Photon-Mass Bound Destroyed by Vortices

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The Particle Data Group gives an upper bound on the photon mass $m < 2 \times 10^{-16}$ eV from a laboratory experiment and lists, but does not adopt, an astronomical bound $m < 3 \times 10^{-27}$ eV, both of which are based on the plausible assumption of large galactic vector potential. We argue that the interpretations of these experiments should be changed, which alters significantly the bounds on m. If m arises from a Higgs effect, both limits are invalid because the Proca vector potential of the galactic magnetic field may be neutralized by vortices giving a large-scale magnetic field that is effectively Maxwellian. If, on the other hand, the galactic magnetic field is in the Proca regime, the very existence of the observed large-scale magnetic field gives $m^{-1} \gtrsim 1$ kpc, or $m \lesssim 10^{-26}$ eV.

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Introduction.—The possibility of a nonzero photon mass remains one of the most important issues in physics, as it would shed light on fundamental questions such as charge conservation, charge quantization, the possibility of charged black holes and magnetic monopoles, etc. The most stringent upper bounds on the photon mass listed by the Particle Data Group [1], $m < 3 \times 10^{-27}$ eV and $m < 2 \times 10^{-16}$, are based on the assumption that a massive photon would cause large-scale magnetic fields to be accompanied by an energy density

$$m_A^2 \tilde{A}_\mu \tilde{A}^\mu \tag{1}$$

associated with the Proca field $\tilde{A}_\mu$ that describes the massive photon [2]. This manifests itself in two different ways. The first limit comes from the potential astrophysical effects [3,4], and the second from an experiment that used a toroidally magnetized pendulum [5] to measure the magnetic field gradient in a magnetically shielded vacuum. A recent experiment [6] using an improved technique obtained a similar result. Both experiments actually measured the product $m^2\tilde{A}$, where the ambient Proca vector potential is presumably dominated by the field of the galaxy. The value assumed in [5,6], $\tilde{A} \sim RB \sim 1\ \mu\text{G} \times$ kpc, is astronomically reasonable as the large-scale, $R \sim 1$ kpc, galactic field has a strength $B \sim 1\ \mu$G.

Let us review the standard arguments behind these bounds, which assume that a massive photon at low energies is described by the Proca field [2]. (Throughout this Letter we denote the Proca field as $\tilde{A}_\mu$, whereas A_μ should be understood as the Maxwellian field.) The Lagrangian density for $\tilde{A}_\mu$ is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2} m_A^2 \tilde{A}_\mu \tilde{A}^\mu + \tilde{A}_\mu J^\mu, \tag{2}$$

where $F_{\mu\nu}$ is the usual field strength and J^μ is the conserved current. The m_A^2 term in Eq. (2) makes $\tilde{A}_\mu$ experimentally observable. Naively, the existence of the Galactic magnetic field $\mathbf{B}_{\text{gal}} = \nabla \times \tilde{\mathbf{A}}_{\text{gal}}$ implies an ambient galactic vector potential

$$\tilde{A}_{\text{gal}} \sim B_{\text{gal}} R_{\text{gal}}. \tag{3}$$

The associated Proca energy $m_A^2 \tilde{\mathbf{A}}^2$ can be detected by direct or indirect observations. Indirect observations rely on the effect the Proca energy would have on the galactic plasma [4], implying the limit $m_A < 3 \times 10^{-27}$ eV. The direct detections [5] are based on measuring the torque on a magnetized ring, which depends on the angle between $\tilde{A}_{\text{gal}}$ and the vector potential of the ring, because the energy density contains a term $\sim m_A^2 \tilde{\mathbf{A}}_{\text{gal}} \cdot \tilde{\mathbf{A}}_{\text{ring}}$. The null result [5] implies the limit $m_A \lesssim 2 \times 10^{-16}$ eV.

