Derivation of Hydrodynamics for the Gapless Mode in the BEC-BCS Crossover from the Exact One-Loop Effective Action

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We derive generalized two-superfluid continuity equations for the BEC-BCS crossover in the presence of a Feshbach resonance at $T = 0$. In addition, we calculate the velocity of sound throughout both BCS and Bose-Einstein condensation (BEC) regimes.

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As the strength of fermionic pairing increases in cold alkali atoms there is a continuous evolution from the BCS-like behavior of Cooper pairs to Bose-Einstein condensation (BEC) of molecules. This crossover phenomenon becomes controllable experimentally [1] when the atoms interact through a Feshbach resonance [2–7]. In both regimes, condensation is a consequence of global $U(1)$ symmetry breaking with a concomitant gapless Goldstone mode (the phonon).

Although the s-wave scattering length diverges at the crossover, the speed of sound changes smoothly, as do the hydrodynamical properties of the condensate. The main result of this Letter is to show that, at $T = 0$, all of these can be determined straightforwardly from the derivative expansion of the exact, fully renormalized one-loop effective action, in the spirit of [7], sidestepping the more complicated derivations based on a full multichannel analysis (e.g., see [4]).

Consider a condensate comprising a mixture of fermionic atoms and molecular bosons. The fermions $\psi_\sigma(x)$, with spin $\sigma = (\uparrow, \downarrow)$, undergo self-interaction through an s-wave BCS-type term. In addition, two fermions can be bound into a molecular boson $\phi(x)$ through a Feshbach resonance effect. To exemplify our method, we take the Lagrangian density to be [2,3] ($U > 0$, g fixed)

$$\begin{aligned} L = &\sum_{\uparrow,\downarrow} \psi^*_\sigma(x)\left[i\partial_t + \frac{\nabla^2}{2m} + \mu \right]\psi_\sigma(x) \\ &+ U\psi^*_\uparrow(x)\psi^*_\downarrow(x)\psi_\downarrow(x)\psi_\uparrow(x) \\ &+ \phi^*(x)\left[i\partial_t + \frac{\nabla^2}{2M} + 2\mu - \nu \right]\phi(x) \\ &- g[\phi^*(x)\psi_\downarrow(x)\psi_\uparrow(x) + \phi(x)\psi^*_\downarrow(x)\psi^*_\uparrow(x)]. \end{aligned} \quad (1)$$

The mass of the bosons is twice that of the fermions, $M = 2m$. The kinetic energy of the fermions is $\epsilon_k = k^2/2m$ and the kinetic energy of the bosons is $k^2/2M + \nu$, where ν is the threshold energy of the Feshbach resonance. To demonstrate the method we restrict ourselves to a narrow resonance approximation [2,3] where quantum loop effects from the molecular boson can be safely ignored [7], although this is often an idealization. The effective four-fermion interaction is determined by the value of ν, which is controllable by an external magnetic field. On tuning the field, the condensate can be varied in form from BCS Cooper pairs to BEC molecules as the scattering length changes sign.

On introducing the auxiliary field $\Delta(x) = U\psi_\downarrow(x)\psi_\uparrow(x)$, a Hubbard-Stratonovich transformation leads to an effective Lagrangian density, to which the fermionic contribution is $\Psi^\dagger(x)G^{-1}\Psi(x)$, where $\Psi(x)$ is the Nambu spinor, $\Psi^\dagger(x) = (\psi^\dagger_\uparrow, \psi_\downarrow)$, and G^{-1} is the inverse Nambu Green function

$$G^{-1} = \begin{pmatrix} i\partial_t - \varepsilon & \tilde{\Delta}(x) \\ \tilde{\Delta}^*(x) & i\partial_t + \varepsilon \end{pmatrix}, \quad (2)$$

