Surface Plasmons Modulate the Spatial Coherence of Light in Young's Interference Experiment

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It is shown how surface plasmons that travel between the slits in Young's interference experiment can change the state of spatial coherence of the field that is radiated by the two apertures. Surprisingly, the coherence can both be increased and decreased, depending on the slit separation distance. This results in a modulation of the visibility of the interference fringes. Since many properties of a light field—such as its spectrum, polarization, and directionality—may change on propagation and are dependent on the spatial coherence of the source, our results suggest that the use of surface plasmons provides a new way to alter or even tailor the statistical properties of a light field.

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The study of surface plasmons [1] has greatly intensified since the observation of enhanced optical transmission through subwavelength apertures in metal films [2]. Surface plasmons that are generated by a field that is incident at one aperture may change back into a freely propagating field at another aperture. For example, in Thomas Young's double-slit experiment [3], the interference of surface plasmons that travel between the slits and the light that is directly transmitted was found to modify the total transmission significantly [4]. It seems worthwhile therefore to ask if surface plasmons can also alter the *state of spatial coherence* of a light field on its passage through a perforated metal screen. Analyzing Young's double-slit setup, we predict that surface plasmons can both increase and decrease the coherence between the fields that emanate from the two slits. This should lead to a modulation of the visibility of the ensuing interference fringes. The state of coherence of an optical source is a fundamental property that determines numerous properties of the generated light field, such as its directionality. Also, the spectrum and the polarization of the field that is generated by a partially coherent source may change on propagation [5,6]. Therefore, any new method to influence the state of coherence (in this case the field radiated by a two-slit configuration) is of fundamental importance. Our results suggest that the use of surface plasmons constitutes a new manner to control the statistical and propagation properties of light.

As pointed out by Zernike [7], a direct measure of the state of coherence of the fields in Young's experiment is the "quality" or visibility of the interference fringes on an observation screen in the far zone. The visibility of the fringes at frequency ω yields information about the spectral degree of coherence of the field [8]. More precisely, if the spectral densities (or "spectral intensities") of the fields at the two slits are equal, then the visibility of the fringes is equal to the modulus of the spectral degree of coherence [9]. That is, at a given frequency ω,

$$\mathcal{V}(\mathbf{r}, \omega) \equiv \frac{I_{\max}(\omega) - I_{\min}(\omega)}{I_{\max}(\omega) + I_{\min}(\omega)}, \tag{1}$$

$$= |\mu_{12}(\mathbf{r}_1, \mathbf{r}_2; \omega)|, \tag{2}$$

where $\mathcal{V}$ is the fringe visibility, and μ_{12} is the spectral degree of coherence of the field at the two apertures at $\mathbf{r}_1$ and $\mathbf{r}_2$. Also, $I_{\min}$ and $I_{\max}$ are the minimum and maximum intensity of the fringes in the immediate vicinity of a point P with position $\mathbf{r}$ on the observation screen. It follows from these two equations that the absolute value of the spectral degree of coherence may be determined in two (equivalent) ways: by calculating the near fields of the two slits, or by calculating the far-field interference pattern. In the analysis that follows, we first present a heuristic analytic model which takes into consideration the contribution from the surface plasmons that are generated at each of the slits. These calculations directly yield the spectral degree of coherence of the field. We then perform rigorous numerical simulations of Young's double-slit setup to obtain the fringe visibility $\mathcal{V}$ in the far field, as well as the spectral degree of coherence μ_{12} based on the values of the electric field in the slit regions. The results from the numerical simulations are then compared with the prediction of the analytic model. Since only TM-polarized ($\mathbf{H}$ perpendicular to the x, z plane) incident fields will excite surface plasmons, we shall consider only the TM case here.

In the analytic model, we take the fields incident on the apertures to be $U_1^{(\mathrm{inc})}(\omega)$ and $U_2^{(\mathrm{inc})}(\omega)$, respectively, with spectral degree of coherence $\mu_{12}^{(\mathrm{inc})}(\omega)$ between these two fields and separation d between the apertures (see Fig. 1). A fraction α of the field incident on a slit is directly transmitted, whereas a fraction $\alpha\beta$ is converted into surface plasmons which travel to the other slit where they reappear as a freely propagating field ($\alpha, \beta \in \mathbb{C}$). Hence, we can write the field in each aperture as a sum of the transmitted field and the surface plasmon contribution as