We claim that photon-mass bounds cannot be established without specifying the microscopic origin of the mass. In particular, if m arises from the commonly accepted Higgs mechanism, the above bounds do not apply over a large portion of the parameter space. It is quite possible for large-scale magnetic fields to be effectively Maxwellian, even if photons are massive. In this case observations of large-scale fields, say from the galaxy or from Jupiter, are not sensitive to m. This leaves us with the upper bound from laboratory tests of Coulomb's law, $m \lesssim 10^{-14}$ eV [7]. It is also possible that the large-scale fields do remain in the Proca regime. But then, the available information about the large-scale magnetic field of the galaxy, and the gas pressure in the galaxy, actually gives a much more stringent bound, $m^{-1} \gtrsim R \sim 1$ kpc, or $m \lesssim 10^{-26}$ eV [3,4].

The Higgs scenario.—If photon has a nonzero mass, there are excellent field-theoretic arguments for thinking it should arise from a Higgs-type effect, in which case the above mass limits are invalid. Note that when estimating the Proca field associated with the galactic magnetic field, one must distinguish the actual Proca field $\tilde{A}_\mu$, which can be measured, from its Maxwellian component A_μ; $\tilde{A}_\mu$ has 3 physical degrees of freedom (polarizations) as opposed to

A_μ, which has only 2. So that by giving a mass term to A_μ, we are supplementing it with an additional degree of freedom. The Proca field can be written as

$$\tilde{A}_\mu = A_\mu - \frac{1}{g}\partial_\mu \psi, \tag{4}$$

where ψ is the additional (longitudinal) polarization. Written in this way, the Proca theory is manifestly gauge invariant under

$$A_\mu \to A_\mu + \frac{1}{g}\partial_\mu \omega, \qquad \psi \to \psi + \omega. \tag{5}$$

The new degree of freedom enters the Proca action only in the mass term; it cancels in $F_{\mu\nu}$ as well as in the couplings to the conserved current. When computing the galactic Proca field, we must be sure it is not compensated by the additional polarization. This is, in fact, what happens in the Higgs scenario.

Proca theory, Eq. (2), can be extended to the Higgs theory, by promoting m_A into a real scalar field $\phi = m_A/g$, with the self-Lagrangian

$$\frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda^2}{2}(\phi^2 - \eta^2)^2. \tag{6}$$

Then, ϕ can be thought of as the modulus of a complex Higgs field $H = \phi e^{i\psi}$, and ψ is its phase (Goldstone boson), which becomes a longitudinal photon. The static energy in the absence of the electric field is

$$\mathcal{E} = \int d^3x\bigg[\frac{B^2}{2} + \frac{g^2}{2}\phi^2\Big(\mathbf{A} - \frac{1}{g}\nabla\psi\Big)^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{\lambda^2}{2}(\phi^2 - \eta^2)^2\bigg]. \tag{7}$$

The two important parameters of the theory are the mass of the photon $m_A = g\eta$, and the mass of the Higgs particle $m_\phi = \lambda\eta$. With frozen $\phi = \eta$, the theory simply reduces to Proca theory with the energy

$$\mathcal{E} = \int d^3x\Big(\frac{B^2}{2} + \frac{g^2}{2}\eta^2\tilde{\mathbf{A}}^2\Big), \tag{8}$$

where now $\tilde{A}$ carries 3 degrees of freedom, only two of which contribute into B.

Naively, one may think that as long as $\lambda^2\eta^4$ is bigger than the energy density of the galactic magnetic field B^2 (with $B \sim \mu\mathrm{G} \sim 10^{-25}\ \mathrm{GeV}^2$), one can ignore the fluctuations in the Higgs field, and the theory should effectively reduce to Proca. This is not correct, because the system has a choice of lowering a huge Proca energy, stored over a large volume, by locally exciting the Higgs field. Even if the Higgs field is much heavier than the value of the magnetic field in question for a big portion of parameter space, the local Higgs energy is in fact less costly than the alternative Proca gauge-field energy. Crudely speaking, if m is due to a Higgs effect, then the Universe is effectively a type II superconductor where magnetic fields create Abrikosov vortices.