with $\varepsilon = -\frac{\nabla^2}{2m} - \mu$. The *combined* condensate of the theory is $\tilde{\Delta}(x)$, given in terms of the bifermion and molecular condensates Δ and ϕ as $\tilde{\Delta}(x) = \Delta(x) - g\phi(x)$. The gapless mode of the theory is encoded in the phases of $\Delta(x)$ and $\phi(x)$ for which we write $\Delta(x) = |\Delta(x)|e^{i\theta_\Delta(x)}$, $\phi(x) = -|\phi(x)|e^{i\theta_\phi(x)}$. The amplitude and phase of $\tilde{\Delta}(x) = |\tilde{\Delta}(x)|e^{i\theta_{\tilde{\Delta}}(x)}$ can be determined from those of $\Delta(x)$ and $\phi(x)$ by its definition above. We now perform a $U(1)$ gauge transformation on the fermion field $\psi_\sigma(x) = e^{i\theta_{\tilde{\Delta}}(x)/2}\chi_\sigma(x)$. Integrating out $\chi_\sigma(x)$ leads to a nonlocal effective action $S_{\rm eff}[\phi, \phi^*, \Delta, \Delta^*]$.

The action possesses a $U(1)$ invariance under the phase change $\theta_\phi \to \theta_\phi + \alpha$ and $\theta_\Delta \to \theta_\Delta + \alpha$ (or $\theta_{\tilde{\Delta}} \to \theta_{\tilde{\Delta}} + \alpha$). This symmetry is spontaneously broken when Δ and ϕ, respectively, acquire the nonvanishing constant values Δ_0 and ϕ_0 determined by the gap equations obtained from extremizing the effective action. In these, $|\tilde{\Delta}_0| = |\Delta_0| + |-g\phi_0|$ satisfies

$$\frac{1}{U_{\rm eff}} = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}, \quad (3)$$

with $U_{\rm eff} = U + g^2/(\nu - 2\mu)$ and $E_p = (\varepsilon_p^2 + |\tilde{\Delta}_0|^2)^{1/2}$, and

$$|\phi_0| = \frac{g}{\nu - 2\mu}\frac{|\tilde{\Delta}_0|}{U_{\rm eff}}. \quad (4)$$

Since the microscopic theory under consideration is Galilean invariant, any effective theory derived from it must respect this symmetry [8]. Consider the dynamics of the phonon carried in the angular variables. To preserve Galilean invariance at each step, the variations in the condensate magnitudes can be written as $|\Delta| = |\Delta_0| + \delta|\Delta|$, $|\phi| = |\phi_0| + \delta|\phi|$ [9–11]. We assume that terms in $\delta|\tilde{\Delta}|$, $\delta|\phi|$, and $(\theta_\Delta - \theta_\phi)^2$ are of the same order in their defining equation, although θ_Δ and θ_ϕ are large variables. We now use the fact that $e^{-i\sigma_3\theta_{\tilde{\Delta}}(x)/2}G^{-1}e^{i\sigma_3\theta_{\tilde{\Delta}}(x)/2} = G_0^{-1} - \Sigma$, where

$$\Sigma = [-i\nabla^2\theta_{\tilde{\Delta}}/4m + (\nabla\theta_{\tilde{\Delta}})(-i\nabla)/2m]\mathrm{I} + [\dot{\theta}_{\tilde{\Delta}}/2 + (\nabla\theta_{\tilde{\Delta}})^2/8m]\sigma_3 - \delta|\tilde{\Delta}|\sigma_1, \quad (5)$$

with $\delta|\tilde{\Delta}|$ defined by $|\tilde{\Delta}| = |\tilde{\Delta}_0| + \delta|\tilde{\Delta}|$ [9]. G_0^{-1} is the free inverse Nambu Green function with the same form as G^{-1} in Eq. (2), except that $\tilde{\Delta}(x)$ is replaced by $|\tilde{\Delta}_0|$. $S_{\rm eff}[\phi, \phi^*, \Delta, \Delta^*]$ then permits the derivative expansion

$$S_{\rm eff} = -i\mathrm{Tr}\ln(G_0^{-1}) + i\mathrm{Tr}\sum_{n=1}^{\infty}\frac{(G_0\Sigma)^n}{n} - \int d^4x\frac{|\Delta|^2}{U} + \int d^4x\phi^*(x)\left(i\partial_t + \frac{\nabla^2}{2M} + 2\mu - \nu\right)\phi(x), \quad (6)$$

the first two terms of which are no more than the expansion of $-i\mathrm{Tr}\ln(G^{-1})$. For our purposes it is sufficient to truncate the sum in n at second order, to yield an action $S_{\rm eff}^{(2)}[\delta|\phi|, \delta|\tilde{\Delta}|, \theta_\phi, \theta_{\tilde{\Delta}}]$ after eliminating Δ in favor of $\tilde{\Delta}$, and θ_Δ in terms of $\theta_{\tilde{\Delta}}$, on using $(\theta_{\tilde{\Delta}} - \theta_\phi) \approx (|\Delta_0|/|\tilde{\Delta}|)\times(\theta_\Delta - \theta_\phi)$. Diagrammatically, this amounts to taking account of fermionic one-loop effects in the effective action.

