

Nonlinear Magnetoconductance of a Classical Ballistic System

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(Received 5 September 2006; published 28 December 2006)

We study nonlinear transport through a classical ballistic system accounting for the Coulomb interaction between electrons. The joint effect of the applied bias V and magnetic field H on the electron trajectories results in a component of the nonlinear current $I(V, H)$ which lacks the $H \rightarrow -H$ symmetry: $\delta I = \alpha_{\text{cl}} V^2 H$. At zero temperature the magnitude of α_{cl} is of the same order as that arising from the quantum interference mechanism. At higher temperatures the classical mechanism is expected to dominate due to its relatively weak temperature dependence.

DOI: 10.1103/PhysRevLett.97.266806

PACS numbers: 73.23.Ad, 73.50.Jt

By Onsager symmetry [1] the electric current through a two terminal device in the linear regime is an even function of the external magnetic field H . The nonlinear current does not necessarily satisfy this symmetry, as was recently observed in experiments [2–6]. In fact, it is violated already in the second order in the external bias V : It was shown in Refs. [7,8] that the nonlinear current acquires a component that is odd in H ,

$$\delta I = \alpha V^2 H. \quad (1)$$

The violation of the $H \rightarrow -H$ symmetry in the nonlinear current is associated with electron-electron interaction.

Electron-electron interactions result in inelastic scattering of electrons. The standard phase space argument shows that the inelastic scattering cross section varies as ϵ^2 , with ϵ being the electron energy, thus contributing to the current only at order V^3 and higher. Therefore, in order to evaluate the coefficient α in Eq. (1) one may neglect inelastic processes and treat electrons as moving independently in the presence of an effective potential which depends on V and H . Within such a treatment [7,8] the bias induces an additional electron density that is linear in the current and has an odd in H component. In the presence of interactions the additional density changes the scattering potential, which results in the nonlinear current (1).

In this approximation electron motion is phase coherent. Therefore a part of the nonlinear current is sensitive to the electron wave interference. The corresponding contribution α_q to the coefficient α in Eq. (1) was studied in Refs. [7–10]. It is a mesoscopic quantity whose H dependence arises from the interference pattern sensitivity to the magnetic flux threading electron trajectories.

In addition to affecting electron interference the magnetic field also bends electron trajectories. The corresponding contribution α_{cl} to the coefficient α is the subject of the present Letter. In order to evaluate it electron motion can be treated as classical.

We focus on the effect of the long range part of the Coulomb interaction and show that its contribution to α_{cl} is

independent of the interaction parameter $e^2/\hbar v_F$ (here e is the electron charge and the v_F is the Fermi velocity). At small $e^2/\hbar v_F$ this is the leading contribution to α_{cl} .

For an open two-dimensional ballistic dot the magnitude of the classical contribution is

$$\alpha_{\text{cl}} \sim \frac{e^4 n d^2}{\epsilon_F^2 m c}, \quad (2)$$

where d is the dot size, and n , ϵ_F , and m are the electron density, Fermi energy, and mass, respectively. In Eq. (2) e^4 arises not from the electron-electron interaction but from the coupling of electrons to the magnetic field H and potential V , and from the definition of current I .

At zero temperature the interference contribution is [7] $\alpha_q \sim \beta \frac{e^4}{\nu E_T \hbar^2 c}$, where ν is the density of states at the Fermi level, β is the dimensionless interaction constant, and E_T is the Thouless energy. In the ballistic case $E_T \sim \hbar v_F/d$ and for $\beta \sim 1$ the coefficients α_{cl} of Eq. (2) and α_q are of the same order. Since α_q decays with temperature on the scale $T \sim E_T$ [6] and α_{cl} is insensitive to T for $T < \epsilon_F$ we expect the classical contribution to Eq. (1) to dominate for $T \gtrsim E_T$ [11].