This effect can be readily demonstrated by taking an example with a constant magnetic field $B_z = 2B$. The corresponding Maxwellian vector potential (up to gauge transformation) is $A_\theta = Br$, and in the absence of a third polarization could naively contribute a divergent Proca energy. However, this energy is canceled by nontrivial winding of longitudinal photon. The Proca energy density is

$$g^2|\phi|^2\Big(Br - \frac{1}{gr}\partial_\theta\psi\Big)^2, \tag{9}$$

which cancels if on average $\partial_\theta\psi = gBr^2$. This configuration of ψ is impossible in the Proca theory, but can occur in the Higgs case if there is an uniform density of zeros of Higgs field ϕ, around which the phase winds nontrivially, producing vortices. The integral around a closed circle of radius r

$$\frac{1}{2\pi}\int \partial_\theta\psi = N(r) \tag{10}$$

defines a winding number $N(r)$ which is equal to the number of vortices located inside the circle. Around each vortex ψ changes by 2π. The system cancels the Proca energy by creating uniform a density of vortices $n = gB/\pi$. The cancellation cannot be exact because $N(r)$ is a discrete number, so the residual Proca energy density is $\sim gB\eta^2$. Equating this to the Higgs energy $gB\eta^2 \sim \lambda^2\eta^4$, we get a critical value of the magnetic field B_c

$$B_c \sim \lambda^2\eta^2/g. \tag{11}$$

For $B > B_c$ it is energetically favorable for ϕ to vanish everywhere, and the theory becomes Maxwellian. The same value of B_c can be obtained by requiring that the Higgs cores overlap. That is, the intervortex distance becomes equal to the inverse Higgs boson mass:

$$\frac{1}{\sqrt{n_c}} \sim \frac{1}{\sqrt{B_c g}} \sim \frac{1}{\lambda\eta} \tag{12}$$

Critical values of B.—There are several interesting critical values of the system parameters. The first is the magnetic field given by Eq. (11). For $B > B_c$ the Higgs VEV vanishes and the photon becomes massless. Then the lab experiments would not measure anything. Even if the galactic magnetic field is above B_c this limit can still be of interest, because the extragalactic magnetic field can be below B_c. Then the photon will be massive outside the galaxy, but massless inside. In such a regime, the information about m can only come from extragalactic observations.

For $B < B_c$, the photon is massive everywhere, and there are two regimes: Proca and non-Proca.

The system can be in the Proca regime (i.e., vortex formation is unfavorable) only when its size satisfies $R \lesssim$

$1/\sqrt{gB}$. For the galaxy, assuming $m_A \sim 10^{-14}$ eV and $\lambda = 1$, this limit requires $\eta \sim 10^{22}$ GeV.

If $R \gtrsim 1/\sqrt{gB}$, vortices are energetically favored, but two subregimes are possible. The first occurs when $R \gtrsim \lambda\eta/gB$. Then the system classically creates vortices out of vacuum and neutralizes the Proca energy. For the galaxy, assuming $m_A \sim 10^{-14}$ eV and $\lambda = 1$, this requires $g \sim 10^{-16}$ and $\eta \sim 10^2$ eV. Such a light, weakly charged Higgs boson is compatible with all existing experimental data and naturalness bounds. At each point, the typical number of magnetically overlapping vortices is $\tilde{N} \sim m_A^{-2} gB$. If $\tilde{N} \gg 1$, the field is effectively Maxwellian. For the galaxy, assuming $m_A \sim 10^{-14}$ eV and $g \sim 10^{-16}$, we get $\tilde{N} \sim 10^5$. (Situations with $\tilde{N} \sim 1$ could provide experimental signatures that would be smoking guns for the Higgs scenario.)