The action of the gapless phonon mode is that part of the quadratic contribution to $S_{\rm eff}^{(2)}$ in which the derivatives of $\delta|\phi|$ and $\delta|\tilde{\Delta}|$ are omitted. After straightforward manipulations this takes the form

$$S_{\rm phonon} = \int d^4x\Big\{\frac{N}{4}\dot{\theta}_{\tilde{\Delta}}^2 - \frac{1}{8m}\rho_0^F(\nabla\theta_{\tilde{\Delta}})^2 - \frac{1}{8m}\rho_0^B(\nabla\theta_\phi)^2 - \frac{1}{2}\Omega^2(\theta_{\tilde{\Delta}} - \theta_\phi)^2 - 2|\phi_0|\delta|\phi|\dot{\theta}_\phi - \alpha\dot{\theta}_{\tilde{\Delta}}\delta|\tilde{\Delta}| + (2\mu - \nu)\frac{U_{\rm eff}}{U}(\delta|\phi|)^2 + \frac{2g}{U}\delta|\tilde{\Delta}|\delta|\phi| - \frac{1}{2}\left(\frac{2}{U} - \beta\right)(\delta|\tilde{\Delta}|)^2\Big\}. \quad (7)$$

This lends itself to a very simple mechanical picture of a coupled "wheel" and "axle". The radius of the wheel is $|\tilde{\Delta}|$ and that of the axle is $|-g\phi|$, measured from the gap values $|\tilde{\Delta}_0|$ and $|-g\phi_0|$ with angles of displacement $\theta_{\tilde{\Delta}}$, θ_ϕ, respectively. There is slippage between the wheel and axle with an elastic restoring force $\Omega^2 = (2g/U)|\phi_0||\tilde{\Delta}_0|$.

The fermion number density at $n = 1$ is $\rho_0 = \rho_0^F + \rho_0^B$, where $\rho_0^F = \int[d^3\mathbf{p}/(2\pi)^3][1 - \varepsilon_p/E_p]$ is the explicit fermion density, and $\rho_0^B = 2|\phi_0|^2$ is due to molecules (two fermions per molecule). The other coefficients are straightforwardly derived as $N = \int[d^3\mathbf{p}/(2\pi)^3](|\tilde{\Delta}_0|^2/2E_p^3)$, $\alpha = \int[d^3\mathbf{p}/(2\pi)^3](|\tilde{\Delta}_0|\varepsilon_p/2E_p^3)$, and $\beta = \int[d^3\mathbf{p}/(2\pi)^3]\times(\varepsilon_p^2/E_p^3)$.

A further straightforward eigenvalue calculation gives the long wavelength dispersion relation for the phonon as $\omega^2 = v^2\vec{k}^2 + O(k^4)$, where

$$v^2 = \frac{\rho_0/2m}{N + A/B} \quad (8)$$

and

$$A = \alpha^2(2\mu - \nu)\frac{U_{\rm eff}}{U} - 2\alpha|\phi_0|\frac{2g}{U} - 2|\phi_0|^2\left(\frac{2}{U} - \beta\right),$$
$$B = \left(\frac{g}{U}\right)^2 + \frac{1}{2}\left(\frac{2}{U} - \beta\right)(2\mu - \nu)\frac{U_{\rm eff}}{U}. \quad (9)$$

Note that v is independent of the slippage strength $\Omega \neq 0$.