The classical and the quantum interference contributions can be distinguished by their respective scales of the magnetic field dependence. The linear H dependence (1) holds as long as the cyclotron radius significantly exceeds the system size d . This limitation yields the magnetic field scale $H_{\text{cl}}^* \sim m v_F c / (e d)$. The interference contribution [7,8] is sensitive to the flux threading a typical electron trajectory. Assuming that electron motion is not chaotic we estimate the area of such a trajectory to be d^2 . Equating the flux piercing a typical trajectory to the flux quantum $\Phi_0 = hc/e$, we find the characteristic field for the interference contribution is much smaller than the classical one, $H_Q^* \sim \Phi_0/d^2 \sim (\lambda_F/d) H_{\text{cl}}^*$, with λ_F being the Fermi wavelength.

Similarly to the quantum interference contribution [7,8] the magnitude and the sign of the classical contribution depend on the sample geometry.

While the existence of the nonlinear current Eq. (1) in a classical system and the estimate, Eq. (2), are quite general, below we concentrate on a specific setup consisting of a ballistic point contact in a two-dimensional electron gas (2DEG) with an adjacent “reflector”; see Fig. 1. The reflector creates the spatial asymmetry necessary for “rectification” current $\sim V^2$. (The linear electron transport in systems of this type was studied experimentally in Ref. [12].) Electron motion in such a system is nonchaotic. This is essential for the validity of our results and the estimate (2). The case of chaotic classical motion is beyond the scope of this work.

In the classical description of electron transport the local momentum distribution function of electrons, $n(\mathbf{p}, \mathbf{r})$, plays a central role. The current I across the contact is

$$I = \int d\mathbf{S} \cdot \mathbf{j}(\mathbf{r}), \quad \mathbf{j}(\mathbf{r}) = \frac{2e}{m} \int \frac{d^2p}{(2\pi\hbar)^2} \mathbf{p}n(\mathbf{r}, \mathbf{p}), \quad (3)$$

where $\mathbf{j}(\mathbf{r})$ is the current density and the vector $d\mathbf{S}$ is normal to the contact line. A voltage bias applied to the contact results in a nonequilibrium, anisotropic electron distribution $n(\mathbf{r}, \mathbf{p})$ yielding a finite current I .

In order to determine the nonlinear I - V characteristic, we need to find the steady-state nonequilibrium distribution function $n(\mathbf{r}, \mathbf{p})$. This can be done by solving the classical equations of motion for electrons moving in the presence of a self-consistent electrostatic potential $\phi(\mathbf{r})$. The initial conditions for the electron dynamics correspond to the equilibrium distributions deep inside the leads with electrochemical potentials differing by eV . Conservation of electron energy enables us to express the electron distribution at any point \mathbf{r} in the form

$$n(\mathbf{r}, \mathbf{p}) = \begin{cases} f(\epsilon_p - \mu_0 + e\phi(\mathbf{r}) - eV), & \mathbf{p} \in L(\mathbf{r}), \\ f(\epsilon_p - \mu_0 + e\phi(\mathbf{r})), & \mathbf{p} \in R(\mathbf{r}). \end{cases} \quad (4)$$

Here $\epsilon_p = p^2/(2m)$ is the electron kinetic energy, μ_0 is the

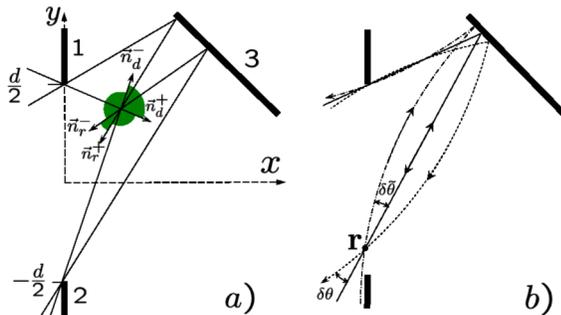


FIG. 1 (color online). Point contact, thick lines 1 and 2, with an adjacent reflector, 3. The reflector line is $y = L - x$ with $(L - d)/2 \leq x \leq (L + d)/2$. (a) Separatrix trajectories and the nonequilibrium electron distribution at $H = 0$. Vectors $\vec{n}_{d/r}^{\pm}$ represent velocity directions on the separatrix trajectories. Angles between $\vec{n}_{d/r}^{\pm}$ and the x axis are denoted by $\theta_{d/r}^{\pm}$, respectively. (b) Original (dashed line) and modified (dash-dotted line) separatrix trajectories at $H \neq 0$, and a separatrix at $H = 0$ (solid line).

equilibrium chemical potential, $f(\epsilon) = [\exp(\epsilon/T) + 1]^{-1}$ is the Fermi function, and we assumed that the voltage eV is applied to the left lead. The solution of the electron equations of motion is encoded in the shapes of the complementary momentum space domains, $L(\mathbf{r})$ and $R(\mathbf{r})$. Electrons with momenta in these domains arrive to \mathbf{r} from the left and right lead, respectively.

