

Quantum Computation using Vortices and Majorana Zero Modes of a $p_x + ip_y$ Superfluid of Fermionic Cold Atoms

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We propose to use the recently predicted two-dimensional “weak-pairing” $p_x + ip_y$ superfluid state of fermionic cold atoms as a platform for topological quantum computation. In the core of a vortex, this state supports a zero-energy Majorana mode, which moves to finite energy in the corresponding topologically trivial “strong-pairing” state. By braiding vortices in the “weak-pairing” state, unitary quantum gates can be applied to the Hilbert space of Majorana zero modes. For readout of the topological qubits, we propose realistic schemes suitable for atomic superfluids.

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Introduction.—Topological quantum computation requires particles that have non-Abelian statistics under interchange and braiding. Under pairwise interchange of particle coordinates, the many-body wave function of particles following non-Abelian statistics transforms via a unitary transformation in the Hilbert space of a degenerate set of wave functions. Such particles can arise as the low-energy excitations of a topological phase of matter. A promising system, other than the recently discussed $\nu = \frac{5}{2}$ fractional quantum Hall state [1], is a spin-triplet ($S = 1$), 2D, $p_x + ip_y$ superconductor, in which certain vortex excitations have zero-energy Majorana modes [2] in their cores, which endow these vortices with non-Abelian statistics [3–5]. For a spin-polarized (spinless) $p_x + ip_y$ superconductor, ordinary vortices with vorticity $N = 1$ have such Majorana modes bound at the core [5]. For a spinful $p_x + ip_y$ superconductor, analogous to the A phase of He 3, only the higher-energy vortices with $N = \frac{1}{2}$ —not the lowest energy $N = 1$ vortices—have the Majorana modes. Nevertheless, it is possible to quench the spin-orbit energy, which acts as a confining potential between two $N = \frac{1}{2}$ vortices, by applying a magnetic field [6], and thus to make the Majorana modes potentially realizable in experiments. Based on this idea, and that strontium ruthenate may be a spinful, quasi-2D, $p_x + ip_y$ superconductor [7], it has been proposed [8] to use thin films of this material as a system which realizes non-Abelian statistics of quasiparticles, associated with these unusual half-quantum (i.e., $N = \frac{1}{2}$) vortices.

Although there is nothing in principle to invalidate such a strategy, difficulties may arise due to the lack of quantum coherent motion of vortices in the films. Moreover, since one needs to apply a threshold magnetic field to quench the spin-orbit energy, there will be a relatively high concentration of the half-quantum vortices, thereby rendering

independent braiding experimentally challenging. Finally, and most importantly, since the quasiparticles of a superconductor are chargeless, and the Majorana modes are also spinless, there is no simple way to couple to the state of a qubit after a braiding operation has been performed. This makes reading out the state of the qubit difficult. Hence, even though the very realization of non-Abelian statistics through the observation of these vortices is an exciting goal in itself, and for the purposes of topological quantum computation several ideas to overcome the difficulties mentioned above were recently proposed [8], it will really pay to have a $p_x + ip_y$ superfluid system where vortex motion is likely to be coherent, $N = 1$ vortices themselves have non-Abelian statistics so that their concentration can be independently kept low, and a natural readout scheme exists.

With the recent observation of a p -wave Feshbach resonance in spin-polarized ^{40}K and ^6Li atoms in optical traps [9–11], just such a system—an “artificially” created $p_x + ip_y$ superfluid of spinless fermions—may now be within experimental reach. Exotic non-Abelian statistics is thus tantalizingly close to fruition in these systems. Since the atoms are in identical spin states, s -wave scattering is Pauli prohibited and a p -wave resonance dominates, allowing the tunability of the atom-atom interaction in $L = 1$ channel. Recently, it has been theoretically shown [12,13] that such interactions have the potential to realize various p -wave superfluid states, among them, a $p_x + ip_y$ state in the so-called “weak-pairing” phase (chemical potential $\mu > 0$) in both 3D and 2D. With Feshbach resonance detuning controlling μ , this phase undergoes a topological quantum transition to the strong-pairing phase ($\mu < 0$) [12]. In 2D, the phase with $\mu > 0$ is topologically non-trivial because it supports zero-energy Majorana modes at vortex cores [5], while they disappear in the topologically

trivial strong-pairing phase. The weak-pairing phase, then, is suitable for use in the hardware of a quantum computer. Vortices in s -wave superfluids have been observed, and p -wave Feshbach resonances have been seen in recent experiments [9–11]. Therefore, establishing a $p_x + ip_y$ superfluid in the optical trap is the next natural step. Since ordinary vortices themselves exhibit non-Abelian statistics in this case, they can be created at a low density, allowing, in principle, coherent independent braiding. Finally, as we discuss later, since the atoms, unlike the electrons in a superconductor, have internal energy levels, this internal structure can be manipulated to read out the states of the qubits after the quantum computation.