In fact, β is UV singular, as are $U_{\rm eff}$, U, and g from the gap equations (3) and (4). Renormalization is implemented by defining the renormalized coupling $\bar{U}_{\rm eff}$ in terms of the s-wave scattering length a_s as

$$-\frac{N_0}{k_F a_s} = \frac{1}{\bar{U}_{\rm eff}} = \frac{1}{U_{\rm eff}} - \int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\epsilon_p} = \int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3}\left[\frac{1}{2E_p} - \frac{1}{2\epsilon_p}\right], \quad (10)$$

where k_F is the Fermi momentum and Λ is a UV cutoff. UV-finite renormalized couplings $\bar{U}$ and $\bar{g}$ are defined similarly in the limit $\Lambda \to \infty$;

$$\frac{1}{\bar{U}} = \frac{1}{U} - \int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\epsilon_p},$$
$$\frac{1}{\bar{g}} = \frac{1}{g} - \frac{U}{g}\int^\Lambda \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2\epsilon_p}. \quad (11)$$

In turn, we now define a renormalized threshold energy $\bar{\nu}$ through

$$\bar{U}_{\rm eff} = \bar{U} + \frac{\bar{g}^2}{\bar{\nu} - 2\mu}. \quad (12)$$

The outcome of this renormalization is that $S_{\rm phonon}$ of (7) and the gap equation for the condensate (4) are rendered UV finite term by term by replacing the unrenormalized U, $U_{\rm eff}$, g, β, ν, by their renormalized counterparts. Henceforth we drop the overbars for simplicity, and understand all quantities in (8) as renormalized. Necessarily, the renormalization prescription we propose above makes not only the gap equations free of UV divergence, but also the sound velocity. Thus, combining the renormalized equations (4) and (10) with the number density allows us to study the behavior of the condensate numerically when the threshold energy varies so that the scattering length goes from $a_s \to 0^+$, the deep BEC regime, to $a_s \to 0^-$, that of deep BCS, through a BCS-BEC crossover. This is shown in the inset in Fig. 1 [6,12]. Solving first for the chemical potential μ as a function of the threshold energy ν, the sound velocity (8) can be computed, as depicted in the main Fig. 1, in agreement with [13].

To understand the numerical behavior shown in Fig. 1 we evaluate v^2 analytically in both deep BCS and BEC regimes. In the deep BCS regime, there is considerable simplification as all momentum integrals are dominated by the $\mathbf{k}$ modes near the Fermi surface. In this limit, $|\phi_0| \approx 0$, where few molecules are present, and $\alpha \approx 0$, the ignorable amplitude-phase coupling due to the particle-hole symmetry near the Fermi surface. We obtain $A/B \ll N$, leading to

$$v^2 \simeq \frac{\rho_0}{2mN}. \qquad (13)$$

With the total fermion density $\rho_0 = k_F^3/3\pi^2$ and $N \approx N_0 = mk_F/2\pi^2$, the fermion density of states at the Fermi surface, the sound velocity is given by $v^2 \simeq v_F^2/3$.

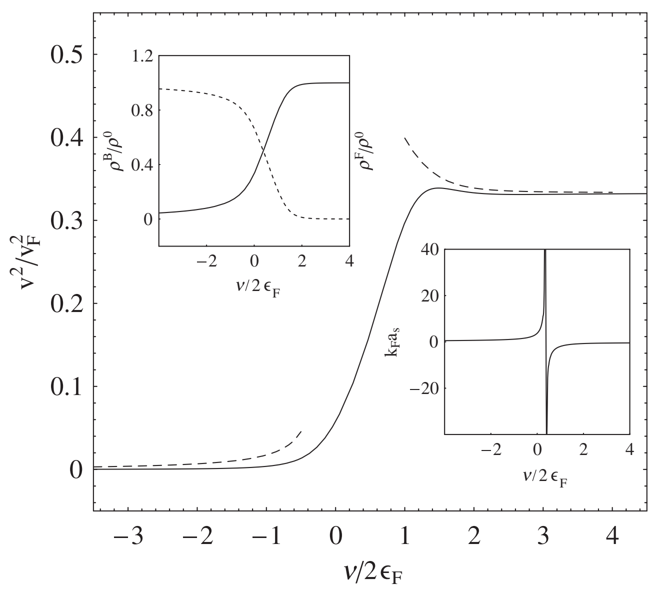

FIG. 1. The behavior of the sound velocity v throughout BEC and BCS regimes as a function of the threshold energy ν. We choose $U = 7.54\epsilon_F/k_F^3$ and $g = 4.62\epsilon_F/k_F^{3/2}$ as an example [12]. The dotted line is obtained with the approximate solution of the sound velocity, Eq. (14) for the BEC regime and Eq. (13) for the BCS regime. The lower inset shows how the scattering length a_s varies from BEC to BCS as the threshold energy increases, while the upper inset reveals the evolution of the fermion density ρ_0^F (solid line) and the molecule density ρ_0^B (dotted line).

