) and orthogonal (o) symmetry classes of RMT afford the asymptotic series

$$C(\epsilon^+) \sim \begin{cases} \frac{1}{2(i\epsilon^+)^2} - \frac{e^{2i\epsilon^+}}{2(i\epsilon^+)^2} & \text{(u)} \\ \frac{1}{(i\epsilon^+)^2} + \sum_{n=3}^{\infty}\frac{(n-3)!(n-1)}{2(i\epsilon^+)^n} & \\ + e^{2i\epsilon^+}\sum_{n=4}^{\infty}\frac{(n-3)!(n-3)}{2(i\epsilon^+)^n} & \text{(o).} \end{cases} \quad (1)$$

In either case, $C(\epsilon^+)$ is a sum of a nonoscillatory part (power series in $1/\epsilon^+$) and an oscillatory one ($e^{2i\epsilon^+}$ times a series in $1/\epsilon^+$). Borel summation of (1) restores the complex correlator whose extrapolation to small positive values of ϵ gives $R(\epsilon) + 1 \propto \epsilon^\beta$, a signature of the level repulsion symptomatic for chaos ($\beta = 1$, 2 for the orthogonal, respectively, unitary symmetry.)

The question to be addressed below is how to obtain the RMT prediction (1) for a concrete chaotic (fully hyperbolic) quantum system. A step in this direction was recently made [4] on the basis of Gutzwiller's semiclassical periodic-orbit theory [5]. Gutzwiller represents the level density $\rho(E)$ as a sum over periodic orbits, whereupon the function $R(\epsilon)$ becomes a sum over orbit pairs [6–8]. Relevant contributions to that double sum were shown [4] to originate from orbit pairs which are identical, mutually time reversed, or differ only by connections in certain close self-encounters. By summing over all distinct families of orbit pairs, the Fourier transform of $R(\epsilon)$, the spectral form factor $K(\tau)$, was found to coincide with the RMT prediction for times $t = \tau T_H$ smaller than the Heisenberg time $T_H = 2\pi\hbar/\Delta$, the time needed to resolve the mean level spacing. The behavior of $K(\tau)$ for $\tau > 1$, also known from RMT, was left unexplained.

We now want to fill the gap left. As a result we will obtain the full expression for the correlation function in the case of unitary symmetry, and an asymptotic $1/\epsilon$ expansion amenable to Borel summation in the orthogonal case. The oscillatory term, Fourier transformed, then complements $K(\tau)$ to its full form at $\tau > 1$. In many respects, our reasoning is inspired by the field theoretical formulation of RMT correlation functions [9], notably the existence of "anomalous saddle points" in the nonlinear σ model [10]. It also affords a new interpretation of ideas underlying the "bootstrapping" [11].

The basic idea of our approach is to consider representations of $C(\epsilon^+)$ different from the standard one in terms of the product of a single retarded and advanced Green function. We start from the generating function

$$Z = \left\langle\frac{\det(E_C^+ - H)\det(E_D^- - H)}{\det(E_A^+ - H)\det(E_B^- - H)}\right\rangle, \quad (2)$$

where $E^\pm_{A,B,C,D}$ are energies in the vicinity of E defined by $E^\pm_{A,B,C,D} = E + \epsilon^\pm_{A,B,C,D}\Delta/2\pi$. From Z, the complex correlator can be accessed as

$$\lim_{\gamma\to 0} C(\epsilon^+) = -\frac{1}{2} + 2\lim_{\gamma\to 0}\frac{\partial^2 Z}{\partial\epsilon_A^+\partial\epsilon_B^-}\bigg|_{\parallel,\times}. \quad (3)$$

The two derivatives produce $\mathrm{Tr}G(E_A^+)\mathrm{Tr}G(E_B^-)$ under the energy average. If we subsequently identify the energies "columnwise" ($\parallel$): $\epsilon_A^+ = \epsilon_C^+ = \epsilon^+, \epsilon_B^- = \epsilon_D^- = -\epsilon^+$, or "crosswise"($\times$): $\epsilon_A^+ = \epsilon^+$, $\epsilon_B^- = -\epsilon^+$, $\epsilon_C^+ = -\epsilon^-$, $\epsilon_D^- = \epsilon^-$ the ratio of determinants approaches unity. The first representation for $C(\epsilon^+)$ does not even require the limit $\gamma \to 0$; it is widely used in RMT. Importantly, the semiclassical approximation of either of these two exact representations misses contributions to Z, and therefore to $C(\epsilon^+)$: the first representation yields only the nonoscillatory contributions, and the second (without "$-\frac{1}{2}$") only the oscillatory ones; adding both we will recover the universal two-point correlator.

