Out-of-Plane Spin Polarization from In-Plane Electric and Magnetic Fields

Hans-Andreas Engel, Emmanuel I. Rashba, and Bertrand I. Halperin
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
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We show that the joint effect of spin-orbit and magnetic fields leads to a spin polarization perpendicular to the plane of a homogeneous two-dimensional electron system with Rashba spin-orbit coupling and in-plane parallel dc magnetic and electric fields, for angle-dependent impurity scattering or nonparabolic energy spectrum, while only in-plane polarization persists for simplified models. We derive Bloch equations, describing the main features of recent experiments, including the magnetic field dependence of static and dynamic responses.

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Generating spin populations at a nanometer scale is one of the central goals of spintronics [1]. Using spin-orbit interaction promises electrical control, allowing to integrate spin generation and manipulation into the traditional architecture of electronic devices. Bulk spin polarization, driven by electron drift in an electric field, was predicted long ago for noncentrosymmetric three- (3D) and two-dimensional (2D) systems [2–7]. In 2D, the polarization is in-plane, typically along the effective spin-orbit field $\mathbf{b}_{\rm dr} = \langle \mathbf{b}_{\rm SO}(\mathbf{k})\rangle \neq 0$, obtained by averaging spin-orbit coupling over the distribution of electron momenta $\hbar\mathbf{k}$ [8]. In-plane polarization components were observed recently in p-GaAs heterojunctions [9], quantum wells [10], and strained n-InGaAs films [11]. Below, we propose a mechanism for out-of-plane spin polarization generated in a homogeneous system by applying an in-plane magnetic field $\mathbf{B}$. This perpendicular polarization allows efficient optical access, e.g., via Kerr rotation. We find that the use of an average field $\mathbf{b}_{\rm dr}$ is not always valid. Naively, one might consider the system as being subject to a total in-plane field $\langle \mathbf{b}\rangle$, given by the sum of $\mathbf{B}$ and $\mathbf{b}_{\rm dr}$, see Fig. 1(a). In steady state, one then expects electrons to be polarized along this total field; in particular, no polarization perpendicular to the ($\mathbf{b}_{\rm dr}$, $\mathbf{B}$) plane. Algebraic addition of these fields worked well in describing Hanle precession of optically oriented 2D electrons in GaAs [12]. However, Kato *et al.* [11] reported a perpendicular spin polarization, which is incompatible with such a naive picture, and emphasized the need of identifying its microscopic mechanisms. A similar polarization was found in ZnSe [13].

The out-of-plane spin polarization in homogeneous systems, which we consider here, may be contrasted with the "spin Hall effect," which can generate out-of-plane polarization only near sample edges [14] or in inhomogeneous systems. That effect can arise when an electric field induces a transverse spin current in the bulk of the sample [15,16]. However, it turns out that there is no bulk spin current, when $\mathbf{B} = \mathbf{0}$, for the linear-in-momentum Rashba spin-orbit coupling, considered here [17–24]. This result holds even in the case of anisotropic impurity scattering [25].

In this Letter, we develop a theory describing the interplay between spin-orbit interaction and external electric and magnetic fields in the presence of impurity scattering, and demonstrate that the concept of average spin-orbit field is subject to severe restrictions. The naive expectation turns out to be correct only in the special case of parabolic bands and isotropic impurity scattering. However, as we show below, for anisotropic scattering (e.g., small-angle scattering), correlations in $\mathbf{k}$ result in a more complex structure of the distribution function and an out-of-plane spin polarization. Concretely, the Bloch equation contains a generation term proportional to $\mathbf{b}_{\rm dr} \times \mathbf{B}$ whose magnitude is controlled by anisotropy of potential scattering and nonparabolicity of the energy spectrum. Remarkably, while anisotropic scattering does not change the symmetry of the Hamiltonian, a perpendicular polarization would also be allowed by symmetry in the special case, but it is absent due to a cancellation. Our results give a microscopic explanation of experiments [11] and provide a novel