The electric potential $\phi(\mathbf{r})$ is determined from the Poisson equation. Its solution is greatly simplified if the screening radius is short compared with the geometrical characteristics of the setup. Under that condition, the charge density is unchanged by the applied bias. A further simplification is possible if the width w of the depletion layers confining the 2DEG is small compared to the geometrical features of the system. The electron density increases from zero in the depletion region to its bulk value n_0 over a distance of order w ; see Ref. [13]. Thus at small w we may assume a constant electron density

$$2 \int \frac{d^2p}{(2\pi\hbar)^2} n(\mathbf{p}, \mathbf{r}) = n_0 \quad (5)$$

inside the 2DEG. This is a reasonable approximation for devices fabricated using the local oxidation method [14]. The latter enables fabrication of structures with very small lateral depletion widths, $w \sim 150 \text{ \AA} \sim \lambda_F$ [14].

In order to find the current with the accuracy $\propto V^2$, we will solve the transport problem defined by Eqs. (3)–(5) using the following iterative procedure. At the first step, we find the electron trajectories at $\phi(\mathbf{r}) = 0$. This defines the zeroth iteration for the momentum domains $L^{(0)}(\mathbf{r})$ and $R^{(0)}(\mathbf{r})$. The distribution function in this approximation depends on $\phi(\mathbf{r})$ only explicitly, through the arguments of the equilibrium distribution functions in Eq. (4). Substituting $n(\mathbf{p}, \mathbf{r})$ into Eq. (5) we obtain an equation for $\phi^{(1)}(\mathbf{r})$ which is valid to the first order in V . After finding $\phi^{(1)}(\mathbf{r})$, we determine corrections to the electron trajectories and the corresponding corrections to the momentum domains. The corrected momentum domains $L(\mathbf{r})$ and $R(\mathbf{r})$ and $\phi^{(1)}(\mathbf{r})$ determine the distribution function via Eq. (4). Substitution of the latter into Eq. (3) gives the current to the second order in V .

For our device geometry each of the domains $L(\mathbf{r})$ and $R(\mathbf{r})$ consists of two separate sectors. The two sectors in $L(\mathbf{r})$ correspond to trajectories arriving to \mathbf{r} from the left lead either directly through the contact, or upon reflection from the obstacle; see Fig. 1. The boundaries of domains $L(\mathbf{r})$ and $R(\mathbf{r})$ are defined by a set of four energy-dependent angles, $\theta_{d/r}^{\pm}(\mathbf{r}, \epsilon_p)$ between the x axis and the velocity vector at point \mathbf{r} for the separatrix trajectories. These trajectories pass through the edge points of the contact and through point \mathbf{r} . The indices $\pm d/r$ denote the trajectories that pass through the top (+) [bottom (–)] edge point of the contact and arrive at \mathbf{r} either directly (d) or upon reflection off the obstacle [r].

To evaluate the current I via Eq. (3), we express the x component of the current density $j_x(\mathbf{r})$ at the contact line