To see the emergence of these structures, let us represent the determinants in (2) as

$$\det(E_A^+ - H)^{-1} = \exp\left\{-\int^{E_A^+} dE \mathrm{Tr} G(E)\right\} \sim \mathrm{const} \times \exp\left(i\pi E_A^+/\Delta + \sum_a F_a e^{iS_a(E_A^+)/\hbar}\right), \tag{4}$$

where the last line invokes the semiclassical expansion of the integrated Green function into a smooth (Weyl) average and a fluctuating (Gutzwiller) part; the latter is a sum of periodic orbits a with action S_a and stability amplitude F_a; for simplicity, we assume the average level density $1/\Delta$ to be constant; the periodic-orbit sum converges for $\mathrm{Im} E_A^+$ large enough; the "const" in (4) comes from the lower limit of the energy integral and cancels from the ratio of determinants in Z.

Expanding the exponential in (4) we get a sum over nonordered sets of periodic orbits. Such sets will be referred to as "pseudo-orbits" and labeled by capital letters. We then obtain

$$\det(E_A^+ - H)^{-1} \sim \mathrm{const} \times e^{i\pi E_A^+/\Delta} \sum_A F_A e^{iS_A(E_A^+)/\hbar}. \tag{5}$$

A pseudo-orbit A may involve any number n_A of component orbits ($n_A = 0$ pertains to the empty set which contributes unity to the sum); F_A is the product of the stability amplitudes and S_A the cumulative action of all component orbits. Expressing all four determinants in (2) similarly to (4) and (5) [e.g., using $\det(E_B^- - H) = (\det(E_B^+ - H))^*$] and writing $S(E + \epsilon\Delta/2\pi) \sim S(E) + T(E)\epsilon\Delta/2\pi$ (T is the period of an orbit, or the sum of periods in a pseudo-orbit) we approximate the generating function as

$$Z \sim e^{i(\epsilon_A^+ - \epsilon_B^- - \epsilon_C^+ + \epsilon_D^-)/2}\left\langle \exp\left(\sum_a F_a e^{iS_a(E)/\hbar + i(T_a/T_H)\epsilon_A^+} + \sum_b F_b^* e^{-iS_b(E)/\hbar - i(T_b/T_H)\epsilon_B^-} - \sum_c F_c e^{iS_c(E)/\hbar + i(T_c/T_H)\epsilon_C^+} - \sum_d F_d^* e^{-iS_d(E)/\hbar - i(T_d/T_H)\epsilon_D^-}\right)\right\rangle \tag{6}$$

$$= e^{i(\epsilon_A^+ - \epsilon_B^- - \epsilon_C^+ + \epsilon_D^-)/2}\left\langle \sum_{A,B,C,D} F_A F_B^* F_C F_D^* (-1)^{n_C + n_D} \times e^{i(S_A(E) - S_B(E) + S_C(E) - S_D(E))/\hbar} \times e^{i(T_A\epsilon_A^+ - T_B\epsilon_B^- + T_C\epsilon_C^+ - T_D\epsilon_D^-)/T_H}\right\rangle. \tag{7}$$

Here, the mean density produces a phase factor $e^{i(\epsilon_A^+ - \epsilon_B^- - \epsilon_C^+ + \epsilon_D^-)/2}$. When representing the correlator as in (3), that phase factor turns into 1 and $e^{2i\epsilon}$ for the columnwise and crosswise identifications of energies, respectively. Indeed, then, we can recover either the nonoscillatory or the oscillatory contributions to $C(\epsilon^+)$.

Another phase factor involves the difference $\Delta S \equiv S_A(E) - S_B(E) + S_C(E) - S_D(E)$ between the cumulative actions of (A, C) and (B, D). Because of this factor, systematic contributions in the limit $\hbar \to 0$ can arise only for quadruplets of pseudo-orbits whose action difference is of the order of $\hbar$ or smaller.