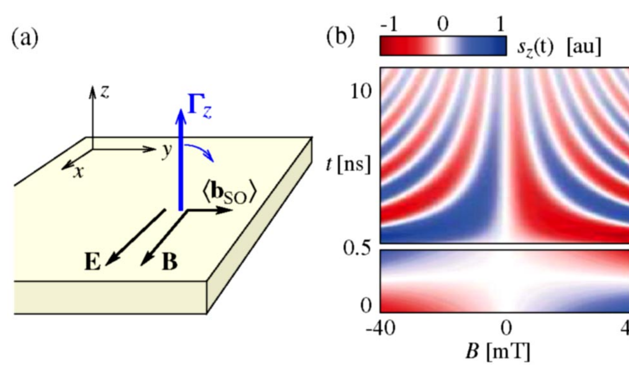

FIG. 1 (color online). (a) Field geometry for $g\mu_B > 0$, $\alpha < 0$. Out-of-plane spin polarization is electrically generated with rate Γ_z due to interplay of spin-orbit interaction, external electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$, and anisotropic impurity scattering. The polarization precesses (curved arrow) in $g^*\mu_B\mathbf{B} + \langle \mathbf{b}_{\rm SO}\rangle$. (b) Dynamics of out-of-plane component of polarization generated by a short electrical pulse of length $t_p \lesssim 1$ ns, for $\gamma_0 = 0.03$, $g^* = 0.65$ and $\tau_z = 5$ ns. This pattern is in agreement with the experimental data of Fig. 4(c) in Ref. [11].

mechanism for generating spin polarization electrically via spin-orbit interaction.

We consider a model of 2D electrons with charge $e < 0$ and (pseudo-) spin $\frac{1}{2}$, obeying a Hamiltonian

$$H = \epsilon_k - \frac{1}{2}\mathbf{b}(\mathbf{k}) \cdot \boldsymbol{\sigma} + V(\mathbf{r}), \tag{1}$$

where ϵ_k is the dispersion law in the absence of spin-orbit coupling, $V(\mathbf{r})$ is the potential due to impurities, $V_i(\mathbf{r})$, plus a small electric field $\mathbf{E}$, $\boldsymbol{\sigma}$ are the Pauli spin matrices, and $\mathbf{b}(\mathbf{k})$ includes both intrinsic spin-orbit field $\mathbf{b}_{\mathrm{SO}}(\mathbf{k})$ and in-plane external field $\mathbf{B}$. We disregard electron-electron interaction. In the following, we study the spin polarization density $\mathbf{s}(\mathbf{r}) = \langle \boldsymbol{\sigma} \rangle n_{2\mathrm{D}}$. Here, $n_{2\mathrm{D}}$ is the electron density and we set $\hbar = 1$.

Kinetic equation.—For a bulk 2D system with only intrinsic spin-orbit interaction, the kinetic equation has been derived [6,26,27]. Following Ref. [27], we may write a spin-dependent Boltzmann equation for the distribution function, represented as a 2×2 spin matrix $\hat{f} = \hat{f}_0(\mathbf{k}) + \frac{1}{2} f_c(\mathbf{k}) \mathbb{1} + \mathbf{f}(\mathbf{k}) \cdot \boldsymbol{\sigma}$, with equilibrium distribution function $\hat{f}_0$, excess particle density f_c, wave vector $\mathbf{k} = (k_x, k_y) = (k \cos\varphi, k \sin\varphi)$, and spin polarization density described by $\mathbf{f}$. Magnetic field and spin-orbit coupling split the energy spectrum into two branches: for a given energy ϵ, there are two Fermi surfaces. Thus, for elastic scattering, energy ϵ is conserved but $|\mathbf{k}|$ is not, e.g., due to interbranch scattering. Instead of using the distribution function $\hat{f}(\mathbf{k})$ as density in $\mathbf{k}$ space, we consider it as a function of energy ϵ and direction φ in $\mathbf{k}$ space. In this representation, $f_c(\mathbf{k}, \epsilon)$ and $\mathbf{f}(\mathbf{k}, \epsilon)$ are transformed into distribution functions $n(\varphi, \epsilon)$ and $\boldsymbol{\Phi}(\varphi, \epsilon)$, respectively; for a detailed derivation, see Ref. [27]. The spin-dependent part of the kinetic equation for $\mathbf{E} = E\hat{\mathbf{x}}$ is [27]

$$\frac{\partial \boldsymbol{\Phi}}{\partial t} + \mathbf{b} \times \left(\boldsymbol{\Phi} - \frac{n}{4 v \epsilon} \frac{\partial \mathbf{b}}{\partial k} \right) - \frac{\beta}{2} \frac{\partial}{\partial \varphi} (\mathbf{b} \sin\varphi) = \left(\frac{\partial \boldsymbol{\Phi}}{\partial t} \right)_{\mathrm{coll.}} \tag{2}$$

