**Radu *et al.* Reply:** In the preceding Comment [1], Mills raises the question of the applicability of the concept of Bose-Einstein condensation (BEC) of the magnon we used [2] when interpreting a magnetic phase transition observed in the quantum antiferromagnet $Cs_2CuCl_4$ at low temperature near the saturation magnetic field. Mills has correctly pointed out that while in statistical mechanics a canonical definition of BEC arises from considering low temperature properties of a bosonic system with a fixed particle number, this constant particle number constraint is not fulfilled in our study. Instead, in [2] an average magnon density is a function of an applied magnetic field (and temperature), which is regarded by us as a bare chemical potential of magnons. Such an assignment for the applied magnetic field is also questioned in [1].

Below we argue that the notion of magnon BEC used in our study [2] was in fact proposed a long time ago and conventionally accepted in the current literature, especially when treating field-induced quantum phase transitions in different classes of quantum antiferromagnets.

Matsubara and Matsuda [3] suggested to study basic properties of the superfluidity and $\lambda$ transition in liquid helium in the framework of a lattice model of hard-core bosons. They mapped the underlying bosonic model onto a quantum spin system and proved an equivalence between grand partition functions of both systems. In this study, the external magnetic field is recognized as a variable chemical potential of interacting bosons, which allows one to control a density of bosons and an appearance of a phase transition. Here, the off-diagonal long-range order of the BEC state is associated with a long-range order of the spin system. Since the magnetic field can be easily controlled, the theory suggested a new possibility for a study of BEC and $\lambda$-like transitions in the grand canonical ensemble with a tunable chemical potential.

Further development of the concept of magnon BEC as a field-induced phase transition in the Heisenberg antiferromagnet was provided by Batyev and Braginskii [4]. By treating the magnon excitations as hard-core bosons, they have shown that near the saturation field and at low temperature the strongly interacting bosons are in the limit of a dilute gas, so that the well-elaborated standard diagrammatic approach can be applied to take into account effects of boson interaction. This theory has been closely used by us [2] for a quantitative description of the phase boundary of the magnon BEC transition in $Cs_2CuCl_4$ that is the Heisenberg antiferromagnet with a sort of easy-plane anisotropy.

Field-induced magnetic ordering within the scenario of low-energy spin excitation BEC has been proposed and extensively studied by Affleck [5] for the haldane-gap antiferromagnet and by Giamarchi and Tsvelik [6] for coupled ladder spin systems. These theories gave rise to intensive experimental investigations of the BEC transition and its characteristic scaling laws in a variety of quantum antiferromagnets with the spin singlet ground state [7].

Finally, it is worth emphasizing that at zero magnetic field the spin ground states and excitation spectra in different classes of quantum antiferromagnets, studied in [2,7], differ dramatically. Nevertheless, their low temperature critical behavior near field-induced phase transitions (i.e., in close vicinity to corresponding quantum critical points) can be described within a unified picture of magnon BEC, which makes this concept a valuable tool of modern condensed matter physics.

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Received 18 April 2006; published 19 January 2007
DOI: 10.1103/PhysRevLett.98.039702
PACS numbers: 75.40.−s, 03.75.Nt, 75.30.Kz, 75.45.+j

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