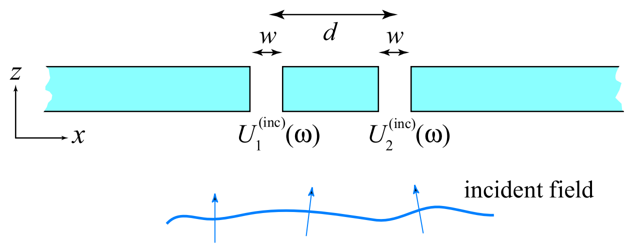

FIG. 1 (color online). Illustrating the geometry.

$$U_1(\omega) = \alpha U_1^{(\mathrm{inc})}(\omega) + \alpha\beta U_2^{(\mathrm{inc})}(\omega)e^{ik_{\mathrm{sp}}d}, \tag{3}$$

$$U_2(\omega) = \alpha U_2^{(\mathrm{inc})}(\omega) + \alpha\beta U_1^{(\mathrm{inc})}(\omega)e^{ik_{\mathrm{sp}}d}, \tag{4}$$

where k_{sp} is the (complex) wave number associated with the surface plasmons. The second-order coherence properties of the fields U_1 and U_2 at frequency ω are characterized by their cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2; \omega)$ [8], viz.

$$W(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle U_1^*(\omega)U_2(\omega)\rangle, \tag{5}$$

where the asterisk indicates complex conjugation, and the angular brackets denote ensemble averaging. The spectral degree of coherence $\mu_{12}(\omega)$ of the fields emanating from the slits is related to the cross-spectral density by the relation

$$\mu_{12}(\omega) = \frac{W(\mathbf{r}_1, \mathbf{r}_2; \omega)}{\sqrt{S_1(\omega)S_2(\omega)}}, \tag{6}$$

where $S_1(\omega) = W(\mathbf{r}_1, \mathbf{r}_1; \omega)$ and $S_2(\omega) = W(\mathbf{r}_2, \mathbf{r}_2; \omega)$ are the spectral densities of the fields at each of the slits. On substituting from Eqs. (3) and (4) into Eq. (5) we obtain

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2; \omega) = |\alpha|^2[\langle U_1^{(\mathrm{inc})^*}(\omega)U_2^{(\mathrm{inc})}(\omega)\rangle + \beta S_1^{(\mathrm{inc})}(\omega) \\ \times \exp(ik_{\mathrm{sp}}d) + \beta^* S_2^{(\mathrm{inc})}(\omega)\exp(-ik_{\mathrm{sp}}^*d) \\ + |\beta|^2\langle U_2^{(\mathrm{inc})^*}(\omega)U_1^{(\mathrm{inc})}(\omega)\rangle\exp(-2k_{\mathrm{sp}}''d)], \end{aligned} \tag{7}$$

where $S_i^{(\mathrm{inc})}(\omega) = \langle U_i^{(\mathrm{inc})^*}(\omega)U_i^{(\mathrm{inc})}(\omega)\rangle$ is the spectral density of the field incident on slit i (with $i = 1, 2$), and $k_{\mathrm{sp}} = k_{\mathrm{sp}}' + ik_{\mathrm{sp}}''$, with $k_{\mathrm{sp}}', k_{\mathrm{sp}}'' \in \mathbb{R}$. We assume, for convenience, that the spectral densities of the two incident fields are identical, i.e.,

$$S_1^{(\mathrm{inc})}(\omega) = S_2^{(\mathrm{inc})}(\omega) = S^{(\mathrm{inc})}(\omega). \tag{8}$$

In that case Eq. (7) simplifies to

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2; \omega) = |\alpha|^2 S^{(\mathrm{inc})}(\omega)\{\mu_{12}^{(\mathrm{inc})}(\omega) + |\beta|^2\mu_{12}^{(\mathrm{inc})^*}(\omega) \\ \times\exp(-2k_{\mathrm{sp}}''d) + 2\mathrm{Re}[\beta\exp(ik_{\mathrm{sp}}d)]\}, \end{aligned} \tag{9}$$

where we have made use of the fact that $\mu_{21}^{(\mathrm{inc})}(\omega) = \mu_{12}^{(\mathrm{inc})^*}(\omega)$, and Re denotes the real part. It is to be noted that Eq. (9) has the form of an interference law in the frequency domain. But whereas the classical spectral interference law [[8], Sec. 4.3.2] pertains to the spectral density at a single point, Eq. (9) pertains to the cross-spectral density of the fields radiated by two slits. It suggests that surface plasmons propagating from one slit to the other can modulate the state of spatial coherence. In other words, the cross-spectral density function of the fields emanating from the two slits can be increased or decreased, according to whether at each slit there is constructive or destructive interference between the directly transmitted field and the field which is due to plasmon generation at the other slit.