The weak and strong pairing phases and the fate of the zero mode.—The BCS Hamiltonian for a system of spinless fermions in a 2D spin-triplet p -wave superfluid state is given by

$$H = \int d^2x d^2x' \psi^\dagger(\vec{x}) \mathcal{H}(\vec{x}, \vec{x}') \psi(\vec{x}'), \quad (1)$$

where $\psi(\vec{x})$ is a two-component column vector, $\psi(\vec{x}) = (c^\dagger(\vec{x}), c(\vec{x}))^T$, and $\mathcal{H}(\vec{x}, \vec{x}')$ is the matrix,

$$\begin{aligned} \mathcal{H}(\vec{x}, \vec{x}') = & \left(\frac{-\nabla^2}{2m} - \mu \right) \delta(\vec{x} - \vec{x}') \sigma_z + \frac{\Delta(\vec{x}, \vec{x}')}{2} \sigma^+ \\ & - \frac{\Delta^*(\vec{x}, \vec{x}')}{2} \sigma^-. \end{aligned} \quad (2)$$

Here, m is the fermion mass, μ is the chemical potential, and $\Delta(\vec{x}, \vec{x}')$ is the gap function. We take $\hbar = k_B = 1$ in this Letter. In a uniform $p_x + ip_y$ state, the gap function takes the form in momentum space, $\Delta(\vec{p}) = \frac{\Delta_0}{v_F} \times (p_x + ip_y)$. Following Ref. [5], we identify the system to be in the strong-pairing phase for $\mu < 0$ and in the weak-pairing phase for $\mu > 0$, the two phases separated by a topological phase transition [14].

We model a vortex by assuming the gap function to be zero inside an area of radius ξ (coherence length). Outside this radius, the gap function takes the form $\Delta(\vec{p}) = \frac{\Delta_0}{v_F} \times \exp(i\theta/2)(p_x - ip_y) \exp(i\theta/2)$, where the total order parameter phase, θ , rotates by 2π . In polar coordinates (ρ, θ) , for $\rho < \xi$, the Bogoliubov–de Gennes (BdG) equations [15–17] take the form

$$\left[-\frac{1}{2m} \left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \theta^2} \right) - \mu \right] \sigma_z \phi(\vec{x}) = 0, \quad (3)$$

where $\phi(\vec{x}) = [u(\vec{x}), v(\vec{x})]^T$. For the zero-energy state, we take the angular momentum operator $l = -i \frac{\partial}{\partial \theta}$ to have eigenvalue zero. The remaining parts of Eq. (3) imply just ordinary Bessel equations of order zero [18] for both u and v . Since one of the two independent solutions is divergent at the origin, we find the solution for ϕ ,

$$\phi(\rho) = A J_0(\sqrt{2m\mu\rho}) \zeta, \quad (4)$$

where J_0 is the Bessel function of the first kind of order zero, A is a constant, and ζ is a constant spinor.

For solutions with $\rho > \xi$, we note that the gap operator can be written in polar coordinates as $-i \frac{\Delta_0}{v_F} \exp(-i\theta/2) \times \left(\frac{\partial}{\partial \rho} - \frac{i}{\rho} \frac{\partial}{\partial \theta} \right) \exp(i\theta/2) = -i \frac{\Delta_0}{v_F} \left(\frac{\partial}{\partial \rho} + \frac{1}{2\rho} \right) - i \frac{\Delta_0}{v_F} l$. Using this, and for zero angular momentum, the Bogoliubov–de Gennes (BdG) equations for the zero-energy state can be written as

$$\left\{ \left[\left(\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \right) + 2m\mu \right] - \frac{2m\Delta_0}{v_F} \left(\frac{\partial}{\partial \rho} + \frac{1}{2\rho} \right) \sigma_y \right\} \phi(\rho) = 0. \quad (5)$$

The solutions to this equation which are well behaved at $\rho \rightarrow \infty$ can be written as $\phi(\rho) = \chi(\rho) \exp(-\frac{\Delta_0}{v_F} \rho) \times (1, -i)^T$, where $\chi(\rho)$ satisfies

