P H Y S I C A L   R E V I E W   L E T T E R S

Optomechanical Entanglement between a Movable Mirror and a Cavity Field

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We show how stationary entanglement between an optical cavity field mode and a macroscopic vibrating mirror can be generated by means of radiation pressure. We also show how the generated optomechanical entanglement can be quantified, and we suggest an experimental readout scheme to fully characterize the entangled state. Surprisingly, such optomechanical entanglement is shown to persist for environment temperatures above 20 K using state-of-the-art experimental parameters.

Entanglement, "*the* characteristic trait of quantum mechanics" [1], has raised widespread interest in different branches of physics. It provides insight into the fundamental structure of physical reality [2], and it has become a basic resource for many quantum information processing schemes [3]. So far, entanglement has been experimentally prepared and manipulated using microscopic quantum systems such as photons, atoms, and ions [3,4]. Nothing in the principles of quantum mechanics prevents macroscopic systems from attaining entanglement. However, the answer to the question as to what extent entanglement should hold when going towards "classical" systems is yet unknown [5]. Therefore, it is of crucial importance to investigate the possibilities to obtain entangled states of macroscopic systems [6] and to study the robustness of entanglement against temperature [7]. Experiments in this direction include single-particle interference of macromolecules [8] and the demonstration of entanglement between collective spins of atomic ensembles [9] and of entanglement in Josephson-junction qubits [10]. Mechanical oscillators are of particular interest, since they resemble a prototype of classical systems. Thanks to the fast-developing field of microfabrication, micro- or nanomechanical oscillators can now be prepared and controlled to a very high precision [11]. In addition, several theoretical proposals exist that suggest how to reach the quantum regime for such systems [12]. Experimentally, quantum limited measurements have been developed that could allow ground state detection [13]. However, quantum effects in mechanical oscillators have not been demonstrated to date.

Optomechanical coupling via radiation pressure [14] is a promising approach to prepare and manipulate quantum states of mechanical oscillators. Proposals range from the quantum state transfer from light to a mechanical oscillator to entangling two such oscillators [15–18]. In this Letter, we propose an experimental scheme to create and probe optomechanical entanglement between a light field and a mechanical oscillator. This is achieved using a bright laser field that resonates inside a cavity and couples to the position and momentum of a moving (micro)mirror. The proposal is based on feasible experimental parameters in accordance with current state-of-the-art optics and microfabrication. In contrast to other proposals [15,18], it requires neither nonclassical states of light nor temperatures close to the oscillator's ground state. Entanglement is shown to persist above a temperature of 20 K. We begin by modeling the system and its coupling to the environment by using the standard Langevin formalism. Then we solve the dynamics and quantify the entanglement generated in the stationary state. Finally, we discuss a suitable experimental apparatus capable of measuring the entanglement.

We consider an optical Fabry-Perot cavity in which one of the mirrors is much lighter than the other, so that it can move under the effect of the radiation pressure force. The motion of the mirror is described by the excitation of several degrees of freedom which have different resonant frequencies. However, a single frequency mode can be considered when a bandpass filter in the detection scheme is used [19] and mode-mode coupling is negligible. Therefore, we will consider a single mechanical mode of the mirror only, which can be modeled as an harmonic oscillator with frequency w_m. The Hamiltonian of the system reads [20]

$$\mathcal{H} = \hbar w_c a^\dagger a + \frac{\hbar w_m}{2}(p^2 + q^2) - \hbar G_0 a^\dagger a q$$
$$+ i\hbar E(e^{-iw_0 t}a^\dagger - e^{iw_0 t}a), \qquad (1)$$

where q and p ($[q, p] = i$) are the dimensionless position and momentum operators of the mirror, respectively, a and $a^\dagger$ ($[a, a^\dagger] = 1$) are the annihilation and creation operators

of the cavity mode, respectively, with frequency w_c and decay rate κ, and $G_0 = (w_c/L)\sqrt{\hbar/mw_m}$ is the coupling coefficient, with L the cavity length in the absence of the intracavity field and m the effective mass of the mechanical mode [19]. The last two terms in Eq. (1) describe the driving laser with frequency w_0, and E is related to the input laser power P by $|E| = \sqrt{2P\kappa/\hbar w_0}$.

