Anomalous Enhancement of a Penguin Hadronic Matrix Element in $B \to K\eta'$

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(Received 11 October 2006; published 29 December 2006)

We estimate the density matrix element for the π^0, η, and η' production from the vacuum in the large-N_c limit. As a consequence, we find that the QCD axial anomaly leads to highly nontrivial corrections to the usual flavor SU(3) relations between $B^0 \to K^0\pi^0$, $B^0 \to K^0\eta$, and $B^0 \to K^0\eta'$ decay amplitudes. These corrections may explain why the $B \to K\eta'$ branching ratio is about 6 times larger than the $B \to K\pi$ one.

DOI: 10.1103/PhysRevLett.97.261804 PACS numbers: 13.25.Hw, 12.39.Fe

First observations of a large branching ratio for $B \to K\eta'$ triggered numerous theoretical investigations within and beyond the standard model. The latest average of the available experimental data [1]

$$Br(B^0 \to K^0\eta') = (64.9 \pm 3.5) \times 10^{-6} \tag{1}$$

definitely confirms a sizable excess of η' compared with π^0, $Br(B^0 \to K^0\pi^0) = (10.0 \pm 0.6) \times 10^{-6}$, and η, $Br(B^0 \to K^0\eta) < 1.9 \times 10^{-6}$. The fact that η' is mostly a flavor $SU(3)$-singlet naturally suggests mechanisms such as the so-called singlet contribution (η' production by gluon-gluon fusion from $b \to sg$ or $b \to sgg$) [2] or intrinsic charm contribution (η' production by $c\bar{c}$ annihilation from $b \to c\bar{c}s$) [3]. However, these do not seem to explain why the $B^0 \to K^0\eta'$ branching ratio is about 6 times larger than the $B^0 \to K^0\pi^0$ one. More recently, the hadronic parameters for η' production have been reexamined, and a couple of different solutions were proposed [4–6]. In this Letter, we analyze an overlooked correction from the axial $U(1)$ anomaly in the hadronic matrix element associated with $b \to s\bar{s}s$.

The derivative of the flavor-singlet axial current is given by

$$\partial_\mu j_0^{\mu 5} = 2i \sum_{q=u,d,s} m_q \bar{q}\gamma_5 q + \frac{3\alpha_s}{4\pi} G_{\alpha\beta}\tilde{G}^{\alpha\beta} \tag{2}$$

where $G_{\alpha\beta}$ is the gluonic field strength tensor and $\tilde{G}^{\alpha\beta}$, its dual. The mass term in the right hand side of Eq. (2) implies explicit flavor $SU(3)$ violation. This breaking alone would lead to the ideal mass relations

$$(M_\eta^2)_{\text{ideal}} = 2M_K^2 - M_\pi^2, \qquad (M_{\eta'}^2)_{\text{ideal}} = M_\pi^2 \tag{3}$$

which are quite successful for the (ϕ, ω, ρ) vector mesons but totally unrealistic for the (η, η', π) pseudoscalar mesons [7]. However, the second term in the r.h.s. of Eq. (2) represents the QCD anomaly which breaks the axial $U(1)$ symmetry to provide the η' with a mass around 1 GeV [8].

The hadronic matrix element $\langle 0|G_{\alpha\beta}\tilde{G}^{\alpha\beta}|\eta^{(\prime)}\rangle$ associated with the anomalous term in Eq. (2) has been considered in relation to the Zweig-suppressed ($J/\psi \to \eta^{(\prime)}\gamma$) radiative decays. In this Letter, we aim at an estimate of the corresponding hadronic matrix elements for the first term in the r.h.s. of Eq. (2). This will then allow us to compute the contribution of the dominant penguin density-density operator to the ($B^0 \to K^0\pi^0$, η, η') hadronic decays in a way fully consistent with the axial $U(1)$ and flavor $SU(3)$ symmetry-breaking requirements.

In a naive quark picture, the hadronic matrix elements $\langle 0|\bar{q}\gamma_5 q|\eta', \eta, \pi^0\rangle$ are simply related through Clebsch-Gordan coefficients (CG). These coefficients are fixed by the $q\bar{q}$ content in the pseudoscalar wave functions. However, corrections due to flavor $SU(3)$ but also to axial $U(1)$ violations have to be taken into account. In particular, we expect the following generic form:

$$\frac{\langle 0|\bar{q}\gamma_5 q|\eta^{(\prime)}\rangle}{\langle 0|\bar{q}\gamma_5 q|\pi^0\rangle}\bigg|_{q=u,d} = \text{CG}\left\{1 + \frac{M_{\eta^{(\prime)}}^2 - M_\pi^2}{\Lambda^2}\right\}. \tag{4}$$