$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} + \left(2m\mu - \frac{\Delta_0^2}{v_F^2} \right) \right] \chi(\rho) = 0, \quad (6)$$

where $v_F = \frac{p_F}{m}$. This is again Bessel equation of order zero. Since both solutions are well behaved at infinity (they are asymptotically sinusoidal), the general solution for $\phi(\rho)$ can therefore be written as

$$\phi(\rho) = [B J_0(\kappa\rho) + C Y_0(\kappa\rho)] \exp\left(i \frac{\pi}{4} - \frac{\Delta_0}{v_F} \rho\right) (1, -i)^T, \quad (7)$$

where Y_0 is the Bessel function of the second kind of order zero, $\kappa = \sqrt{2m\mu - \Delta_0^2/v_F^2}$, B, C are constants, and the phase factor $e^{i(\pi/4)}$ is for equal distribution of phase between ϕ and ϕ^\dagger (see below). Next, to get a complete solution for the zero-energy state, one needs to match the wave function and its derivative at $\rho = \xi$, and also normalize the function in all space. These conditions will provide three equations for the three constants A, B , and C , which can then be straightforwardly solved in terms of the known parameters. Once the solution $\phi(\rho)$ and, in turn, $u(\vec{x}), v(\vec{x})$ are known, the quasiparticle operator for the zero-energy state can be written as

$$\gamma_0^\dagger = \int d^2x [u(\vec{x}) c^\dagger(\vec{x}) + v(\vec{x}) c(\vec{x})]. \quad (8)$$

For the overall phase choice $e^{i(\pi/4)}$ in Eq. (7), one can see from Eq. (8) that $\gamma_0^\dagger = \gamma_0$: the zero-energy state is a self-Hermitian Majorana state. As has been discussed before [3], this zero-energy state is protected from weak perturbations including external potential, electromagnetic vector potential, and local deformations of the order parameter. Note that the nonuniform trapping potential ~ 5 Hz in an optical trap can be considered a weak external potential since the gap to the first excited state in the vortex core $\omega_0 \sim \frac{\Delta_0^2}{\epsilon_F}$, where the superfluid gap $\Delta_0 \sim \epsilon_F$ in the Feshbach superfluids and $\epsilon_F \sim 5$ kHz [19], can be much larger, $\omega_0 \sim 5$ kHz. Therefore, the zero mode will exist even in the presence of the trapping potential. The existence of the zero mode has also recently been shown to follow from an index theorem [20] circumventing any

approximations made in the Bogoliubov–de Gennes (BdG) equations.

To see what happens to the zero-energy state in the strong-pairing phase, this phase can be most simply accessed, in the spirit of Ref. [5], by continuing to use the weak-coupling mean-field BCS theory even in the presence of strong attractive coupling and taking $\mu < 0$. Even in this heuristic spirit, it will be amusing to find below that no consistent zero-energy bound state is possible for negative μ . By extending the mean-field theory to $\mu < 0$ [15,21], and replacing μ by $-|\mu|$, Eq. (3) (for zero angular momentum) and Eq. (6) now imply *modified* Bessel equations of order zero for inside and outside the core, respectively. The solution $\phi'(\rho)$ is now given by only one of the two modified Bessel functions of order zero in each case, $I_0(\sqrt{2m|\mu|}\rho)$ for $\rho < \xi$, and $K_0(\kappa'\rho)$ for $\rho > \xi$, since the other one is divergent in the relevant region [18]. Here, $\kappa' = (2m|\mu| + \frac{\Delta_0^2}{v_F^2})^{1/2}$. The corresponding constants multiplying the solutions drop out when one matches the solutions and their derivatives at $\rho = \xi$, and divide one equation by the other. For generic values of the parameters, the resulting equation does not have a solution [21], and therefore, we do not expect a zero-energy state in the strong-pairing phase.