The most basic of quadruplets have each of the component orbits of A and C repeated in either B or D, such that $\Delta S = 0$. These "diagonal quadruplets" may be summed by a lowest-order cumulant expansion: denoting the periodic-orbit sums in the exponent of (6) by X, we may write $\langle e^X\rangle_{\mathrm{diag}} = \exp\{\langle X^2\rangle_{\mathrm{diag}}/2\}$, wherein $\langle X^2\rangle_{\mathrm{diag}}$ contains only pairs of identical orbits. We find

$$Z_{\mathrm{diag}} \sim \exp\langle X^2\rangle_{\mathrm{diag}}/2 \times \exp\left\langle \sum_a |F_a|^2 \left(e^{i(T_a/T_H)(\epsilon_A^+ - \epsilon_B^-)} - e^{i(T_a/T_H)(\epsilon_A^+ - \epsilon_D^-)}\right) - \sum_c |F_c|^2 \left(e^{i(T_c/T_H)(\epsilon_C^+ - \epsilon_B^-)} - e^{i(T_c/T_H)(\epsilon_C^+ - \epsilon_D^-)}\right)\right\rangle. \tag{8}$$

Relying on ergodicity, the resulting sums over orbits may be evaluated by the sum rule of Hannay and Ozorio de Almeida [12], $\sum_a |F_a|^2(\cdot) \approx \int_{T_0}^{\infty} \frac{dT}{T}(\cdot)$; the lower limit of the integration is some minimal period T_0 . By this rule, e.g., the first sum turns into $-\ln(i(\epsilon_A^+ - \epsilon_B^-)) + \mathrm{const} + \mathcal{O}(\hbar)$. All four sums yield

$$Z_{\mathrm{diag}} \sim e^{i(\epsilon_A^+ - \epsilon_B^- - \epsilon_C^+ + \epsilon_D^-)/2}\left(\frac{(\epsilon_A^+ - \epsilon_D^-)(\epsilon_C^+ - \epsilon_B^-)}{(\epsilon_A^+ - \epsilon_B^-)(\epsilon_C^+ - \epsilon_D^-)}\right)^{\kappa}, \tag{9}$$

with $\kappa = 1$ for the unitary class. For the orthogonal class we must also consider pairs of mutually time-reversed orbits. Therefore, each sum in (8) must be multiplied by 2 whereupon in the final result (9) we have $\kappa = 2$.

Taking derivatives and columnwise identified energies, we recover the leading nonoscillatory contribution to the two-point correlator $(i\epsilon^+)^{-2}/\beta$. Crosswise identified energies yield the oscillatory contribution $-e^{2i\epsilon}(i\epsilon)^{-2}/2$ for $\beta = 2$ [thus completely reproducing (1)], while for $\beta = 1$ we get zero; i.e., no oscillatory term arises up to $\mathcal{O}(\epsilon^{-2})$.

Going beyond the above level of approximation, we note that small phases may also arise from component orbits B and D differing from A and C in topology, but only weakly in action. The key notion of the theory is an "encounter."

By that we mean a close approach of two or more mutually almost parallel stretches of the same orbit or different periodic orbits; examples are highlighted by thick arrows in Fig. 1. Because of the Lyapunov divergence an encounter may last only for a relatively short time such that we can speak of its beginning and end. The orbit pieces outside the encounters will be called "links" [13]; the duration of links is extremely large compared with the encounter duration. Encounters are switch boxes of the hyperbolic dynamics: Orbits and pseudo-orbits with encounters have "partner" (pseudo-) orbits with practically the same links differently connected within these encounters. Since almost all of the orbit duration is concentrated in links, these partners have nearly the same action as the "original" (pseudo-) orbit; the contributions of original and partner orbits can thus effectively interfere [8].

The example in Fig. 1(a) involves a pseudo-orbit, say, A which contains just one orbit with an encounter of two stretches; C is empty. Reconnecting the links of A within the encounter we obtain two disjoint orbits [dashed in Fig. 1(a)], each containing one link and one encounter stretch; their cumulative action is almost the same as for A. These two orbits can either both be included in B such that D remains empty, or vice versa, or one is included in B and the other one in D. At any rate the cumulative action of (A, C) and (B, D) almost coincide.

A second example, a so-called Sieber-Richter pair [8], is shown in Fig. 1(b): Here the encounter consists of two almost mutually time-reversed stretches which avoid a crossing in one orbit and cross in the other one. The original orbit is included in A or C, whereas its partner is included in B or D. Such orbit pairs exist only in time-reversal invariant systems since the motion along one of the links is reverted in time in the partner orbit.

More complicated quadruplets involve any number of orbits, and (A, C) and (B, D) can differ in any number of encounters where arbitrarily many stretches come close in phase space (modulo time-reversal for time-reversal invariant systems); encounters may be self-encounters within periodic-orbit components in a pseudo-orbit, mutual encounters of different periodic orbits within one pseudo-orbit, or even from different pseudo-orbits.