A proper analysis of the system must include photon losses in the cavity and the Brownian noise acting on the mirror. This can be accomplished by considering the following set of nonlinear Langevin equations, written in the interaction picture with respect to $\hbar w_0 a^\dagger a$:

$$\dot{q} = w_m p, \tag{2a}$$

$$\dot{p} = -w_m q - \gamma_m p + G_0 a^\dagger a + \xi, \tag{2b}$$

$$\dot{a} = -(\kappa + i\Delta_0)a + iG_0 aq + E + \sqrt{2\kappa}a^{in}, \tag{2c}$$

where $\Delta_0 = w_c - w_0$ and γ_m is the mechanical damping rate. We have introduced the vacuum radiation input noise a^{in}, whose only nonzero correlation function is [21]

$$\langle a^{in}(t)a^{in,\dagger}(t')\rangle = \delta(t - t'), \tag{3}$$

and the Hermitian Brownian noise operator ξ, with correlation function [22]

$$\langle \xi(t)\xi(t')\rangle = \frac{\gamma_m}{w_m}\int \frac{d\omega}{2\pi}e^{-i\omega(t-t')}\omega\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right) + 1\right] \tag{4}$$

(k_B is the Boltzmann constant and T is the mirror temperature). We can always rewrite each Heisenberg operator as a c-number steady-state value plus an additional fluctuation operator with zero-mean value, $a = \alpha_s + \delta a$, $q = q_s + \delta q$, $p = p_s + \delta p$. Inserting these expressions into the Langevin equations of Eqs. (2), these latter decouple into a set of nonlinear algebraic equations for the steady-state values and a set of quantum Langevin equations for the fluctuation operators [23]. The steady-state values are given by $p_s = 0$, $q_s = G_0|\alpha_s|^2/w_m$, $\alpha_s = E/(\kappa + i\Delta)$, where the latter equation is, in fact, a nonlinear equation determining the stationary intracavity field amplitude α_s, since the effective cavity detuning Δ, including radiation pressure effects, is given by $\Delta = \Delta_0 - G_0^2|\alpha_s|^2/w_m$. The parameter regime relevant for generating optomechanical entanglement is that with a very large input power P, i.e., when $|\alpha_s| \gg 1$. In this case, one can safely neglect the nonlinear terms $\delta a^\dagger \delta a$ and $\delta a \delta q$ and gets the linearized Langevin equations

$$\delta\dot{q} = w_m \delta p, \tag{5a}$$

$$\delta\dot{p} = -w_m\delta q - \gamma_m\delta p + G\delta X + \xi, \tag{5b}$$

$$\delta\dot{X} = -\kappa\delta X + \Delta\delta Y + \sqrt{2\kappa}X^{in}, \tag{5c}$$

$$\delta\dot{Y} = -\kappa\delta Y - \Delta\delta X + G\delta q + \sqrt{2\kappa}Y^{in}, \tag{5d}$$

where we have chosen the phase reference of the cavity field so that α_s is real, and we have defined the cavity field

quadratures $\delta X \equiv (\delta a + \delta a^\dagger)/\sqrt{2}$ and $\delta Y \equiv (\delta a - \delta a^\dagger)/i\sqrt{2}$ and the corresponding Hermitian input noise operators $X^{in} \equiv (a^{in} + a^{in,\dagger})/\sqrt{2}$ and $Y^{in} \equiv (a^{in} - a^{in,\dagger})/i\sqrt{2}$. What is relevant is that the quantum fluctuations of the field and the oscillator are now coupled by the much larger *effective* optomechanical coupling $G \equiv G_0\alpha_s\sqrt{2}$, so that the generation of significant optomechanical entanglement becomes possible.