$x = 0$ in terms of the angles $\theta_{d/r}^{\pm}(\mathbf{r}, \epsilon)$:

$$j_x(\mathbf{r}) = j_x^d(\mathbf{r}) + j_x^r(\mathbf{r}), \quad (6a)$$

$$j_x^d(\mathbf{r}) = \frac{e\nu_0}{\pi} \int_{\epsilon_-}^{\epsilon_+} d\epsilon \sqrt{\frac{2\epsilon}{m}} \sin\theta \Big|_{\theta_d^+(\mathbf{r}, \epsilon)}^{\theta_d^-(\mathbf{r}, \epsilon)}, \quad (6b)$$

$$j_x^r(\mathbf{r}) = \frac{e\nu_0}{\pi} \int_{\epsilon_-}^{\epsilon_+} d\epsilon \sqrt{\frac{2\epsilon}{m}} \sin\theta \Big|_{\theta_r^+(\mathbf{r}, \epsilon)}^{\theta_r^-(\mathbf{r}, \epsilon)}, \quad (6c)$$

and treat I as the difference:

$$I = I_d - I_r, \quad I_d = \int dS j_x^d(\mathbf{r}), \quad I_r = - \int dS j_x^r(\mathbf{r}). \quad (7)$$

Here $\nu_0 = \frac{m}{2\pi\hbar^2}$ is the electron density of states per spin projection, $\epsilon_- = \mu_0 - e\phi(\mathbf{r})$, $\epsilon_+ = \mu_0 - e\phi(\mathbf{r}) + eV$, and the integrals over dS go along the contact line. We set $T = 0$ for brevity. The generalization to finite temperature is straightforward; the scale for the temperature dependence of the current is set by the Fermi energy μ_0 .

Initiating the iterations, we find the angles $\theta_{d/r}^{\pm} = \theta_{d/r}^{\pm(0)}(\mathbf{r}, \epsilon)$ at $V = 0$. It is clear from Eq. (6) that to find the linear term in the I - V characteristic, it is sufficient to know $\theta_{d/r}^{\pm(0)}(\mathbf{r}, \epsilon)$ only at the Fermi energy. Equation (6) also indicates that the spatial dependence $\phi(\mathbf{r})$ does not affect the linear conductance [15].

Next, using Eqs. (4) and (5) we obtain the first iteration for the potential $\phi(\mathbf{r})$,

$$\frac{\phi^{(1)}(\mathbf{r})}{V} = 1 - \frac{\theta_d^{+(0)}(\mathbf{r}) - \theta_d^{- (0)}(\mathbf{r}) - \theta_r^{+(0)}(\mathbf{r}) + \theta_r^{- (0)}(\mathbf{r})}{2\pi}. \quad (8)$$

Here the angles θ are evaluated at the Fermi level $\epsilon = \mu_0$. This simplification is possible because the electron distributions in the domains $L(\mathbf{r})$ and $R(\mathbf{r})$ differ from each other only within a narrow energy strip of width eV around the Fermi energy.

Finding the electric field from Eq. (8) we determine the correction to electron trajectories in the linear order in V . This defines the first iteration of the angles $\theta_{d/r}^{\pm(1)}(\mathbf{r}, \epsilon)$. To find the current to the order V^2 we substitute

$$\theta_{d/r}^{\pm}(\mathbf{r}, \epsilon) = \theta_{d/r}^{\pm(0)}(\mathbf{r}, \epsilon) + \theta_{d/r}^{\pm(1)}(\mathbf{r}, \mu_0) \quad (9)$$

into Eqs. (6) and (7). Note that we set $\epsilon \rightarrow \mu_0$ in $\theta^{(1)}$ because $\theta^{(1)}$ is already proportional to V .

In the absence of magnetic field and at zero bias the electron trajectories are straight lines. Therefore the angles $\theta^{(0)}$ do not depend on the energy and can be easily found from the geometric construction; see Fig. 1:

$$\theta_d^{+(0)}(\mathbf{r}) = \text{Im} \ln[x + iy + id/2], \quad (10a)$$

$$\theta_r^{+(0)}(\mathbf{r}) = \text{Im} \ln[x + iy - L(1+i) \pm d/2]. \quad (10b)$$

