Comment on "Can Two-Photon Correlation of Chaotic Light Be Considered as Correlation of Intensity Fluctuations?"

A recent Letter [1] presents an experiment of ghost imaging with chaotic light. We definitely disagree with two main claims of the accompanying theory. The first one is "the explanation in terms of statistical correlation of intensity fluctuations would not give an acceptable interpretation for this experiment." In the experiment of [1] (analogous to the ghost image experiment of [2]), a beam of chaotic light from a pseudothermal source is divided by a beam splitter (BS). Beam 1 probes an object at distance d_A from the source, and the light is collected into a bucket detector. Beam 2 is detected at a distance $d_B = d_A$ from the source. Two-photon coincidences are registered, whose rate is proportional to $G^2(\mathbf{x}_1, \mathbf{x}_2) = \langle I_1(\mathbf{x}_1) I_2(\mathbf{x}_2)\rangle$, I_i being the intensities of fields E_i and $\langle \cdots \rangle$ a statistical average. By using the (classical) Siegert relation for chaotic statistics and the BS transformation, we find (see, e.g., [3])

$$G^2(\mathbf{x}_1, \mathbf{x}_2) = \langle I_1(\mathbf{x}_1)\rangle\langle I_2(\mathbf{x}_2)\rangle + |\langle E_2^*(\mathbf{x}_1)E_1(\mathbf{x}_2)\rangle|^2. \quad (1)$$

Let I_1' be the intensity in arm 1 at the plane just before the object. Following [4], the source is described as a surface of roughness small on a wavelength scale, illuminated by a beam of transverse distribution $\langle I_s(\mathbf{x})\rangle$ of width D_s. By inserting the free Fresnel propagators and the BS relations, we get $\langle \delta I_1'(\mathbf{x}_1)\delta I_2(\mathbf{x}_2)\rangle = |\langle E_2^*(\mathbf{x}_2)E_1'(\mathbf{x}_1)\rangle|^2 \propto |\int d\mathbf{x}' \exp[i\frac{2\pi}{\lambda d_A}(\mathbf{x}_1 - \mathbf{x}_2)\cdot \mathbf{x}']\langle I_s(\mathbf{x}')\rangle|^2$. This well-known result [4] implies that the two intensities at the object plane (arm 1) and at the detection plane (arm 2) are correlated over a length $\Delta x \simeq \lambda d_A/D_s$. For the distances in [1], $d_A = 139$ mm, and taking, e.g., $D_s \approx 5$ mm, we get $\Delta x \approx 17\ \mu$m, which is much smaller than the object size. Hence, to a good approximation, the intensity fluctuations of the two beams are δ correlated in space: $\langle \delta I_1'(\mathbf{x}_1)\delta I_2(\mathbf{x}_2)\rangle \simeq C\delta(\mathbf{x}_1 - \mathbf{x}_2)$. The object is described by its transmission function, so that in the plane beyond it $\delta I_1(\mathbf{x}) = |T(\mathbf{x})|^2 \delta I_1'(\mathbf{x})$. By performing a bucket detection in arm 1, and an average over the product of intensity fluctuations, one measures $\int d\mathbf{x}_1 \langle \delta I_1(\mathbf{x}_1)\delta I_2(\mathbf{x}_2)\rangle \propto \int d\mathbf{x}_1 |T(\mathbf{x}_1)|^2 \delta(\mathbf{x}_1 - \mathbf{x}_2) = |T(\mathbf{x}_2)|^2$. Hence the presence of intensity correlations before the object perfectly explains the appearance of the object image.

The second claim we disagree with is "two-photon correlation phenomena have to be described quantum mechanically, regardless if the source of radiation is classical or quantum" [1]. This claim is based on the result in Eq. (8) of [1], and follows from the highly nonclassical model the authors assume to describe their light; see Eq. (5) of [1]. However, the same Eq. (8) can be obtained by describing the light within a classical stochastic formalism. Following [1], we model the source as an incoherent superposition of plane waves: $\langle E_s^*(\mathbf{q})E_s(\mathbf{q}')\rangle = \langle I_s(\mathbf{q})\rangle \delta(\mathbf{q} - \mathbf{q}')$, $\mathbf{q}$ being the transverse wave vector. By using the relation between the fields at the source and detection planes given by propagators h_i, $E_i(\mathbf{x}_i) = \int d\mathbf{q}_i h_i(\mathbf{x}_i, \mathbf{q}_i)E_s(\mathbf{q}_i)$, we recast Eq. (1) as

$$\begin{aligned} G^2(\mathbf{x}_1, \mathbf{x}_2) = & \int d\mathbf{q}_1 |h_1(\mathbf{x}_1, \mathbf{q}_1)|^2 \langle I_s(\mathbf{q}_1)\rangle \\ & \times \int d\mathbf{q}_2 |h_2(\mathbf{x}_2, \mathbf{q}_2)|^2 \langle I_s(\mathbf{q}_2)\rangle \\ & + \left| \int d\mathbf{q}_1 h_2^*(\mathbf{x}_2, \mathbf{q}_1) h_1(\mathbf{x}_1, \mathbf{q}_1)\langle I_s(\mathbf{q}_1)\rangle \right|^2 \quad (2) \end{aligned}$$

$$\begin{aligned} = & \frac{1}{2}\int d\mathbf{q}_1 \int d\mathbf{q}_2 |h_1(\mathbf{x}_1, \mathbf{q}_1)h_2(\mathbf{x}_2, \mathbf{q}_2) \\ & + h_1(\mathbf{x}_1, \mathbf{q}_2)h_2(\mathbf{x}_2, \mathbf{q}_1)|^2 \langle I_s(\mathbf{q}_1)\rangle\langle I_s(\mathbf{q}_2)\rangle. \quad (3) \end{aligned}$$

Equation (3) is basically identical to Eq. (8) of [1]. The term $h_1(\mathbf{x}_1, \mathbf{q}_1)h_2(\mathbf{x}_2, \mathbf{q}_2) + h_1(\mathbf{x}_1, \mathbf{q}_2)h_2(\mathbf{x}_2, \mathbf{q}_1)$ was interpreted in [1] as a superposition of possibilities for photon paths. In our classical formalism, this term can be ascribed to the mutual phase coherence between pairs of modes (in arms 1 and 2, respectively) with the same $\mathbf{q}$. Because of spatial incoherence each mode $\mathbf{q}$ has chaotic and independent fluctuations; however, modes with the same $\mathbf{q}$ in the two arms have correlated phase fluctuations, because the BS transformation imposes a precise phase relation to the outgoing beams. This is the mechanism which allows the second term on the right-hand side of Eq. (2) to be nonzero, and is at the origin of the "superposition" term in Eq. (3). To derive Eq. (3), we used only the Siegert relation and the BS transformation: thus, any implementation of thermal ghost imaging, ranging from the low-intensity regime of [1] to the bright beams used in [2], has a very natural description in terms of classical coherence of radiation.

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