When the system is stable, it reaches a unique steady state, independently of the initial condition. Since the quantum noises ξ and a^{in} are zero-mean quantum Gaussian noises and the dynamics is linearized, the quantum steady state for the fluctuations is a zero-mean bipartite Gaussian state, fully characterized by its 4×4 correlation matrix $V_{ij} = (\langle u_i(\infty)u_j(\infty) + u_j(\infty)u_i(\infty)\rangle)/2$, where $u^T(\infty) = (\delta q(\infty), \delta p(\infty), \delta X(\infty), \delta Y(\infty))$ is the vector of continuous variables (CV) fluctuation operators at the steady state ($t \to \infty$). Defining the vector of noises $n^T(t) = (0, \xi(t), \sqrt{2\kappa}X^{in}(t), \sqrt{2\kappa}Y^{in}(t))$ and the matrix

$$A = \begin{pmatrix} 0 & w_m & 0 & 0 \\ -w_m & -\gamma_m & G & 0 \\ 0 & 0 & -\kappa & \Delta \\ G & 0 & -\Delta & -\kappa \end{pmatrix}, \tag{6}$$

Eqs. (5) can be written in compact form as $\dot{u}(t) = Au(t) + n(t)$, whose solution is $u(t) = M(t)u(0) + \int_0^t ds M(s)n(t - s)$, where $M(t) = \exp\{At\}$. The system is stable and reaches its steady state when all of the eigenvalues of A have negative real parts so that $M(\infty) = 0$. The stability conditions can be derived by applying the Routh-Hurwitz criterion [24], yielding the following two nontrivial conditions on the system parameters:

$$2\gamma_m\kappa[\Delta^4 + \Delta^2(\gamma_m^2 + 2\gamma_m\kappa + 2\kappa^2 - 2w_m^2) + (\gamma_m\kappa + \kappa^2 + w_m^2)^2] + w_mG^2\Delta(\gamma_m + 2\kappa)^2 > 0, \tag{7a}$$

$$w_m^2(\Delta^2 + \kappa^2) - w_mG^2\Delta > 0, \tag{7b}$$

which will be considered to be satisfied from now on. When the system is stable, one gets

$$V_{ij} = \sum_{k,l}\int_0^\infty ds\int_0^\infty ds' M_{ik}(s)M_{jl}(s')\Phi_{kl}(s - s'), \tag{8}$$

where $\Phi_{kl}(s - s') = (\langle n_k(s)n_l(s') + n_l(s')n_k(s)\rangle)/2$ is the matrix of the stationary noise correlation functions. Due to Eq. (4), the mirror Brownian noise $\xi(t)$ is not delta-correlated and therefore does not describe a Markovian process [22]. However, quantum effects are achievable only using oscillators with a large mechanical quality factor $Q = w_m/\gamma_m \gg 1$. In this limit, $\xi(t)$ becomes delta-correlated [25]:

$$\langle \xi(t)\xi(t') + \xi(t')\xi(t)\rangle/2 \simeq \gamma_m(2\bar{n} + 1)\delta(t - t'), \tag{9}$$

where $\bar{n} = (\exp\{\hbar w_m/k_BT\} - 1)^{-1}$ is the mean thermal excitation number, and one recovers a Markovian process. As a consequence, and using the fact that the three com-

ponents of $n(t)$ are uncorrelated, we get $\Phi_{kl}(s - s') = D_{kl}\delta(s - s')$, where $D = \text{Diag}[0, \gamma_m(2\bar{n} + 1), \kappa, \kappa]$ is a diagonal matrix, and Eq. (8) becomes $V = \int_0^\infty ds M(s)DM(s)^T$. When the stability conditions are satisfied, $M(\infty) = 0$ and one gets the following equation for the steady-state correlation matrix (CM):

$$AV + VA^T = -D. \tag{10}$$

Equation (10) is a linear equation for V and can be straightforwardly solved, but the general exact expression is too cumbersome and will not be reported here. In order to establish the conditions under which the optical mode and the mirror vibrational mode are entangled, we consider the logarithmic negativity $E_{\mathcal{N}}$, a quantity which has been already proposed as a measure of entanglement [26]. In the CV case, $E_{\mathcal{N}}$ can be defined as [27]

$$E_{\mathcal{N}} = \max[0, -\ln 2\eta^-], \tag{11}$$

where $\eta^- \equiv 2^{-1/2}\{\Sigma(V) - [\Sigma(V)^2 - 4\det V]^{1/2}\}^{1/2}$, with $\Sigma(V) \equiv \det A + \det B - 2\det C$, and we have used the 2×2 block form of the CM

$$V \equiv \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}. \tag{12}$$

Therefore, a Gaussian state is entangled if and only if $\eta^- < 1/2$, which is equivalent to Simon's necessary and sufficient entanglement nonpositive partial transpose criterion for Gaussian states [28], which can be written as $4\det V < \Sigma - 1/4$.