As stated above, it is the spectral degree of coherence, the normalized version of the cross-spectral density function, which is observable in Young's interference experiment. To obtain an expression for $\mu_{12}(\omega)$, we must evaluate $S(\omega) = \langle U_1^*(\omega)U_1(\omega)\rangle = \langle U_2^*(\omega)U_2(\omega)\rangle$. On using Eq. (3) we find that

$$\begin{aligned} S(\omega) = |\alpha|^2 S^{(\mathrm{inc})}(\omega)\{1 + |\beta|^2\exp(-2k_{\mathrm{sp}}''d) \\ + 2\mathrm{Re}[\beta\mu_{12}^{(\mathrm{inc})}(\omega)\exp(ik_{\mathrm{sp}}d)]\}. \end{aligned} \tag{10}$$

On substituting from Eqs. (9) and (10) into Eq. (6), we obtain the expression

$$\mu_{12}(\omega) = \frac{\mu_{12}^{(\mathrm{inc})}(\omega) + |\beta|^2\mu_{12}^{(\mathrm{inc})^*}(\omega)\exp(-2k_{\mathrm{sp}}''d) + 2\mathrm{Re}[\beta\exp(ik_{\mathrm{sp}}d)]}{1 + |\beta|^2\exp(-2k_{\mathrm{sp}}''d) + 2\mathrm{Re}[\beta\mu_{12}^{(\mathrm{inc})}(\omega)\exp(ik_{\mathrm{sp}}d)]}. \tag{11}$$

This formula demonstrates that the spectral degree of coherence of the field that is radiated by the apertures is not equal to the spectral degree of coherence of the incident field. Because of the presence of the oscillating terms in this equation, the modulus of the former can either be larger or smaller than that of the latter. In other words, varying the distance that separates the two slits will modulate the visibility of the interference fringes. That this effect is solely due to the action of surface plasmons is easily verified by setting β, their relative contribution strength, equal to zero. In that case Eq. (11) reduces to

$$\mu_{12}(\omega) = \mu_{12}^{(\mathrm{inc})}(\omega), \tag{12}$$

i.e., the spectral degree of coherence of the radiated field is then equal to that of the incident field.

It is helpful to consider the effect of the plasmons on the state of coherence in the limiting cases $\mu_{12}^{(\mathrm{inc})}(\omega) = 1$ and $\mu_{12}^{(\mathrm{inc})}(\omega) = 0$. For $\mu_{12}^{(\mathrm{inc})}(\omega) = 1$, the numerator and denominator of Eq. (11) are equal and we find that

$\mu_{12}(\omega) = 1$. In other words, if the incident field illuminating the slits is fully coherent and in phase, the plasmons will not modify the state of coherence. For $\mu_{12}^{(\mathrm{inc})}(\omega) = 0$, i.e., an incident incoherent field, Eq. (11) reduces to

$$\mu_{12}(\omega) = \frac{2\mathrm{Re}[\beta \exp(ik_{\mathrm{sp}}d)]}{1 + |\beta|^2 \exp(-2k''_{\mathrm{sp}}d)}. \tag{13}$$

It is to be noted that this formula suggests that not only can the spatial coherence of the output field be greater than that of the input field, it may also switch signs, resulting in the field at the two slits being anticorrelated. Setting the plasmon decay constant k''_{sp} to zero for the moment, the maximum value of $|\mu_{12}(\omega)|$ is approximately given by

$$|\mu_{12}^{(\max)}(\omega)| = \frac{2|\beta|}{1 + |\beta|^2} < 1. \tag{14}$$

The spatial coherence of the output field increases with increasing value of $|\beta|$, in this case.

Next we compare the results from the numerical simulations with the analytic expression (11). In our simulations, the two slits are illuminated separately with a synthesized Gaussian beam based on the angular spectrum representation [[10], Sec. 5.1]. The beamwidth and the wavelength are taken to be 750 and 600 nm, respectively. The slit separation used in the simulations is at least 1000 nm, to avoid joint illumination of the slits with a single beam. The electric and magnetic fields in the far field and in the vicinities of the apertures are evaluated using a Green's tensor formalism [11,12], which allows for an exact numerical solution of Maxwell's equations. The coherence properties and intensity of the resulting partially coherent field are evaluated by combining the individual field contributions in accordance with the spectral interference law [[8], Sec. 4.3.2]. The illuminating fields are taken to be uniformly partially coherent [i.e., a single spectral degree of coherence $\mu_{12}^{(\mathrm{inc})}(\omega)$ characterizes the correlation between the two illuminating fields.]

