

## Falsifying Models of New Physics via WW Scattering

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We show that the coefficients of operators in the electroweak chiral Lagrangian can be bounded if the underlying theory obeys the usual assumptions of Lorentz invariance, analyticity, unitarity, and crossing to arbitrarily short distances. Violations of these bounds can be explained by either the existence of new physics below the naive cutoff of the effective theory, or by the breakdown of one of these assumptions in the short distance theory. As a corollary, if no light resonances are found, then a measured violation of the bound would falsify generic models of string theory.

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The standard model (SM) is only an effective field theory, a good approximation only at energies below some scale  $\Lambda$ . This scale, however, is still undetermined. If, as naturalness arguments indicate, new physics is required to explain the relative smallness of the weak to Planck scale ratio, then we would expect the theory to break down at energies of about 1 TeV. However, even if naturalness arguments fail, we still know that the SM, augmented by Einstein gravity, must break down at the scale of quantum gravity, where predictive power is lost.

In searching for low-energy effects of the physics which underlies the SM, it is prudent to take the model independent approach of adding operators of dimension higher than four to the SM Lagrangian and parameterizing the new physics by their coefficients. Dimensional analysis dictates that these coefficients contain inverse powers of  $\Lambda$ , so the precision with which we must extract them grows with the scale of new physics. This decoupling phenomena makes falsifying theories of the underlying short distance interactions [the ultraviolet (UV)] extremely difficult. Indeed, if the scale of quantum gravity is as high as the Planck scale, it becomes interesting to ask the question as to whether or not the theory is, even in principle, falsifiable. One possibility is that the mathematical structure leads to unique low-energy predictions. However, in the case of string theory, recent progress seems to indicate that this is not a likely scenario. Another possibility is that there are low-energy, non-Planck suppressed, consequences of some underlying symmetries. Symmetries link the UV and the infrared (IR) by distinguishing between universality classes. However, string theory does not seem to have any problems generating the low-energy symmetries manifested at energies presently explored. Indeed, given the enormous number of string vacua, it may be that string theory can accommodate whatever new physics is found at the TeV scale by the Large Hadron Collider (LHC).

Thus it seems that decoupling may have the effect of rendering string theory unfalsifiable. However, dispersion relations can be used to establish bounds which, if violated,

imply that the underlying theory cannot obey the usual assumptions of Lorentz invariance, crossing, unitarity, and analyticity. This type of bound was raised a long time ago in the context of chiral perturbation theory [1–3] and was recently revisited in [4]. In this Letter, we will show that such assumptions in general lead to bounds on the values of coefficients of higher dimension operators in the SM. (This possibility was raised in [4].) As we shall see, the utility of these bounds depends upon the value of the Higgs boson mass.

In the absence of a light Higgs particle, symmetry considerations dictate that the electroweak symmetry breaking sector of the SM be described by a chiral Lagrangian of the nonlinearly realized spontaneously broken  $SU(2) \times U(1)$ . We will derive bounds on certain parameters in the Lagrangian which are not well constrained from oblique corrections. For simplicity, we will assume that the strongly coupled dynamics responsible for electroweak symmetry breaking preserves a custodial  $SU(2)$  symmetry. This assumption, which is empirically validated by the fact that the  $\rho$  parameter is so close to unity, drastically reduces the number of terms in the effective Lagrangian. The Lagrangian we consider [5] contains, in addition to the usual field strength terms for the electroweak gauge bosons, a derivative expansion in the  $SU(2)$  nonlinear sigma model fields,

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{gauge}} - \frac{1}{4}v^2\text{Tr}(V_\mu V^\mu) + \frac{1}{2}\alpha_1 g g' \text{Tr}(B_{\mu\nu} T W^{\mu\nu}) \\ & + \frac{1}{2}i\alpha_2 g' \text{Tr}(T[V^\mu, V^\nu])B_{\mu\nu} + i\alpha_3 g \text{Tr}(W_{\mu\nu}[V^\mu, V^\nu]) \\ & + \alpha_4 (\text{Tr}(V_\mu V_\nu))^2 + \alpha_5 (\text{Tr}(V_\mu V^\mu))^2 \end{aligned} \quad (1)$$