The opposite case occurs when $R \lesssim \lambda\eta/gB$. Vortices are still energetically favorable, but the system cannot create them classically so that their existence will depend on preexisting conditions such as phase transitions in the early Universe. Because of its very small charge, the Higgs field decoupled from ordinary matter very early so that a phase transition with vortex formation could have preceded formation of the magnetic field. The evolution of such vortices is not yet understood, but is expected to be different from more conventional cosmic-string networks [8].

Although we focused on Proca-Higgs cases, our analysis can be extended to alternative gauge-invariant, ghost-free theories of the photon mass [9] in which the E field of a point charge for $r \ll m^{-1}$ is not screened but rather modified to a higher inverse power law $\sim 1/r^3$. In such cases the constraints may be even milder.

Primordial magnetic field.—As an interesting by-product, the nonzero photon mass naturally predicts generation of a self-sustained primordial magnetic field in the early Universe. Indeed, if the photon acquires mass by the Higgs mechanism, then the thermal phase transition in ϕ would inevitably form vortices by the Kibble mechanism [10].

Because of the small charge, ϕ is never in thermal equilibrium with the standard model species, but because of the large self-coupling, it is in thermal equilibrium with itself. Thus, due to the usual high-temperature symmetry restoration, the expectation value of ϕ had to vanish at early times. The only situation in which ϕ would not vanish would be if it never were in a thermal equilibrium since inflation. This is unlikely: even if ϕ had no direct coupling with inflaton, it would still be produced gravitationally with a Gibbons-Hawking temperature ($T_{GH} \sim 10^{14}$ GeV for the standard inflation), unless $m_\phi > T_{GH}$.

The vortices are produced when the temperature of the ϕ field drops to $T_\phi \sim m_\phi$. The standard big bang nucleosynthesis requires that the temperature in ϕ quanta be smaller than the temperature in the standard model sector $T_\phi < T_{SM}$, and the vortex network would form before galaxies if

$$m_\phi > \frac{T_\phi}{T_{SM}} 10^{-3} \text{ eV}. \tag{13}$$

Thus, formation of a primordial magnetic field is a direct consequence of the photon mass.

The Proca regime.—If the galaxy is in the Proca regime the averaged magnetic pressure is (see below)

$$p_{\text{magnetic}} = \frac{\mathbf{B}^2}{24\pi} - \frac{m^2\tilde{\mathbf{A}}^2}{24\pi}. \tag{14}$$

(The electric pressure is much smaller because the interstellar medium is a good conductor.) In a stable system, this magnetic pressure must be counterbalanced by the plasma pressure and/or the plasma kinetic energy. The interstellar medium of our galaxy is in approximate equipartition assuming conventional electrodynamics [11]; the kinetic energy density of the plasma, the plasma pressure, and the standard magnetic energy density $B^2/(8\pi)$ are comparable to each other. Therefore, the "massive" part of the full magnetic pressure cannot exceed the standard part $m^2\tilde{\mathbf{A}}^2 \lesssim \mathbf{B}^2$, which, together with the estimate $\tilde{A} \sim RB$, gives our bound

$$m \lesssim R^{-1} \lesssim 10^{-26} \text{ eV}. \tag{15}$$

The estimate $\tilde{A} \sim RB$ simply means that $\tilde{A}$ is the integral of B, and we should not worry about the Proca magnetohydrodynamics being largely unexplored.

The upper bound (15), derived from energy equipartition, was already discussed by Yamaguchi [3], but then dismissed because the energy source of the magnetic field is unknown [2]. But, no matter what the source of the energy, the interstellar medium of the galaxy must provide the pressure support against the anomalous negative magnetic pressure, and the Yamaguchi estimate is, in fact, correct.