The numerically obtained fringe visibility $\mathcal{V}$ and the spectral degree of coherence calculated based on the electric field in the slit regions are plotted against the analytically calculated spectral degree of coherence, for a slit width $w = 200$ nm in Fig. 2. The coupling constant β in the analytic model is set for a phase shift of $\arg(\beta) = 180°$, while $|\beta| \sim 0.39$ and 0.33 for $\mathcal{V}$ and $|\mu_{12}(\omega)|$, respectively. As predicted by Eq. (14), the maximum value of $|\mu_{12}(\omega)|$ for incoherent illumination is also approximately 0.67 and 0.60. The fringe visibility $\mathcal{V}$ is calculated from the spectral densities of the fields in the far zone, with the values of $I_{\max}$ and $I_{\min}$ taken from the central fringe. The numerically obtained spectral degree of coherence $\mu_{12}(\omega)$ is calculated using the x component of the electric field near the slits. Because the field in the vicinity of the slits can vary rapidly with position, $\mu_{12}(\omega)$ was calculated for a number of closely spaced points around the pair of slits, and the mean of these values is then used as the final result. The average standard deviation between the numerically calculated μ_{12}'s at the different positions for $0 \le \mu_{12}^{(\mathrm{inc})}(\omega) \le 1$ was found to be 0.014, which represents approximately a 10% deviation when $|\mu_{12}(\omega)| = 0.1$. The results in Fig. 2 show very good agreement between the analytical and numerical results, hence we can use the fringe visibility $\mathcal{V}$ or the numerically obtained $|\mu_{12}(\omega)|$ to study the modulation of the coherence of the fields at the two slits.

To see how the coupling constant β changes with the slit width, we performed the simulations for slit widths $w = 100$ nm and $w = 50$ nm. The results are shown in Fig. 3.