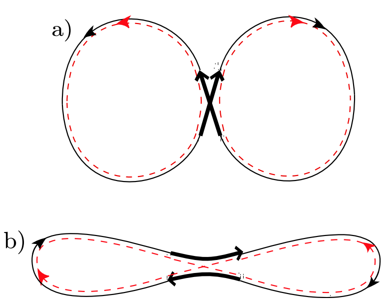

FIG. 1 (color online). Thin full lines: sketches of periodic orbits in configuration space. Thick: "encounters" where two orbit stretches come close; arrows indicate sense of motion. Dashed: partner orbits with changed connections in encounters.

As in the diagonal approximation some of the component orbits of (A, C) may just be repeated in (B, D). To evaluate the generating function, we must sum over all quadruplets of this type. We can split the sum into one over the "diagonal" parts of these quadruplets and one over the orbits differing in encounters. The first subsum yields Z_{diag} as in (9) such that $Z = Z_{\text{diag}}(1 + Z_{\text{off}})$ with

$$Z_{\text{off}} = \sum_{\substack{A,B,C,D \\ \text{diff. in enc.}}} \langle F_A F_B^* F_C F_D^* (-1)^{n_C+n_D} e^{i\Delta S/\hbar} \times e^{i(T_A\epsilon_A^+ - T_B\epsilon_B^- + T_C\epsilon_C^+ - T_D\epsilon_D^-)/T_H}\rangle. \quad (10)$$

To find Z_{off} we must classify and count quadruplets with encounters. Their topological "structure" must be dealt with first. To that end we number all encounter stretches (stretches, for short) of A and C, ordering the orbits by number of stretches and the stretches inside an orbit by order of traversal starting from an arbitrary one. This leaves $\prod_\mu \mu^{\omega_\mu}\omega_\mu!$ equivalent ways to label the stretches (where ω_μ is the number of orbits in A, C with μ stretches) and we shall later have to divide by this number. Each structure now corresponds to one way of (i) grouping these numbered stretches into encounters, (ii) choosing their mutual orientation if the system is time-reversal invariant, (iii) changing connections inside the encounters, and (iv) dividing the original orbits among A and C and the partner orbits among B and D.

Next, pseudo-orbit quadruplets are characterized by phase-space separations between the encounter stretches. To measure separations for an encounter of l stretches, we introduce a Poincaré surface of section orthogonal to the original orbit in an arbitrary point in one of the stretches. The other stretches pierce through the same section in $(l - 1)$ further points; their phase-space separation from the first piercing can be decomposed into components u_i and s_i along the unstable and stable manifolds. As shown in Ref. [4], the encounter contributes to the action difference with $\sum_j s_j u_j$ and has a duration $t_{\text{enc}} = \frac{1}{\lambda}\ln[c^2/(\max_j|s_j| \times \max_k|u_k|)]$, where λ is the Lyapunov exponent and c a constant whose value is unimportant.

The sum over A, B, C, D in (10) can be written as a sum over structures and an integral over s, u and the link durations t. The measure to be used [4,14] obtains a factor $1/(\Omega^{l-1}t_{\text{enc}})$ from each encounter of l stretches, with Ω the volume of the energy shell. The factor $1/\Omega^{l-1}$ gives the uniform ergodic probability density for the $l - 1$ later piercings to have given s, u; the factor $1/t_{\text{enc}}$ compensates an overcounting due to the fact that the Poincaré section may be placed anywhere inside the encounter.

We now split the phase-space integral into factors representing the links and the encounters. To do so, we write the time T_A as a sum of durations of all links and encounter stretches which belong to A before reconnection; T_B, T_C, and T_D are decomposed similarly. We then obtain an integral $\int_0^\infty dt e^{it(\epsilon^+_{A\text{ or }C} - \epsilon^-_{B\text{ or }D})/T_H}$ for each link (belonging to A or C before reconnection and to B or D

afterwards), and an integral $\int d^{l-1}s d^{l-1}u \frac{1}{\Omega^{l-1}t_{\mathrm{enc}}} \times e^{i\sim_j s_j u_j/\hbar} e^{i(l_A\epsilon_A^+ - l_B\epsilon_B^- + l_C\epsilon_C^+ - l_D\epsilon_D^-)t_{\mathrm{enc}}/T_H}$ for each encounter (with l_A, l_B, l_C, l_D the numbers of stretches of the encounter belonging to A, B, C, D, and $l_A + l_C = l_B + l_D$.) Evaluating these integrals as in [14] we obtain a factor $i(\epsilon^+_{A\,\mathrm{or}\,C} - \epsilon^-_{B\,\mathrm{or}\,D})^{-1}$ for each link, and a factor $i(l_A\epsilon_A^+ - l_B\epsilon_B^- + l_C\epsilon_C^+ - l_D\epsilon_D^-)$ for each encounter, while T_H cancels out. In this way, Z_{off} becomes the sum over structures