where  $T \equiv 2\Sigma T^3 \Sigma^\dagger$ ,  $V_\mu \equiv (D_\mu \Sigma) \Sigma^\dagger$ , and  $D_\mu \Sigma = \partial_\mu \Sigma + \frac{1}{2}igW_\mu^a \tau^a \Sigma - \frac{1}{2}ig'B_\mu \Sigma \tau^3$ , with  $\Sigma(x) = \exp(i\pi^a(x)\tau^a/v)$  and  $\tau^a$  the Pauli matrices. The “pion” fields here play the role of the would-be Goldstone bosons arising from the broken gauge symmetry. Had we not imposed the custodial symmetry, we would have included

six additional operators. The coefficient  $\alpha_1$  is strongly constrained by virtue of its contribution to the gauge boson self energies at tree level [6]. The coefficients  $\alpha_2$  and  $\alpha_3$  contribute at tree level to the anomalous three gauge boson vertices which have been studied at LEP [6]. Given the constraints on these parameters, they will not be considered in our analysis, as their effect on our bounds are small, although their inclusion is straightforward. The final two coefficients,  $\alpha_4$  and  $\alpha_5$ , contribute to two scattering at tree level, and bounds on them arising from loop corrections to the  $T$  parameter are rather weak [7]. It is these coefficients that we bound below.

The bounds on these couplings are obtained by considering longitudinal  $ZZ \rightarrow ZZ$  and  $WZ \rightarrow WZ$  scattering. Assuming Lorentz invariance, analyticity, and unitarity, the forward scattering amplitude  $T$  satisfies the twice subtracted dispersion relation

$$\frac{d^2 \hat{T}(s)}{ds^2} = 2! \int_{4m^2}^{\infty} \frac{dx}{\pi} \sqrt{x(x-4m^2)} \times \left( \frac{\sigma(x)}{(x-s)^3} + \frac{\sigma_u(x)}{(x-4m^2+s)^3} \right). \quad (2)$$

Here and below,  $s$ ,  $t$ , and  $u$  are Mandelstam variables. We have used the optical theorem to express the discontinuity across the cut in terms of the scattering cross section  $\sigma$  and the crossed channel cross section  $\sigma_u$ . For simplicity, we have denoted by  $m$  the mass of the scattering particles, but more precisely  $2m^2$  should be replaced by  $2m_Z^2$  and  $m_Z^2 + m_W^2$  for  $ZZ \rightarrow ZZ$  and  $ZW \rightarrow ZW$ , respectively. We have introduced  $T = \hat{T} +$  pole term, where the pole term arises from the exchange of a gauge boson. The  $s$ -channel poles are of the form  $p(s)/(s-m^2)$ , where  $p$  is a polynomial. Since the degree of  $p$  is at most 3, the pole cancels from both sides of a twice subtracted dispersion relation. The  $t$  channel poles vanish upon differentiation. Note that the existence of a long range force renders the charged particle scattering total cross section divergent (a pole at  $t=0$ ). Our bounds will rely only on interactions which contain no Coulomb singularities. To obtain bounds, we will use the Equivalence Theorem (ET) to approximate the scattering amplitude of longitudinally polarized massive vector bosons,  $\hat{T}$ , by that of pseudo Goldstone bosons [8–10]. The ET has been studied extensively. It is by now well understood, in a loop expansion, how an amplitude for scattering of (longitudinal) vector particles in a gauge theory can be reproduced by that of pseudoscalars in a nonlinear sigma model [11–14] to leading order in an expansion in  $m^2/s$  and  $g^2$ .

The ET approximation is valid provided  $s \gg m^2$ . Hence we take  $s \sim v^2 + i0$  in the dispersion relation. In this regard, we deviate from the classical analysis, which takes  $s$  below threshold,  $s < 4m^2$ , and real. We then break the integral in Eq. (2) into two terms, the integrals from  $4m^2$  to  $kv^2$  and from  $kv^2$  to  $\infty$ . For the latter, we use the fact that the cross section is positive definite, while the former is computed using the ET to evaluate the cross section.

The constant  $k$  is chosen to minimize the error introduced by our approximations while keeping the right side of (2) positive. One loop electroweak corrections to the amplitude scale as

$$\delta_{ew} T \propto O\left(\frac{g^2 s}{(4\pi v)^2} \ln(s/\mu^2)\right) \quad (3)$$

while chiral corrections scale as

$$\delta_\chi T \propto O\left(\frac{s^3}{(4\pi)^4 v^6} \ln(s/\mu^2)\right). \quad (4)$$