Non-Abelian statistics and unitary operators in the Hilbert space.—When the system is in the weak-pairing superfluid phase, a dilute gas of vortices can be created. For $2n$ vortices, there are $2n$ Majorana fermions, which we denote by γ_i , where i counts the vortices. The Majorana fermions can be combined pairwise to create n complex fermionic states, $c_i = \gamma_{2i} + i\gamma_{2i-1}$. These complex fermionic states can be occupied or unoccupied, giving rise to 2^n -fold degeneracy protected by the gap ω_0 . Two states of a qubit are identified with the absence ($|0\rangle$) or the presence ($c_i^\dagger|0\rangle$) of a superfluid quasiparticle in the fermionic state constructed from γ_{2i-1} and γ_{2i} . Note that the two states are degenerate and are not associated with any particular vortices. It is this nonlocality that protects the qubits from decoherence due to the environment, which acts through local operators. In Ref. [22] Semenoff and Sodano show that nonlocality is a consequence of Majorana fermion zero modes. They show that if one looks at a 1D system with a single such mode (by ignoring its partner far away), then the ground state will violate fermion parity. However, if both zero modes are kept, fermion parity is conserved. This continues to hold in our 2D case as well. For initialization of the qubits, note that creating vortices in pairs from the vacuum will always put each pair in the $|0\rangle$ state at zero temperature (T). At finite T , there is always a nonzero probability that a fermionic quasiparticle will end up on a vortex pair. Since we can read these nondestructively (see below), we can correct or discard the $|1\rangle$'s. We now briefly describe the physics [3] behind the braiding-induced unitary transformations in this space.

Using the property that $\gamma \rightarrow -\gamma$ for an order parameter phase change of 2π , it follows that upon interchange of two

vortices 1 and 2, $\gamma_1 \rightarrow \gamma_2$, but $\gamma_2 \rightarrow -\gamma_1$ [3]. The corresponding unitary operator in the 2D Hilbert space is

$$T_1 = \exp\left(\frac{\pi}{4}\gamma_2\gamma_1\right) = \exp\left[i\frac{\pi}{4}(2c^\dagger c - 1)\right], \quad (9)$$

where $c = \gamma_1 + i\gamma_2$. This operator can be written as a 2×2 matrix in the space of $|0\rangle$ and $c^\dagger|0\rangle$. Likewise, for four vortices $\gamma_1, \gamma_2, \gamma_3$, and γ_4 , the unitary braiding operators can be written as 4×4 matrices in the space of $|0\rangle, c_1^\dagger|0\rangle, c_2^\dagger|0\rangle$, and $c_1^\dagger c_2^\dagger|0\rangle$, where $c_1^\dagger = \gamma_1 + i\gamma_2$, and $c_2^\dagger = \gamma_3 + i\gamma_4$. In the case of $2n$ vortices, the braiding operators are $(2^n \times 2^n)$ -dimensional matrices: they form a matrix representation of the braid group in 2D. Upon braiding of two vortices, an initial state, which is now a 2^n -dimensional vector, is multiplied by these matrices and gets transformed to another vector in this space. It is these unitary transformations that can be utilized to build unitary quantum gates, and this is the essence of topological quantum computation. Exactly which braid gives rise to which quantum gate can be deduced for the Majorana fermions as it was done for Fibonacci anyons in Ref. [23]. Note that, in optical traps $T \ll \omega_0 \sim \epsilon_F$ [19], so only the Majorana modes are preferentially occupied. In Fermi superfluids, optical tweezers can be used to pin and braid the vortices. Although the optical tweezers would modify the effective μ at the vortex site, they would not affect the Majorana states as long as the pinning potential is smaller than μ . For the braiding, the positions of the vortices need to be known so that they can be translated in a known way. This can be achieved using phase-contrast imaging, which probes the positions of vortices without destroying them [24]. Because of the relatively large energy gap $\omega_0 \sim 5$ kHz, we estimate that the optical tweezers can be adiabatically moved around in a period of about 20 ms, leading to small excitation probability $\sim 10^{-4}$. Such a period is much shorter than the typical coherence time ~ 10 s of Fermi superfluids. Below, we show that the atoms in optical traps also offer a natural strategy for determining the state of a qubit after a computation has been performed.

Reading out the states of the qubits.—A central question in the above scheme is how to determine the state of a qubit after a computation has been performed. The two states of the qubit are distinguished by the presence or absence of a superfluid quasiparticle at the complex fermionic state when two vortices are fused. However, since these are quasiparticles of a spinless superfluid, they are chargeless and spinless, and hence difficult to couple by an external probe. Our method, suitable for atomic (or molecular) superfluids only, is to use the internal energy levels of the atoms themselves. The basic point is that, if there is an unpaired atom at the core of the composite vortex (the qubit is in the state $c^\dagger|0\rangle$), photons from a laser can be absorbed to excite the atom to an appropriately chosen excited level. If there is no quasiparticle there (the qubit is in the state $|0\rangle$), there will be no absorption at this frequency. Note that, during this process, one might end

up exciting Cooper pairs from outside the core as well. However, to excite an atom bound in a Cooper pair with another atom in an identical internal state, one first needs to break the pair, costing an energy $2\Delta_0$. Thus, from this process, photons can only be absorbed at a frequency shifted by $2\Delta_0$. Since the typical spontaneous emission rate, $\sim \mathcal{O}(2\pi \times 1 \text{ MHz})$, in such a detection process is much larger than $\Delta_0 \sim 2\pi \times 5 \text{ kHz}$ [19], this method can be applied only through intermediate states which induce a much larger energy splitting between paired and unpaired atoms. Here we illustrate this reading out scheme using 40 K atoms, although the technique is applicable to other species as well.