$$Z_{\mathrm{off}} \sim \sum_{\mathrm{struct}} \frac{\kappa^{n_B+n_D}\prod_{\mathrm{enc}} i(l_A\epsilon_A^+ - l_B\epsilon_B^- + l_C\epsilon_C^+ + l_D\epsilon_D^-)}{(-1)^{n_C+n_D}\prod_{\mu}\mu^{\omega_\mu}\omega_\mu!\prod_{\mathrm{links}}(-i)(\epsilon^+_{A\,\mathrm{or}\,C} - \epsilon^-_{B\,\mathrm{or}\,D})},$$

where for $\kappa = 2$ the factor $\kappa^{n_B+n_D}$ accounts for the two different senses of motion on the "reconnected" orbits. Summarily referring to the linear combinations of the $\epsilon^{\pm}_{A,B,C,D}$ in the link and encounter factors as ϵ, we infer that Z_{off} is a power series in $1/\epsilon$. The term $(1/\epsilon)^m$ is provided by all structures with $m = L - V$, with V the number of encounters and L the number of stretches in a structure; note $L > V$. This remark allows to draw all "diagrams" contributing to each of the first few orders of the expansion and to evaluate their contributions.

For instance, the order $m = 1$ is determined by the two diagrams in Fig. 1, whereas for $m = 2$ we need quadruplets with two 2-encounters or one 3-encounter. In the unitary case, all these quadruplets either yield vanishing contributions to Z (after summing over all possible assignments of orbits to A, B, C, D) or mutually cancel. Reassuringly, this complies with the fact that for $\beta = 2$ the diagonal approximation exhausts the RMT result.

In the orthogonal case, off-diagonal contributions remain and the nonoscillatory and the oscillatory parts of the correlator $C(\epsilon)$ are obtained according to (3). Not surprisingly, the nonoscillatory terms are determined only by pairs of orbits [such as Fig. 1(b)] known from the previous work on the small-time form factor; in the present language, either A or C, and either B or D are empty. All genuine pseudo-orbits end up contributing nothing. The first oscillatory term, $\propto e^{2i\epsilon}/\epsilon^4$, does involve nontrivial pseudo-orbit quadruplets. It can be attributed to quadruplets of orbits involving two Sieber-Richter pairs; further (mutually canceling) diagrams are archived in [15]. Proceeding to all orders we get the full asymptotic expansions (1) in the manner of RMT.

We conclude with the following remarks. Implicit to our present analysis is a specific order of two limits. These are the semiclassical limit which brings in the periodic-orbit sum in the manner of Gutzwiller in (4), alluded to as $\lim_{\Delta\to 0}$ below, and the vanishing of the imaginary part $\mathrm{Im}E^+ = \gamma\Delta/2\pi$ of the complex energies and of $\mathrm{Im}\epsilon^+ = \gamma$, i.e., $\lim_{\gamma\to 0}$. We need the condition $\gamma\Delta/\Delta = \gamma > 1$ to make the periodic-orbit contributions to our asymptotic expansions well defined. It is worth noting that this condition effectively limits the orbit periods as $T < T_H$; see Eq. (10) where the final exponential includes a damping $e^{-\gamma(T_A+T_B+T_C+T_D)/T_H}$. In our limit sequence (first $\Delta \to 0$ with $\gamma > 1$ fixed, then $\gamma \to 0$) the two representations, $\times$ and $\|$, become inequivalent, and resolve different parts of the two-point function (oscillatory and nonoscillatory); separate asymptotic expansions for both are required to recover the full information. This complementarity is reflected in the structure of the field theoretical approach to spectral statistics as well: the functional integral representation of Z in RMT is controlled by two saddle points [10]. In the limit $\gamma \ll 1$, both saddles equally contribute and give C in full in either representation, $\|$ and $\times$; one saddle provides the nonoscillatory part of C, the other the oscillatory part. However, for $\gamma > 1$, the oscillatory part gets suppressed as $e^{-\gamma}$ in the $\|$ representation, while the $\times$ representation has the nonoscillatory part exponentially damped. The $1/\epsilon$ expansions of the surviving parts coincide with the present results, to all orders in $1/\epsilon$.

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