We have made a careful analysis in a wide parameter range and found a parameter region very close to that of recently performed optomechanical experiments [29], for which a significant amount of entanglement is achievable. Figure 1 shows $E_{\mathcal{N}}$ versus the normalized detuning Δ/w_m for two different masses, 5 and 50 ng: Optomechanical entanglement is present only within a finite
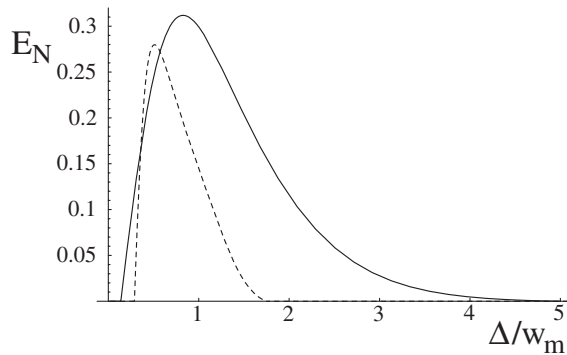

FIG. 1.   Plot of the logarithmic negativity $E_{\mathcal{N}}$ as a function of the normalized detuning Δ/w_m in the case of an optical cavity of length $L = 1$ mm, driven by a laser with wavelength 810 nm and power $P = 50$ mW. The mechanical oscillator has a frequency $w_m/2\pi = 10$ MHz, a damping rate $\gamma_m/2\pi = 100$ Hz, and its temperature is $T = 400$ mK. The solid line refers to a mass $m = 5$ ng and finesse $\mathcal{F} = 1.07 \times 10^4$, while the dashed line refers to a mass $m = 50$ ng and finesse $\mathcal{F} = 3.4 \times 10^4$.

interval of values of Δ around $\Delta \simeq w_m$. The robustness of such an entanglement with respect to the mirror's environmental temperature is shown in Fig. 2. The relevant result is that for the 5 ng mirror optomechanical entanglement persists for temperatures above 20 K, which is several orders of magnitude larger than the ground state temperature of the mechanical oscillator. For the 50 ng mirror, entanglement vanishes at lower temperatures (Fig. 2). Figures 1 and 2 refer to $Q = 10^5$, but we found that entanglement persists even for $Q \simeq 10^4$, although it becomes much less robust against temperature. In this case, entanglement persists up to 3 (1) K for a 5 (50) ng mirror.

We finally discuss the experimental detection of the generated optomechanical entanglement. In order to measure $E_{\mathcal{N}}$ at the steady state, one has to measure all ten independent entries of the correlation matrix V. This has been recently experimentally realized [30] for the case of two entangled optical modes at the output of a parametric oscillator. In our case, the measurement of the field quadratures of the cavity mode can be straightforwardly performed by homodyning the cavity output using a local oscillator with an appropriate phase. Measuring the mechanical mode is less straightforward. However, if we consider a second Fabry-Perot cavity C_2, adjacent to the first one and formed by the movable mirror and a third fixed mirror (see Fig. 3), it is possible to adjust the parameters of C_2 so that both the position and the momentum of the mirror can be measured by homodyning the C_2 output. In fact, assuming that the movable mirror has unit reflectivity at both sides so that there is no light coupling the two cavities, the annihilation operator of the second cavity a_2 obeys an equation analogous to the linearized version of Eq. (2c),

$$\delta\dot{a}_2 = -(\kappa_2 + i\Delta_2)\delta a_2 + iG_2\alpha_2\delta q + \sqrt{2\kappa_2}a_2^{\text{in}}(t), \tag{13}$$

where κ_2, Δ_2, α_2, and $a_2^{\text{in}}(t)$ are the bandwidth, the effective detuning, the intracavity field amplitude, and the input noise of C_2, respectively. Moreover, $G_2 = (w_{c2}/L_2) \times \sqrt{\hbar/mw_m}$, where w_{c2} and L_2 are the frequency and the
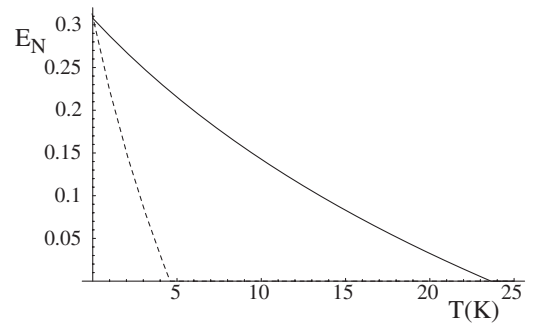