The second term on the left-hand side (lhs) of Eq. (2) describes the spin precession in the momentum dependent field $\mathbf{b}$, and the third term is the driving term, given in lowest order in $\mathbf{E}$ with $\beta = (eE/16\pi^2 v_\epsilon)(-\partial f_0/\partial \epsilon)$ and Fermi distribution function f_0. Equation (2) was derived for $b \ll E_F$ and should be evaluated at $k = k_\epsilon$, where k_ϵ is such that $\epsilon_{k_\epsilon} = \epsilon$ and the spin-independent velocity contribution is $v_\epsilon = \epsilon'_k$. Further, the kinetic equation for the charge distribution is the same as for $b = 0$, with solution $n = 8\beta\tau v_\epsilon k \cos\varphi$, where τ is the transport lifetime. The collision integral on the right-hand side (rhs) of Eq. (2) was found in Born approximation by golden rule. In the absence of $\mathbf{b}$, it is given by the usual relaxation term $\int_0^{2\pi} d\varphi' K(\vartheta)[\boldsymbol{\Phi}(\varphi') - \boldsymbol{\Phi}(\varphi)]$, with kernel $K(\vartheta) = \langle |V_i(\mathbf{q})|^2 \rangle k / 2\pi v_\epsilon$, scattering angle $\vartheta = \varphi' - \varphi$, and momentum transfer $q = 2k \sin(|\vartheta|/2)$. Coupling of spins via $\mathbf{b}$ leads to two corrections to the collision integral, arising from the spin dependences of the density of states and momentum transfer for a fixed energy ϵ. These contributions are proportional to $K(\vartheta)$ and $\tilde{K}(\vartheta) \equiv (dK/d\vartheta) \times \tan(\vartheta/2)$, respectively [27,28]. Like the third term on the lhs of Eq. (2), these are source terms, proportional to β, which do not involve $\boldsymbol{\Phi}$. Note that $\tilde{K}$ is a distinctive feature of anisotropic scattering.

We now consider Rashba spin-orbit interaction, and choose the x axis along the field $\mathbf{B}$, i.e.,

$$\mathbf{b}(\mathbf{k}) = 2\alpha \hat{\mathbf{z}} \times \mathbf{k} + \Delta_x \hat{\mathbf{x}}, \qquad \Delta_x = g^* \mu_B B, \tag{3}$$

with Zeeman splitting Δ_x. Thus, $\mathbf{b}(\mathbf{k})$ is in-plane and $\mathbf{E}$ and $\mathbf{B}$ are parallel, see Fig. 1(a). (For $\mathbf{E} = E\hat{\mathbf{y}}$ there is yz mirror symmetry and s_z vanishes. Therefore, the s_z term linear in $\mathbf{E}$ is determined only by the component E_x parallel to $\mathbf{B}$.)

The effective field $\mathbf{b}(\mathbf{k})$ for a 2DEG with pure linear Dresselhaus coupling, on the (001) surface of a III-V material, is obtained by replacing $\mathbf{k}$ on the rhs of Eq. (3) by $\mathbf{q} \equiv \mathcal{R}\mathbf{k}$, where $\mathcal{R}$ denotes reflection through the (110) crystal plane. Our results for the polarization s_z (see below) can be applied to this case if we replace E_x by the component of the electric field along the direction $\mathbf{B}' \equiv \mathcal{R}\mathbf{B}$. For general forms of the spin-orbit coupling, we note that the C_{2v} symmetry of the system ensures that if $B = 0$, there can be no term in s_z linear in $\mathbf{E}$. However, there could be terms nonlinear in $\mathbf{E}$, if $\mathbf{E}$ is not parallel to a symmetry direction [110] or [1$\bar{1}$0], e.g., $s_z \propto E_x^2 - E_y^2$ where x refers to the [100] crystal axis, which would then give an all-electrical mechanism for generating out-of-plane spin polarization.

Next, we write the kinetic Eq. (2) in Fourier space by expanding the azimuthal dependence as $f(\varphi) = \sum_{m=-\infty}^{\infty} e^{im\varphi} f_m$. We assume that $\mathbf{B}$ is time independent and any time dependence of $\mathbf{E}$ is slow compared to τ^{-1}. Combining the in-plane spin distribution as $\Phi^x(\varphi) + i\Phi^y(\varphi) = \sum_m e^{im\varphi} \Psi_m$, and inserting Eq. (2) we find [28]

$$\dot{\Psi}_m = i\Delta_x \Phi_m^z - 2\alpha k \Phi_{m-1}^z + 4ik_2 \alpha k \beta \delta_{m,2} + \Delta_x \beta (1 + \gamma_0) \delta_{|m|,1} - \tau^{-1} k_m \Psi_m, \tag{4}$$