Hence the optimal choice of  $k$  should be a number of order unity. It is easy to see that this can be achieved while keeping the right side of (2) positive. In fact, calculating  $\pi^0 \pi^0$  and  $\pi^0 \pi^+$  scattering ( $Z_L^0 Z_L^0$  and  $Z_L^0 W_L^+$  scattering in the ET approximation), we find that the values of  $k$  for which the real part of the integrals from  $4m^2$  to  $kv^2$  vanish,

$$\text{Re} \left[ \int_{4m^2}^{kv^2} \frac{dx}{\pi} \sqrt{x(x-4m^2)} \left( \frac{\sigma(x)}{(x-s)^3} + \frac{\sigma_u(x)}{(x-4m^2+s)^3} \right) \right] = 0, \quad (5)$$

at a fixed value of  $s + i0$ , are well fit by

$$k = 5.1(s/v^2) - 0.4 \quad (6)$$

$$k = 5.0(s/v^2) - 0.2 \quad (7)$$

for  $\pi^0 \pi^0$  and  $\pi^0 \pi^+$  scattering, respectively. Restricting  $k$  so that  $kv^2 \log(s/\mu^2)/(4\pi v)^2 \leq \delta$  determines how large  $s$  may be taken in the dispersion relation while keeping the errors from the chiral expansion under control. For our numerical estimates, we take  $\delta = 1/5$  and explore the range from  $\delta = 1/2 \times (1/5)$  to  $2 \times (1/5)$ .

Bounds on  $\alpha_4, \alpha_5$  follow from positivity of the right hand side of the dispersion relation. The left hand side of (2) may be approximated using the ET. We use the results for one loop pion scattering calculated in Ref. [15]. Up to second order in the chiral expansion,

$$\frac{d^2 \hat{T}}{ds^2}(s) = \frac{1}{24\pi^2 v^4} \left[ \sum_{i=1,2} c^i \hat{\alpha}_i + \tilde{T}(s) \right], \quad (8)$$

The expressions for the  $c^i, \tilde{T}$  depend on which physical amplitude is considered. The coefficients  $\hat{\alpha}_{4(5)}$  are defined in a similar fashion as the  $\bar{\ell}_i$  in [15], except that the renormalization point is taken at  $\mu = v$ :

$$\alpha_i^r(\mu) = \frac{\gamma_i}{96\pi^2} \left[ \hat{\alpha}_i + \frac{1}{4} \ln(v^2/\mu^2) \right], \quad (9)$$

with  $\gamma_5 = 1$  and  $\gamma_4 = 2$ .

It is now straightforward to obtain the bounds by computing the real part on the LHS of (2). Figure 1 shows the bounds from  $Z_L^0 Z_L^0$  and  $W_L^+ Z_L^0$  scattering for the chosen value of  $s$ . The best bound (largest  $s$ ) is obtained by allowing  $k$  as large as allowed by the restriction on chiral corrections,  $k \log(s/\mu^2)/16\pi^2 \leq \delta$ , at  $\mu = v$ . We find

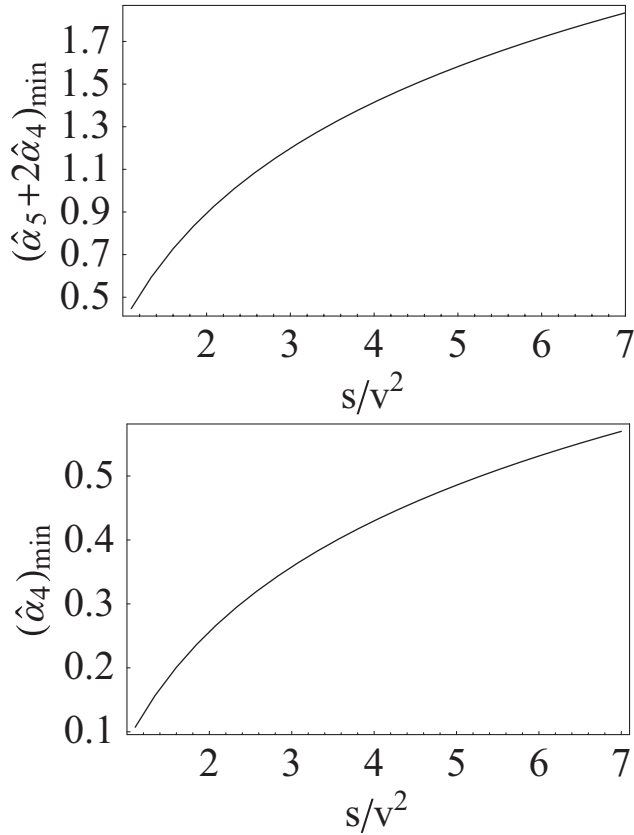


FIG. 1. Bounds on electroweak chiral parameters from  $Z_L^0 Z_L^0$  and  $W_L^+ Z_L^0$  scattering as a function of  $s$  in the dispersion relation, Eq. (2).