The first iteration $\theta^{(1)}(r)$ is determined from the condition that the velocity direction at point \mathbf{r} must change in the

presence of the external force $\nabla\phi^{(1)}$ in such a way that the edge point of the contact still belongs to the separatrix trajectory. A perturbative solution of Newton's equations of motion gives

$$\theta^{(1)} = \frac{1}{2l_0\nu_F^2} \frac{e}{m} \int_0^{l_0} dl \int_0^l dl' \nabla_{\perp} \phi^{(1)}(l'), \quad (11)$$

where l_0 is the length of the unperturbed trajectory, l is the coordinate along the trajectory, and $\nabla_{\perp} \phi^{(1)}(l')$ is the component of the electric potential gradient that is perpendicular to the trajectory. Substitution of the angles Eq. (10) found in the zeroth order into Eq. (8) and subsequent solution of Eq. (11) yield corrections $\theta_{d/r}^{\pm(1)}$ which are proportional to V and depend on the geometry of the device. Substitution of the found $\theta_{d/r}^{\pm(0)}$ and $\theta_{d/r}^{\pm(1)}$ into Eq. (6) yields the I - V characteristic with linear and $\propto V^2$ terms:

$$I = G_{\text{Sh}} V \left[1 - \frac{d}{4\sqrt{2}L} \left(1 + \frac{1}{\pi\sqrt{2}} \frac{eV}{\mu_0} \right) \right], \quad (12)$$

where $G_{\text{Sh}} = 2e^2 m \nu_F d / \pi \hbar^2$ is the Sharvin conductance of the point contact [16]. The rectification ($\propto V^2$) term and the corrections to the linear Sharvin conductance exist only due to the electron trajectories which are reflected by the obstacle back into the contact. The relative weight of such trajectories, contributing to Eq. (6c), is $\propto d/L$. In writing Eq. (12) we took the limit $d \ll L$, in order to simplify the result. In this limit the rectification term arises solely from the energy dependence of the velocity $\sqrt{2\epsilon/m}$ in the integrand of Eq. (6).

Now we evaluate the influence of the magnetic field, assumed to be perpendicular to the plane of motion. In this case it is convenient to evaluate the ‘‘back-current’’ I_r in Eq. (7) in a different way. Equations (6c) and (12) express the ‘‘back current’’ in terms of the directions of velocities of electrons that are already reflected from the obstacle and head toward the contact. We may, instead, evaluate the back current by accounting for those trajectories of electrons that head toward the obstacle and will subsequently scatter back in the contact. We introduce the angles $\tilde{\theta}_r^{\pm}(\mathbf{r})$ between the x axis and the velocity direction at point \mathbf{r} for the modified separatrix trajectories that start from \mathbf{r} and, upon reflection from the barrier, arrive to the top or bottom edge point of the contact; see Fig. 1(b). In terms of these angles, the back current is

$$I_r' = \frac{e\nu_0}{2\pi} \int dS \int_{\epsilon_-}^{\epsilon_+} d\epsilon \sqrt{\frac{2\epsilon}{m}} \sin\theta \Big|_{\tilde{\theta}_r^+(\mathbf{r}, \epsilon)}^{\tilde{\theta}_r^-(\mathbf{r}, \epsilon)}. \quad (13)$$

Here the integration is performed along the contact line. The stationary-state current conservation law dictates $I_r' = I_r$. It is convenient for us to replace I_r in Eq. (7) by $(I_r + I_r')/2$. Upon this substitution, we find

$$I = \frac{e\nu_0}{2\pi} \int dS \int_{\epsilon_-}^{\epsilon_+} d\epsilon \sqrt{\frac{2\epsilon}{m}} \left[4 - \sin\theta \left| \frac{\theta_r^-(\mathbf{r}, \epsilon)}{\theta_r^+(\mathbf{r}, \epsilon)} \right| - \sin\theta \left| \frac{\tilde{\theta}_r^+(\mathbf{r}, \epsilon)}{\tilde{\theta}_r^-(\mathbf{r}, \epsilon)} \right| \right]. \quad (14)$$

To arrive at this equation we used the fact that on the contact line $\theta_d^\pm(\mathbf{r}) = \pm\pi/2 + O(H, V)$. This enabled us to set $\sin\theta_d^\pm(\mathbf{r}, \epsilon) = \mp 1$ in Eq. (6b).