$$\dot{\Phi}_m^z = i\frac{\Delta_x}{2}(\Psi_m - \Psi_{-m}^*) + \alpha k (\Psi_{1+m} + \Psi_{1-m}^*) + \alpha k \Delta_x \tau \beta (2\delta_{m,0} + \delta_{|m|,2}) - \tau^{-1} k_m \Phi_m^z, \tag{5}$$

with inverse transport time $\tau^{-1} = 2\pi(K_0 - K_1)$ and $k_m = (K_0 - K_m)/(K_0 - K_1)$. Finally, we define

$$\gamma_0 = \zeta + 2\frac{\tilde{K}_1 - \tilde{K}_0}{K_0 - K_1}, \tag{6}$$

parametrizing how scattering in the magnetic field leads to spin-dependent corrections to the collision integral. The first term in Eq. (5) is the band nonparabolicity $\zeta = (k/v_\epsilon)(\partial v_\epsilon / \partial k) - 1$, i.e., $\zeta = 0$ for parabolic bands. The second term quantifies the effect of anisotropic scattering. In the limit of small-angle scattering, we find $\gamma_0 = \zeta + 3$.

Isotropic scattering, parabolic bands, stationary regime.—First, we assume isotropic scattering and parabolic bands, thus $k_m = 1 - \delta_{m,0}$, $\tilde{K}_m = 0$, and $\gamma_0 = 0$. In this regime, we solve the kinetic equations (3) and (4) exactly in the static case $\partial\mathbf{\Phi}/\partial t = 0$. The stationary solution is

$$\Psi_m^{\rm is} = \Delta_x\tau\beta\delta_{|m|,1} + 4i\alpha k\tau\beta(\delta_{m,0} + \delta_{m,2}), \qquad (7)$$

$$\Phi_m^{z,\rm is} = 0, \qquad (8)$$

which can be checked by inspection. The total spin polarization density is $\mathbf{s} = \mathbf{s}^{\rm eq} + \mathbf{s}^E$, with equilibrium contribution $\mathbf{s}^{\rm eq} \parallel \mathbf{B}$ and nonequilibrium contribution $s_\mu^E = 4\pi\int d\epsilon\Phi_0^\mu$. Thus, the out-of-plane polarization vanishes, $s_z = 0$, as one would expect from the above naive argument—even though the symmetry allows $s_z \neq 0$. Hence, vanishing s_z is a property of the specific model of isotropic scattering and parabolic bands.

Now we are in a position to develop a physical picture of the different mechanisms generating spin polarization. Polarization s_x arises from Pauli paramagnetism $s_x^{\rm eq} = \frac{1}{2}\nu\Delta_x$, with the density of states $\nu = k_F/\pi v_F = m^*/\pi$, Fermi momentum k_F, Fermi velocity v_F, and effective mass m^*. This spin polarization does not depend on the electric field, thus $\Phi_0^x = 0$. However, the electric field causes drift, producing an average spin-orbit splitting, $b_{\rm dr}^y = \langle b_y\rangle = (1/n_{\rm 2D})\iint d\epsilon d\varphi n b_y = 2\alpha e E_x\tau$. From Eq. (6), we find $s_y^E = \alpha e E_x\tau\nu$, in agreement with results known for $B = 0$ [3,5–7,17]. Because $s_y^E = \frac{1}{2}\nu\langle b_y\rangle$, we can understand that s_y is produced by the drift field $\langle b_y\rangle$, in analogy to Pauli paramagnetism. Therefore, for isotropic scattering and parabolic bands, the in-plane polarization can be described in terms of the average spin-orbit field. Remarkably, even for this model, our solution is $\mathbf{\Phi}(\varphi) = 4\tau\beta[\mathbf{b}(\varphi) - \frac{1}{2}\Delta_x\hat{\mathbf{x}}]\cos\varphi$, i.e., in addition to the total field $\mathbf{b}$, there is a correction $-\frac{1}{2}\Delta_x\hat{\mathbf{x}}$. Thus in $\mathbf{\Phi}(\varphi)$ the spin-orbit and external magnetic fields cannot be added. However, this correction does not contribute to the spin polarization.