$$\hat{\alpha}_5 + 2\hat{\alpha}_4 \geq 1.08 \quad (10)$$

$$\hat{\alpha}_4 \geq 0.31 \quad (11)$$

These are the bounds which correspond to  $\delta = 1/5$ . If we vary  $\delta$  by a factor of 2 or  $1/2$ , the first bound changes by  $+0.37$  and  $-0.35$ , while the second by  $+0.13$  and  $-0.12$ . Note that we consider the choice  $\delta = 1/5$  to be quite conservative since the integral from  $kv^2$  to  $\infty$ , which we have neglected, is positive definite. The bounds are shown in the  $\hat{\alpha}_4$  vs  $\hat{\alpha}_5$  plane in Fig. 2. It should be kept in mind that while the fractional uncertainty in our bounds seem large, the relevant parameters for  $WW$  scattering is the renormalized coupling constant at a scale comparable to the  $W$  and  $Z$  masses. The large uncertainty quoted above corresponds to 25% and 21% uncertainty in the bounds for the renormalized couplings at  $\mu^2 = 4m_W^2$ . The dominant error on these bounds is due to electroweak loop corrections whose contributions are down by  $g^2 v^2/s \approx 0.4g^2$ , where the last equality is from the numerical value of  $v^2/s$  obtained by restricting the chiral corrections to be smaller than  $\delta = 1/5$ . These corrections, which are not included in (10) and (11), are unlike the chiral corrections in that they are calculable and will appear in a subsequent publication [16]. Alternatively, we can estimate the uncertainty intro-

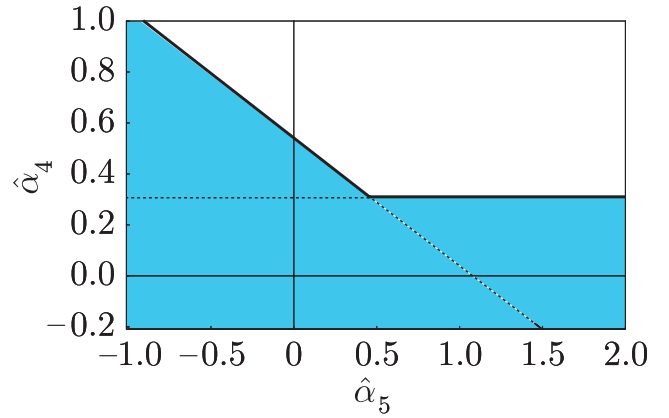


FIG. 2 (color online). Bounds on the coefficients of the electroweak chiral Lagrangian,  $\hat{\alpha}_4$  and  $\hat{\alpha}_5$  renormalized at the scale  $v$ .

duced by our approximations by recomputing the bounds retaining the subleading terms in  $m^2/s$  in the pion scattering amplitude. The result is to move down the bound on  $\hat{\alpha}_5 + 2\hat{\alpha}_4$  in (10) to 1.04 while the bound on  $\hat{\alpha}_4$  in (11) stays at 0.31, consistent with the error estimates above.

In the future it may be possible to measure these coefficients through measurements of  $WW$  or  $ZZ$  scattering at the LHC or the NLC. Studies suggest a sensitivity to  $(\alpha_4, \alpha_5)$  in  $WW$  and  $ZZ$  scattering at a linear collider that could well establish a result in contradiction with our bounds [17,18]. The bounds can also be tested in QCD. In the limit  $g, g' \rightarrow 0$ , the EW chiral Lagrangian of (1) reduces to the hadronic chiral Lagrangian[15], and the parameters of the former,  $\alpha_{4,5}$ , tend towards those of the latter,  $\ell_{1,2}$ . Working above threshold,  $s > 4m_\pi^2$ , as we have done here, is probably not trustworthy for bounds on parameters of the hadronic chiral Lagrangian since in that case  $m_\pi/f_\pi \gtrsim 1$  (as compared with  $m_W/v \lesssim 1/3$ ), and thus the validity of the chiral approximation used for the right hand side of the dispersion relation comes into question. Nevertheless, bounds on  $\bar{\ell}_1$  and  $\bar{\ell}_2$  derived from (10) and (11) using  $\bar{\ell}_1 = 4\bar{\alpha}_5$  and  $\bar{\ell}_2 = 4\bar{\alpha}_4$ , with  $\bar{\alpha}_i = \hat{\alpha}_i + \frac{1}{4} \ln(v^2/m^2)$  are consistent with the experimental values quoted in [19]. It is not necessary to choose  $s$  above threshold in the dispersion relation to bound  $\ell_{1,2}$ , since there is no need to invoke the ET to perform the calculation. The optimum bound obtained by working below threshold and for nonforward scattering,  $t > 0$  [20], reproduces the somewhat weaker bounds found in [3].