The modified separatrix trajectories coincide with the time-reversed original separatrix trajectories in the opposite magnetic field and unchanged electric potential. Therefore, the modified angles can be expressed in terms of the original ones as $\tilde{\theta}_r^\pm(\mathbf{r}, H) = \theta_r^\pm(\mathbf{r}, -H) - \pi$. Thus Eq. (14) explicitly shows the $H \rightarrow -H$ symmetry of the linear current, in agreement with the Onsager relations.

In the presence of a magnetic field, the trajectories become curved even at zero bias. We denote the deviation of the angle θ from its value at $H = 0$ by $\delta\theta$. For a trajectory originating from \mathbf{r}_i and arriving at \mathbf{r}_f without reflection off the barrier in a weak magnetic field we have

$$\delta\theta \approx |\mathbf{r}_f - \mathbf{r}_i|/(2R_c), \quad (15)$$

where $R_c = mvc/(eH)$ is the cyclotron radius. The angles $\theta_d^{\pm(0)}(z)$ acquire the correction $\delta\theta_d^{\pm(0)}(\mathbf{r}) = |x + iy \mp id/2|/(2R_c)$. Though the expressions for the corresponding corrections to the reflected angles $\delta\theta_r^{\pm(0)}$ are more cumbersome they can be straightforwardly obtained from geometric considerations using Eq. (15). Substituting these corrections into Eq. (8) we obtain the correction to the induced potential due to the magnetic field. It has an especially simple form on the contact line, $x = 0$,

$$\delta\phi_H^{(1)}(x = 0, y) = \frac{V}{4\pi R_c} \left[2y + \frac{3d}{2\sqrt{2}} \right]. \quad (16)$$

Although the second term here is caused by the reflected trajectories, the length L defining the barrier position drops out for $L \gg d$. Indeed, from Eq. (15) it follows that the corrections to the reflected angles are $\delta\theta_r^\pm \sim L/R_c$, whereas their difference entering the potential via Eq. (8) has an additional smallness of d/L .

Next we evaluate the current across the contact using Eq. (14). It depends on magnetic field due to the H dependence of the integration limits ϵ_\pm as described by Eq. (16), and of the angles $\theta, \tilde{\theta}$ entering in the integrand. The finite- H corrections to the angles θ and $\tilde{\theta}$ arise from two effects: (i) the direct bending of electron trajectories by the Lorentz force, and (ii) the H dependence of the electric potential, Eq. (16). For our geometry these effects are small: The corrections to θ and $\tilde{\theta}$ due to the first effect are opposites of each other, as illustrated in Fig. 1(b), and drop out from Eq. (14). The correction due to the second effect arises only in the first iteration $\theta^{(1)}$, and can be estimated as $\frac{eV}{\mu_0} \frac{d}{R_c} \frac{d}{L}$. This turns out to be smaller than the correction arising from change of the energy integration limits ϵ_\pm in Eq. (14) by a factor of d/L . Therefore, in order to find the coefficient α_{cl} of the V^2H term in the nonlinear

current to the leading order in d/L we may replace the argument of the sines in Eq. (14) by their $H = 0$ values. Doing so we obtain in the leading order in d/L

$$\alpha_{cl} = \frac{3}{16\pi^2\sqrt{2}} \frac{e^4 n_0 d^2}{\mu_0^2 mc} \quad (17)$$

in agreement with the estimate Eq. (2).

While the ‘‘rigid-wall’’ confinement model applies to some setups [3], the use of gates and scanning probes results in a ‘‘soft’’ confining potential changing over length scales of the order of the size of the contact [12]. This difference affects only the numerical coefficient in α_{cl} .

In summary, we have shown that bending of classical electron trajectories by a magnetic field H results in a component of the nonlinear current δI that is odd in H , Eq. (1). For a ballistic structure in which electron motion is not chaotic the low temperature classical contribution to δI , Eq. (2), is of the same order as the one arising from quantum interference [7,8]. At higher temperatures the classical contribution is dominant.

This work was supported in part by NSF Grants No. DMR 02-37296 and No. DMR 04-39026 (L.I.G.) and by the David and Lucille Packard Foundation (A.V.A.). We are grateful to M. Büttiker and B. Spivak for useful discussions. The hospitality of MPIPKS-Dresden and Aspen Center for Physics is gratefully acknowledged.

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