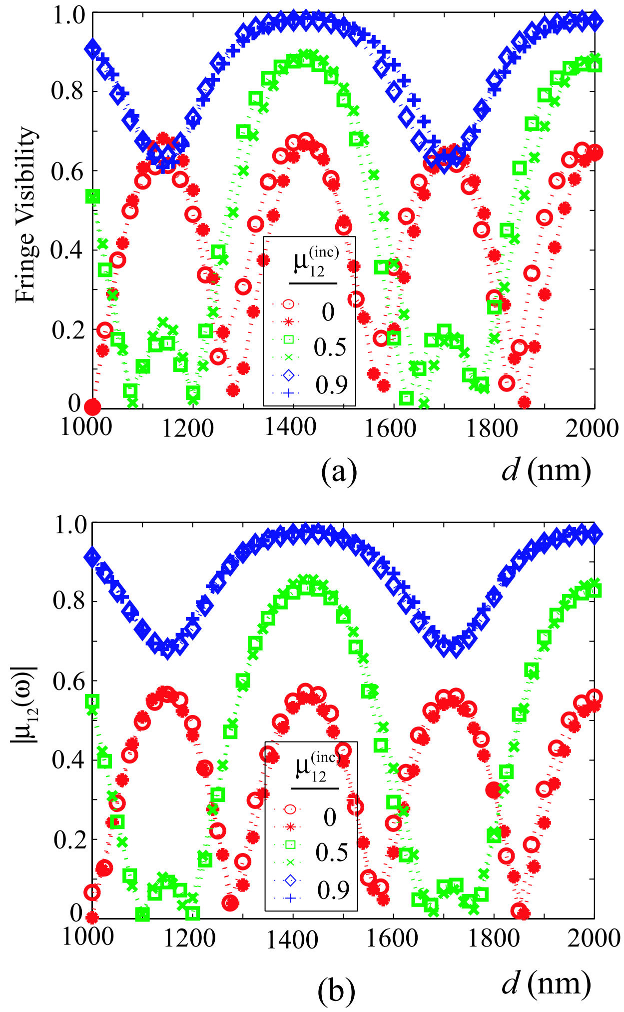

FIG. 2 (color online). Plots of: (a) fringe visibility $\mathcal{V}$, (b) absolute value of the spectral degree of coherence $|\mu_{12}(\omega)|$, as a function of $\mu_{12}^{(\mathrm{inc})}(\omega)$, the spectral degree of coherence of the incident field. Markers (*, ×, +) indicate analytic results, while the unfilled shapes (○, □, ◇) indicate numeric results. In this case the slit width $w = 200$ nm. In all examples the wavelength $\lambda = 600$ nm, and the thickness of the gold film $t = 200$ nm. The refractive index was taken to be $n = 0.21 + i3.27$ and hence $k_{\mathrm{sp}} = (1.099 + i0.007\,21) \times 10^7\ \mathrm{m}^{-1}$.

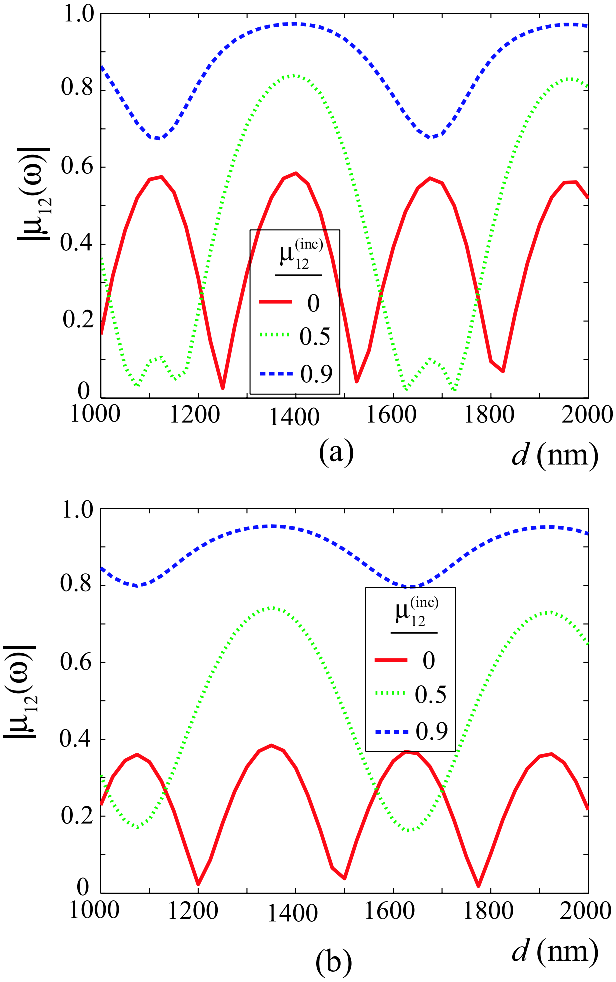

FIG. 3 (color online). The absolute value of the spectral degree of coherence $|\mu_{12}(\omega)|$ as a function of $\mu_{12}^{(\mathrm{inc})}(\omega)$, for slit widths: (a) $w = 100$ nm, (b) $w = 50$ nm. For the sake of clarity, only numeric results are shown.

When the slit width is 100 nm, the phase shift $\arg(\beta) = 205°$, and $|\beta| \sim 0.33$. When the slit width is 50 nm, the phase shift $\arg(\beta) = 225°$, and $|\beta| \sim 0.21$. From the plots, the maximum value of $|\mu_{12}(\omega)|$ is approximately 0.60 and 0.40, respectively, in agreement with the prediction of Eq. (14). These results also suggest that the argument of β varies nontrivially with the width of the slits (cf. Ref. [13]). It is to be noted that the previously studied modulation of the total optical transmission with varying slit separation [4] is less complicated than that of the spectral degree of coherence.

In conclusion, we have shown that surface plasmons propagating between the slits in Young's experiment modulate the fringe visibility of the interference fringes, and hence the spectral degree of coherence of the fields at the two slits. The spectral degree of coherence can be increased or decreased depending on whether there is constructive or destructive interference, which depends on the slit separation in the case of the Young's experiment. The modulation is attributed to surface plasmons propagating between the two slits, as indicated by the good agreement between the analytic model and the rigorous numerical results.

These results allow the possibility of developing new "coherence converting" optical devices, in which the spatial coherence of an incident field can be modified by a suitable array of subwavelength-sized holes. Since various properties of a partially coherent wave field such as the spectral density and the degree of polarization change upon propagation, such a device may prove to be extremely useful in optical systems.

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