This is the result we obtain from conventional BCS theory with no Feshbach resonance (e.g., see [9]), for which $g = 0$ and $|\phi_0| = 0$.

On the other hand, in the deep BEC limit $U \ll g^2/|\nu - 2\mu|$, which gives rise to $|\nu - 2\mu| \approx g^2 N_0/k_F a_s$, as obtained from $U_{\rm eff}$. To maintain this relation for small a_s, U cannot be too large. From Eqs. (9), the sound velocity v^2 can be approximated as

$$v^2 \simeq \frac{1}{8m}\frac{|\tilde{\Delta}_0|^2}{|\mu|} \qquad (14)$$

in this deep BEC regime as a result of $A/B \gg N$. We find that $|\phi_0|$ increases as ν goes increasingly negative, while the combined condensate $|\tilde{\Delta}_0|$ decreases to keep the number of fermions fixed. As a result $|\tilde{\Delta}_0|^2 \simeq g^2|\phi_0|^2 \simeq (g^2/2)\rho_0$ for largely negative ν (i.e., $a_s \to 0^+$) when all fermions are in the form of molecules as seen in the inset in Fig. 1. Thus, the behavior of v^2, which is found to approach zero at small a_s, is now determined by how the chemical potential μ increases negatively as a_s deceases.

In the central region, the behavior is smooth across the "unitarity limit" at $|a_s| \to \infty$ when $\nu = 2\mu$. In this case, the gap and number equations are reduced to involving only one parameter g in the one-channel formulation of Ref. [14]. If g is large, "universal behavior" is found where $v^2 \approx 0.2 v_F^2$. Nevertheless, the renormalization of the molecular boson is expected to contribute sizable corrections to the sound velocity obtained above in such a strongly coupled regime [15].

More generally, the BEC-BCS system permits a hydrodynamic interpretation as a two-component superfluid. The crucial ingredient is the Galilean invariance of the derivative expansion. It is not difficult to rederive the relevant angular parts of $S_{\rm eff}^{(2)}$ from $S_{\rm phonon}$. We restore Galilean invariance by the substitutions

$$\dot{\theta} \to \dot{\theta} + \frac{(\nabla\theta)^2}{4m}, \qquad \frac{(\nabla\theta)^2}{4m} \to \dot{\theta} + \frac{(\nabla\theta)^2}{4m}, \qquad (15)$$

for both phase angles.

Taking the variation of $S_{\rm eff}^{(2)}$ with respect to the condensate phase $\theta_{\tilde{\Delta}}$ leads to a single mean-field equation of motion which can be rewritten in terms of the explicit fermion number density ρ_F:

$$\frac{\partial}{\partial t}\rho_F + \nabla\cdot\mathbf{j}_F - 2\Omega^2(\theta_{\tilde{\Delta}} - \theta_\phi) = 0, \qquad (16)$$

where

$$\rho_F = \rho_F^0 - N_0\left(\dot{\theta}_{\tilde{\Delta}} + \frac{(\nabla\theta_{\tilde{\Delta}})^2}{4m}\right) + 2\alpha\delta\tilde{\Delta} \qquad (17)$$

and $\mathbf{j}_F = \rho_F \nabla\theta_{\tilde{\Delta}}/2m$. There is a similar equation for the fermion number density due to molecules:

$$\frac{\partial}{\partial t}\rho_B + \nabla\cdot\mathbf{j}_B + 2\Omega^2(\theta_{\tilde{\Delta}} - \theta_\phi) = 0, \qquad (18)$$

where (to lowest order)

$$\rho_B = \rho_B^0 + 4|\phi_0|\delta\phi = 2|\phi|^2 \tag{19}$$

and $\mathbf{j}_B = \rho_B \nabla\theta_\phi/4m$. Putting these together gives the continuity equation for total fermion number

$$\frac{\partial}{\partial t}\rho + \nabla\cdot\mathbf{j} = 0, \tag{20}$$

describing a coupled two-component superfluid, where $\rho = \rho_F + \rho_B$ and $\mathbf{j} = \mathbf{j}_F + \mathbf{j}_B$. The behavior of the mean explicit and molecular fermion number densities ρ_F^0 and ρ_B^0 as ν varies is given in the inset of Fig. 1.