The vanishing of the perpendicular polarization s_z results from a cancellation of precession terms, as given by Eq. (4), for $m = 0$. The first term on the rhs describes the precession of s_y (induced by $b_{\rm dr}^y$) in the magnetic field $\mathbf{B}$, producing an out-of-plane polarization. However, this polarization is exactly canceled by the second and third terms, which describe the precession of the equilibrium polarization (induced by $\mathbf{B}$) in the field $\mathbf{b}_{\rm dr} \propto \alpha k$. Also, note that the third term in Eq. (5), which results from the $\mathbf{b}\times(\partial\mathbf{b}/\partial k)$ contribution to the kinetic Eq. (2), is important for the cancellation, and, in particular, does not lead to a finite s_z.

Anisotropic scattering and Bloch equation.—Now we consider anisotropic scattering and/or nonparabolic bands, and also include transients. We consider the "dirty limit," $2|\alpha|k \ll \tau^{-1} \ll E_F$ with constant $\mathbf{B}$ such that

$$|\omega|, \tau_z^{-1}, |\Delta_x| \ll 2|\alpha|k \ll \tau^{-1}. \qquad (9)$$

Here, ω is the characteristic frequency of the field $\mathbf{E}$, and $\tau_z = \frac{1}{2}\tau_{xy} = \tau^{-1}(2\alpha k)^{-2}$ are the Dyakonov-Perel spin relaxation times. In this regime, Φ_m^z and Ψ_m decay exponentially fast with increasing $|m|$, since $\Phi_m^z/\Phi_{m-1}^z \sim \tau^2|\alpha|k\Delta_x \ll 1$ for $m \geq 2$, and similarly for Ψ_m. This allows us to solve kinetic equations (3) and (4) order-by-order in the small parameter $(\tau/\tau_z)^{1/2}$. In lowest nonvanishing order, it is sufficient to retain only equations for $|m| \leq 2$. Eliminating the $m = \pm 1, \pm 2$ components yields the equations of motion for $\mathbf{\Phi}$ up to order $(\tau/\tau_z)^{1/2}$ [28].

Finally, we evaluate the equations of motion for the total polarization $\mathbf{s}$ at low temperature T, taking all parameters at the Fermi level. We obtain the *Bloch equation*

$$\dot{\mathbf{s}} = \langle\mathbf{b}\rangle\times\mathbf{s} - \overleftrightarrow{\tau}_s^{-1}\mathbf{s} + \mathbf{\Gamma}, \qquad (10)$$

where the spin relaxation tensor $\overleftrightarrow{\tau}_s^{-1}$ is diagonal with components $\{\tau_{xy}^{-1}, \tau_{xy}^{-1}, \tau_z^{-1}\}$ and

$$\mathbf{\Gamma} = (\tfrac{1}{2}\nu\Delta_x\tau_{xy}^{-1}, \tfrac{1}{2}\nu\langle b_y\rangle\tau_{xy}^{-1}, \tfrac{1}{4}\nu\Delta_x\langle b_y\rangle\gamma_0). \qquad (11)$$

Note that our proof of Eq. (10) is valid only in linear order in $\mathbf{E}$ [cf. Eq. (2)], i.e., products $\langle b_y\rangle s_\mu^E$ were disregarded.

To develop a physical picture for this central result, we note that Eq. (10) is a Bloch equation, where polarization $\mathbf{s}$ is generated with a rate Γ and then precesses in the total field $\langle\mathbf{b}\rangle = g^*\mu_B\mathbf{B} + \mathbf{b}_{\rm dr}$ (Hanle effect). What is remarkable is that for anisotropic scattering and/or band nonparabolicity, the combined effect of spin-orbit and external fields generates a component of spin polarization along the z axis with rate $\Gamma_z = \frac{1}{4}\nu g^*\mu_B(\mathbf{B}\times\langle\mathbf{b}_{\rm SO}\rangle)_z\gamma_0 = \frac{1}{2}\nu\alpha e E_x\tau\Delta_x\gamma_0$, i.e., perpendicular to both magnetic and spin-orbit fields. The physical mechanism for this may be understood as follows. Because of the Zeeman field, there will be different Fermi radii, $k_\uparrow$ and $k_\downarrow$, for spins aligned parallel and antiparallel to the x direction, in the absence of spin-orbit coupling. The electric field causes a net drift in the x direction, and in the case of anisotropic scattering, the scattering rates for $k_\uparrow$ and $k_\downarrow$ will be different, because of different momentum transfers. This leads to a φ-dependent x polarization, which is given by the term proportional to γ_0 in Eq. (4). [The similar term $\Delta_x\beta\delta_{|m|1}$ is not due to scattering; it describes the acceleration in the electric field and contributes to the cancellation explained above.] On a time scale of τ, this polarization then precesses around the y component of $\mathbf{b}_{\rm SO}$, as described by the second term of Eq. (5). Because $b_y \propto k_x$, the precession frequency depends on φ; so if the spins initially aligned along $\mathbf{B}$ precess faster (say, because they are predominantly scattered in the forward direction) than the antialigned spins, these two precession contributions do not cancel (even when averaged over φ) and a finite s_z polarization is produced.