Ref. [4] has proposed that constraints on  $\ell_{1,2}$  can also be obtained by requiring the absence of superluminal propagation. When the chiral Lagrangian is expanded about the classical background  $\Sigma = \exp(icx\tau^3)$ , for some constant vector,  $c^\mu$ , the absence of superluminal excitations gives  $\ell_2^{\text{cl}} > 0$ ,  $\ell_1^{\text{cl}} + \ell_2^{\text{cl}} > 0$ . Classical propagation in a nontrivial background is tantamount to studying forward scattering off that background, and, for this process, chiral loops are generally as important as the tree-level contributions of

$\ell_{1,2}$ . Including chiral loop corrections shifts the bounds from forward scattering: up  $\bar{\ell}_2 \geq (39\pi - 92)/48$  and down  $\bar{\ell}_1 + 2\bar{\ell}_2 \geq (9\pi - 36)/32$ . Moreover, a third bound appears,  $\bar{\ell}_1 + 3\bar{\ell}_2 \geq 0.91$ , changing the shape of the excluded region. Note that these are not the strongest bounds obtainable on  $\bar{\ell}_{1,2}$ . Stronger bounds can be obtained from dispersion relations in the unphysical ( $t \rightarrow 4m^2$ ) regime (see, e.g., [3]). But they do demonstrate the unreliability of the classical approximation. Neglecting the chiral loops is tantamount to making an additional assumption about the underlying UV theory, namely, that it is weakly-coupled. This is an unwarranted assumption about the nature of the UV physics since it cannot be justified from considerations of the low-energy effective theory alone.

Let us now consider the implications of our new bounds. Suppose that no light Higgs boson is found and the bounds are violated. There are then two logical possibilities. Either the cutoff is lower than expected,  $\Lambda \ll 4\pi v$ , and the subsequent power corrections, of order  $(s/\Lambda)^2$  invalidate the bounds, or the underlying theory has an  $S$  matrix which does not have the usual analytic properties we associate with causal, unitary theories. The former possibility is what we would expect if the underlying strong dynamics leading to electroweak symmetry breaking were a large- $N$  gauge theory. In the large- $N$  limit, the masses of resonances are suppressed by  $1/N$  (holding the confinement scale fixed) and, as such, the cutoff is effectively reduced. The masses of the resonances would have to be sufficiently light to invalidate the bounds. It is interesting to note that this is exactly the situation one would expect in a Randall-Sundrum scenario where the gravitational theory is dual to a large- $N$  gauge theory. In principle, one could retool the bounds in this case, by including the resonances in the effective theory, thus raising the cutoff scale. One could then test whether this new effective theory is the low-energy limit of a theory with an analytic  $S$ -matrix. In the absence of a light Higgs boson or other light resonances, a violation of the bound on  $(\hat{\alpha}_4, \hat{\alpha}_5)$  would indicate a breakdown of one or more basic properties of the  $S$ -matrix. The assumptions used in obtaining the dispersion relation are Lorentz invariance: the amplitude can only depend upon the three Mandelstam invariants. Analyticity and crossing: the cuts lie on the real axis, with no singularities on the physical sheet off the real axis. Unitarity: the imaginary part of the scattering amplitude along the cuts is positive. String theory, which is designed to be valid at all distance scales, is constructed to produce an  $S$ -matrix with precisely these properties. More generally, if the bounds are violated, whatever underlying dynamics is responsible for the electroweak chiral Lagrangian must not satisfy these basic properties of  $S$ -matrix theory. Theories which could violate the bound include those which violate Lorentz invariance [21], or unitarity [22].

There remains, however, the question of what energy scale the new physics (which violates one or more of the

above assumptions) enters and what the nature of that new physics is. It is tempting to assert that the scale of the unconventional new physics should not be too far above the cutoff of the electroweak chiral Lagrangian,  $\Lambda \sim 4\pi v$ . But, absent a better characterization of the nature of this new physics (which, by definition, differs from that of conventional quantum field theory or string theory), it would be hard to present a proof of that assertion.

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