We can repeat the above analysis in terms of the Lorentz force. In Proca theory, one can still calculate magnetic fields from Ampere's law, if a new current density $m^2\tilde{\mathbf{A}}$ is added to the usual electric current $\mathbf{j}$. Then, approximately, $B \sim Rj + m^2R^2B$. Assume for the moment that m is equal to the Particle Data Group upper bound, $\sim 10^{-16}$ eV. Then $m^2R^2 \gg 1$, and the usual current must precisely balance the "massive current," $4\pi\mathbf{j} = m^2\tilde{\mathbf{A}}$. But this current results in a huge Lorentz force density jB, which cannot be possibly counterbalanced by the large-scale pressure gradients or accelerations.

Finally, the same upper bound can be presented as a virial theorem [4]. The virial theorem relates mean values of different forms of energy for a system of particles and fields executing a bound motion. For the Proca theory, the energy-momentum tensor is

$$T^\mu_\nu = \frac{1}{4\pi}\left(-F^{\mu\alpha}F_{\nu\alpha} + \frac{1}{4}\delta^\mu_\nu F^2\right) + \frac{m^2}{4\pi}\left(\tilde{A}^\mu\tilde{A}_\nu - \frac{1}{2}\delta^\mu_\nu\tilde{A}^2\right). \quad (16)$$

The magnetic pressure is defined as the magnetic part of T^i_i, and gives Eq. (14). The virial theorem, proved along standard lines [12], is

$$\mathcal{E} = -T - \frac{m^2}{4\pi}\int d^3r\langle\tilde{A}^2\rangle, \quad (17)$$

where $\mathcal{E}$ is the energy of the system, T is the mean kinetic energy of the particles, and $\langle\tilde{A}^2\rangle = \langle\Phi^2\rangle - \langle\tilde{\mathbf{A}}^2\rangle$ is the mean squared 4-potential. Assuming that B fields are much larger than the E fields, we obtain

$$-U_g + \frac{m^2}{8\pi}\int d^3r\langle\tilde{\mathbf{A}}^2\rangle = 2T + \frac{1}{8\pi}\int d^3r\langle\mathbf{B}^2\rangle, \quad (18)$$

which shows that, for virialized motion, kinetic energy (which includes plasma pressure and kinetic energy of the macroscopic motion) plus Maxwellian magnetic energy is equal to the gravitational energy plus the Proca part of the magnetic energy. The bound in Eq. (15) assumes that the virial theorem can be applied, approximately, to the random part of the galactic motion, after the mean rotation of the galaxy has been excluded.

Conclusions.—When trying to measure m one must distinguish between measurements performed on large and small scales. If the photon acquires mass by the Higgs mechanism, the large-scale behavior of the photon might be effectively Maxwellian. If, on the other hand, one postulates the Proca regime for all scales, the very existence of the galactic field implies $m < 10^{-26}$ eV, as correctly calculated by Yamaguchi and Chibisov [3,4].

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[1] K. Hagiwara *et al.* (Particle Data Group), Phys. Rev. D **66**, 010001 (2002).
[2] A. S. Goldhaber and M. M. Nieto, Rev. Mod. Phys. **43**, 277 (1971).
[3] Y. Yamaguchi, Prog. Theor. Phys. Suppl. **11**, 33 (1959).
[4] G. V. Chibisov, Sov. Phys. Usp. **19**, 624 (1976).
[5] R. Lakes, Phys. Rev. Lett. **80**, 1826 (1998).
[6] J. Luo, L.-C. Tu, Z.-K. Hu, and E.-J. Luan, Phys. Rev. Lett. **90**, 081801 (2003).
[7] E. R. Williams, J. E. Faller, and H. A. Hill, Phys. Rev. Lett. **26**, 721 (1971).
[8] A. Vilenkin, Phys. Rep. **121**, 263 (1985).
[9] G. Dvali, G. Gabadadze, and M. Shifman, Phys. Lett. B **497**, 271 (2001).
[10] T. W. B. Kibble, J. Phys. A **9**, 1387 (1976).
[11] F. Shu, *The Physical Universe* (University Science Books, California, 1982).
[12] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Butterworth-Heinemann, Oxford, 2002).