FIG. 2.   Plot of the logarithmic negativity $E_{\mathcal{N}}$ versus the mirror temperature. The solid line refers to a mass $m = 5$ ng, detuning $\Delta = w_m$, and finesse $\mathcal{F} = 1.07 \times 10^4$; the dashed line refers to a mass $m = 50$ ng, $\Delta = w_m/2$, and finesse $\mathcal{F} = 3.4 \times 10^4$. The other parameters are those of Fig. 1.
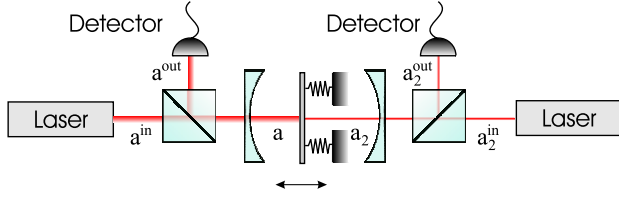
FIG. 3 (color online).   Schematic description of the proposed experiment, including the second Fabry-Perot cavity on the right for the detection of the mechanical motion.

length of C_2, respectively. The presence of the second cavity affects the mirror dynamics, which is no more exactly described by Eqs. (5). However, if C_2 is driven by a much weaker intracavity field so that $|\alpha_2| \ll |\alpha_s|$, its backaction on the mechanical mode can be neglected and the relevant dynamics is still well described by Eqs. (5).

If we now choose parameters so that $\Delta_2 = w_m \gg k_2$, $G_2|\alpha_2|$, we can rewrite Eq. (13) in the frame rotating at $\Delta_2 = w_m$ for the slow variables $\delta\tilde{o}(t) \equiv \delta o(t)\exp\{-iw_m t\}$ and neglect the terms rapidly oscillating at the frequency $2w_m$, so to get [15,18]

$$\delta\dot{\tilde{a}}_2 = -\kappa_2\delta\tilde{a}_2 + i\frac{G_2\alpha_2}{\sqrt{2}}\delta\tilde{b} + \sqrt{2\kappa_2}\tilde{a}_2^{\text{in}}(t), \qquad (14)$$

where $\delta b = (i\delta p + \delta q)/\sqrt{2}$. If $\kappa_2 \gg G_2|\alpha_2|/\sqrt{2}$, the cavity mode adiabatically follows the mirror dynamics and one has $\delta\tilde{a}_2 \simeq i(G_2\alpha_2/\kappa_2\sqrt{2})\delta\tilde{b} + \sqrt{2/\kappa_2}\tilde{a}_2^{\text{in}}(t)$. Using $\tilde{a}_2^{\text{out}} = \sqrt{2\kappa_2}\delta\tilde{a}_2 - \tilde{a}_2^{\text{in}}$ [21], we finally get

$$\tilde{a}_2^{\text{out}} = i\frac{G_2\alpha_2}{\sqrt{\kappa_2}}\delta\tilde{b} + \tilde{a}_2^{\text{in}}(t), \qquad (15)$$

showing that, in the chosen parameter regime, the output light of C_2 gives a direct measurement of the mirror dynamics. By changing the phases of the two local oscillators and by measuring the correlations between the two cavity outputs, one can determine all of the entries of the CM V and from them numerically extract the logarithmic negativity $E_{\mathcal{N}}$ by means of Eq. (11).

In conclusion, we have shown that a Fabry-Perot cavity with an oscillating micromirror and driven by coherent light can produce robust and stationary entanglement between the optical intracavity mode and the mechanical mode of the mirror. The amount of entanglement is quantified by the logarithmic negativity and surprisingly robust against increasing temperature: For experimental parameters close to those of recently performed experiments [29], entanglement may persist above 20 K in the case of a 5 ng mechanical oscillator. Finally, we suggest a readout scheme that allows a full experimental characterization of the CV Gaussian steady state of the system and, hence, a measurement of the generated entanglement.

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