Next we consider the dc case $\dot{\mathbf{s}} = 0$. In the lowest order in $\mathbf{E}$, the total spin polarization is $s_x = \frac{1}{2}\nu\Delta_x$,

$$s_y = \frac{1}{2}\nu\alpha e E_x\tau\left[2 + \frac{\Delta_x^2\tau_{xy}\tau_z}{1 + \Delta_x^2\tau_{xy}\tau_z}\gamma_0\right], \qquad (12)$$

$$s_z = \frac{1}{2}\nu\alpha e E_x \tau \frac{\Delta_x \tau_z}{1 + \Delta_x^2 \tau_{xy} \tau_z} \gamma_0. \quad (13)$$

The first term of Eq. (12) arises from Eq. (7), while the second term and s_z are due to anisotropic scattering or nonparabolic bands. The dependence of s_z on Δ_x is in agreement with the data in Fig. 1(c) of Ref. [11], where $\tau_s = (\tau_{xy}\tau_z)^{1/2} \approx 5$ ns, suggesting that our microscopic model might explain the experimental observations.

Spin dynamics.—Even for isotropic scattering $\gamma_0 = 0$, a time-dependent electric field leads to an out-of-plane polarization $s_z(\omega) = \frac{i}{2}\omega\nu\Delta_x b_{\rm dr}^y(\omega)/[\Delta_x^2 - \omega^2 + \tau_s^{-2} - i(\tau_{xy}^{-1} + \tau_z^{-1})\omega]$; however, it has no static component $s_z(0)$. Similar results were found for $|\alpha|k \ll \Delta_x, \tau^{-1}$ [29].

Spin dynamics is accessible in a pump-probe scheme [11]. Namely, spins can be pumped by applying a short electric pulse of duration $t_p \ll \tau_z, \Delta_x^{-1}$. Then, according to Eq. (10), the spin polarization immediately after the pulse is $s_z(0) = t_p\Gamma_z \propto \Delta_x\gamma_0$, i.e., $s_z(0)$ is an odd function of Δ_x. Solving the Bloch equation (10), we get

$$s_z(t) = s_z(0)e^{-3t/4\tau_z}\left[\cos\Omega t - \frac{1 + 2/\gamma_0}{4\Omega\tau_z}\sin\Omega t\right] \quad (14)$$

with frequency $\Omega = \sqrt{(4\Delta_x\tau_z)^2 - 1}/4\tau_z$ of the Hanle oscillations (for consistency, we only consider terms linear in **E**). We plot $s_z(t)$ in Fig. 1(b), taking the parameters of Ref. [11] and with a choice of $\gamma_0 = 0.03$, and find qualitative agreement with the experiment. The experimental data show that the sign of s_z depends on the sign of Δ_x, already on time scales much shorter than $|\Delta_x|^{-1}$. Therefore, the sign of s_z cannot be due to spin precession in the external magnetic field, implying that a polarization generation mechanism such as the one described above was experimentally observed in Ref. [11].

Strictly speaking, quantitative comparison with the data of Ref. [11] cannot be performed because the films were of low mobility $E_F\tau \sim 1$, violating the assumptions of our Boltzmann description, and were in 3D regime (a coupling $k_y\sigma_x - k_x\sigma_y$ occurs here due to strain). Furthermore, in models with a more complicated spin-orbit interaction than the Rashba coupling, other sources of z polarization might become important. However, Eq. (9) was satisfied, because $\hbar/\tau_z \sim 3 \times 10^{-8}$ eV; $|\Delta_x| \lesssim 10^{-6}$ eV; $|b_\alpha| \sim 10^{-5}$ eV; and $\hbar/\tau \sim 2 \times 10^{-3}$ eV [11,30].

In conclusion, we proposed a mechanism for generating bulk spin populations polarized perpendicularly to magnetic and spin-orbit fields; for 2D systems this is an out-of-plane polarization. It relies on anisotropic impurity scattering and/or band nonparabolicity and provides a new method for electrical control of electron spins. Our model is derived for 2D systems, but the results should have a more general validity, and they agree with recent observations of combined effects of the external magnetic and spin-orbit fields in 3D samples.

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