Suppose the atoms in the superfluid are in the $4^2S_{1/2}$ hyperfine ground state $|i\rangle \equiv |F = 9/2, m_F = -7/2\rangle$ in the case of p -wave resonance [9]. To determine whether there is an unpaired atom inside a composite vortex, a two-photon Raman pulse is applied that transfers the unpaired atom to another hyperfine state $|j\rangle \equiv |F = 7/2, m_F = -5/2\rangle$. The frequency difference between the two Raman lasers is adjusted to be resonant with the hyperfine splitting between states $|i\rangle$ and $|j\rangle$ for the unpaired atom, but has a $2\Delta_0$ detuning for paired atoms due to the energy cost to break a pair. The lasers have maximal intensities located at the core of the vortex, and their beam waist width $w \approx 1.5 \mu\text{m}$ is much smaller than the typical distance ($\geq 10 \mu\text{m}$) between vortices [25], allowing individual access to the qubits. The Rabi frequency of the Raman pulse is chosen to have a Gaussian shape $\Omega = \Omega_0 \exp(-\omega_0^2 t^2)$ ($-t_f \leq t \leq t_f$) to reduce the impact on paired atoms [26]. For a set of parameters $\Delta_0 = 2\pi \times 5 \text{ kHz}$, $\omega_0 = \Delta_0/2$, and $\Omega_0 = 1.77\omega_0$, $t_f = 5/\omega_0$ [26], we find that the unpaired atom is completely transferred from state $|i\rangle$ to $|j\rangle$ by the Raman pulse, while the probability for the paired atoms to be excited to state $|j\rangle$ is about 6×10^{-6} and may therefore be neglected.

To obtain a cycling transition necessary for the detection of the unpaired atom, π Raman pulses are applied to transfer the unpaired atom to state $|k\rangle \equiv |F = 9/2, m_F = 9/2\rangle$. Because of large Zeeman splitting between different magnetic sublevels, these Raman pulses may be performed in a short period (no longer than $100 \mu\text{s}$). A focused σ^+ -polarized detection laser resonant with the cycling transition $|k\rangle \rightarrow |l\rangle \equiv |5^2P_{3/2}: F = 11/2, m_F = 11/2\rangle$ is then applied to detect atoms at state $|k\rangle$. Here we choose the $4S \rightarrow 5P$ instead of $4S \rightarrow 4P$ transition for the detection laser to obtain smaller diffraction limit as well as smaller spontaneous decay rate. In the experiment [9], the magnetic field $B \approx 200 \text{ G}$ for the p -wave Feshbach resonance, which yields an effective detuning $\delta \approx 2\pi \times 170 \text{ MHz}$ for the paired atoms at state $|i\rangle$. The ratio between the number of the spontaneously emitted photons by paired and unpaired atoms is estimated to be $(\Gamma/2\delta)^2 \approx 1.2 \times 10^{-5}$, where $\Gamma \approx 2\pi \times 1.2 \text{ MHz}$ is the decay rate for the excited state $|l\rangle$. Therefore, in the detection process, the impact on paired atoms may be neglected, and resonant

fluorescence is observed if and only if initially there is one unpaired atom inside the vortex at state $|i\rangle$. For this readout scheme, T should be as low as possible, so that there are no other unpaired atoms in the bulk around the vortex cores. As mentioned before, the lowest T achievable in the optical traps should be sufficient for this purpose. Moreover, the thermally excited quasiparticles are expected to occur near the trap edges, and so by focusing the lasers on the vortices near the center of the trap, one should be able to significantly suppress the unwanted signals. Finally, we mention that the resonant detection laser may be replaced with a resonant multiphoton ionization process of the unpaired atom, yielding a single ionized electron that can be detected with essentially unit efficiency [27].

In conclusion, we have proposed to use the 2D, spin-polarized, $p_x + ip_y$ atomic resonance-superfluid in the weak-pairing phase, potentially realizable in optical traps, in a suitable hardware for topological quantum computation. We have given a realistic readout scheme for the topological qubits, a major hurdle in this field, using the internal states of the constituent atoms.

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