Our earlier definitions give $\Omega^2(\theta_{\tilde{\Delta}} - \theta_\phi) \propto |\Delta_0||\phi_0|(\theta_\Delta - \theta_\phi)$, showing that the coupling between the superfluid components is due to the difference in the phase of the fermionic pairs Δ and the molecular field ϕ. It vanishes in both the deep BCS and BEC regimes, when $|\phi_0|$ and $|\Delta_0|$ tend to zero, respectively. Further, with $\rho_F \gg \rho_B$ and $|\mathbf{j}_F| \gg |\mathbf{j}_B|$ in the deep BCS regime and $\rho_F \ll \rho_B$ and $|\mathbf{j}_F| \ll |\mathbf{j}_B|$ in the deep BEC regime, the system is described by a single fluid in each case. Away from these extremes the situation gets more complicated, with the coupling strongest in the transition regime, but still tractable for vortices with phase coupling $\theta_\Delta = \theta_\phi$, whose properties will be pursued elsewhere.

Finally, we briefly consider Gross-Pitaevskii (GP) equations in the BEC regime. (For the BCS regime the results of [9] can be simply extended to $g \neq 0$). The relationship of superfluid equations to GP equations is well established, in principle. To each $(\rho, \mathbf{j})$ pair there is allocated a complex GP (or nonlinear Shrödinger) field Ψ. In our case the Gross-Pitaevskii fields underlying (16) and (18) are $\Psi_F = \sqrt{\rho_F}e^{i\theta_{\tilde{\Delta}}}/\sqrt{2}$ and $\Psi_B = \phi = \sqrt{\rho_B}e^{i\theta_\phi}/\sqrt{2}$. Although, in general, Ψ_F has the phase of the combined condensate, but the magnitude due to the explicit fermion density only, it happens that, in the BEC regime,

$$|\Psi_F|^2 = \frac{1}{32\pi}\frac{(2m|\mu|)^{3/2}}{|\mu|^2}|\tilde{\Delta}|^2. \tag{21}$$

That is, now $\Psi_F \propto \tilde{\Delta}$, linking its phase to the condensate density.

Furthermore, $S_{\rm phonon}$ is all that is needed to extract the coupling constant of two-body interactions between the condensate. The form of this two-body interaction is

$$\lambda_{\tilde{\Delta}}(\tilde{\Delta}^\dagger\tilde{\Delta})^2 = \lambda_{\tilde{\Delta}}(|\tilde{\Delta}_0|^4 + 4|\tilde{\Delta}_0|^2(\delta|\tilde{\Delta}|)^2 + \ldots). \tag{22}$$

From the second term we can read off $\lambda_{\tilde{\Delta}}$ directly from $S_{\rm phonon}$ as $\lambda_{\tilde{\Delta}} = (2/U - \beta)/8|\tilde{\Delta}_0|^2$. Again in the BEC regime, we find

$$\lambda_{\tilde{\Delta}} \simeq -\frac{1}{256\pi}\frac{(2m|\mu|)^{3/2}}{|\mu|^3}. \tag{23}$$

Let us rewrite $\lambda_{\tilde{\Delta}}(\tilde{\Delta}^\dagger\tilde{\Delta})^2$ as a GP self-interaction $\lambda|\Psi_F|^4$, using the renormalization of (21). On using (23) we find a weakly repulsive interaction $\lambda = -2\pi|a_{\tilde{\Delta}}|/M$, where $a_{\tilde{\Delta}} = 2a_s$ and $M = 2m$, as follows from the strong-coupling Bogoliubov–de Gennes equations [16].

The extension of our approach to nonzero temperature is straightforward, in principle, and will be considered elsewhere. It has yet to be seen whether, in general, the effect of Landau damping can be interpreted as a normal fluid component in addition to the coupled superfluids of (20), as happens for pure BCS theory [17].

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