The appearance of the physical η' mass may, at first sight, be surprising. But this is in fact required by the axial $U(1)$ symmetry. Indeed, in the absence of the axial anomaly, the ideal mass relations given in Eq. (3) would consistently imply that the η'/π^0 ratio in Eq. (4) is equal to $+1$ if $q = u$ and -1 if $q = d$ since the η' and π^0 wave functions have the same quark content as the isosinglet ω and the isotriplet ρ in this fictitious world. The axial anomaly calls thus for a sizable correction to the $\langle 0|\bar{q}\gamma_5 q|\eta'\rangle$ hadronic matrix elements if the cutoff scale is what we naturally expect from the real QCD dynamics, namely $\Lambda = \mathcal{O}(1)$ GeV. So, these matrix elements have to be consistently extracted from a low-energy effective theory of QCD.

Again with reference to the observed pattern for the pseudoscalar mass spectrum, let us consider the large-N_c limit at each order in the (squared) momentum p^2. The genuine $U(3)_L \times U(3)_R$ chiral invariant structure of the effective nonlinear theory implies then the following hierarchy [9]:

$$\mathcal{O}(p^0, 1/N_c) > \mathcal{O}(p^2, 1/\infty) > \mathcal{O}(p^4, 1/\infty) \cdots \tag{5}$$

such that the full pseudoscalar mass spectrum naturally arises in three steps. The leading (p^0) term ensures the breaking of the $U(1)_A$ symmetry and provides the flavor-singlet η_0 with a mass around 1 GeV. The next-to-leading (p^2) term implies the usual $SU(3)_V$ mass splitting among the (π, K, η_8) flavor-octet. Eventually, next-to-next-leading (p^4) terms are needed to precisely reproduce the observed $\eta - \eta'$ mass splitting. This hierarchy in the flavor symmetry breakings is compatible with more dynamical approaches based on instantons [10] and recent lattice studies [11].

Starting from this effective theory of QCD in the large-N_c limit, we may consistently express the anomalous operator of Eq. (2) purely in terms of the flavor-singlet field η_0. This allows us to extract the $\eta - \eta'$ mixing angle associated with the diagonalization of the $\eta_8 - \eta_0$ squared mass matrix

$$\eta = \eta_8 \cos\theta - \eta_0 \sin\theta \qquad \eta' = \eta_8 \sin\theta + \eta_0 \cos\theta \quad (6)$$

from the well-measured $(J/\psi \to \eta^{(\prime)}\gamma)$ radiative decays [9]. The extracted value is $\theta = -(22 \pm 1)°$. Let us emphasize once again that in the absence of the axial anomaly, θ would have been equal to the ideal octet-singlet mixing angle $+35.3°$ of the $\phi - \omega$, i.e.,

$$\cos\theta_{\text{ideal}} = \sqrt{2/3}, \qquad \sin\theta_{\text{ideal}} = \sqrt{1/3}. \qquad (7)$$

since the ideal mass relations given in Eq. (3) correspond to the wave functions $\eta = -s\bar{s}$ and $\eta' = (u\bar{u} + d\bar{d})/\sqrt{2}$. This illustrates how $\eta^{(\prime)}$ masses and mixing are strongly correlated through the $U(1)_A$ symmetry.

Similarly, from the same effective theory of QCD, we may also express the density operator of Eq. (2) in terms of the flavor-nonet field π

$$\bar{q}^a \gamma_5 q^b \supset i\frac{fr}{2\sqrt{2}}\left[1 - \frac{\partial_\mu \partial^\mu}{\Lambda_0^2} + r(m_a + m_b)\left(\frac{1}{\Lambda_1^2} - \frac{1}{4\Lambda_2^2}\right)\right]\pi^{ba} \quad (8)$$

where $a, b = 1, 2, 3$ are the flavor indices. The scales $\Lambda_{0,2}$ and Λ_1 are associated with the kinetic and mass terms surviving at $\mathcal{O}(p^4, 1/\infty)$, respectively. Consequently, the parameters r and f are related to the physical masses and decay constants as (for more details, see [9])

$$M_\pi^2 = rm_q\left[1 + M_\pi^2\left(\frac{2}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right] \quad (9)$$

$$M_K^2 = \frac{r(m_q + m_s)}{2}\left[1 + M_K^2\left(\frac{2}{\Lambda_1^2} - \frac{1}{\Lambda_2^2}\right)\right] \quad (10)$$

and

$$f_\pi = f\left[1 + M_\pi^2\left(\frac{1}{\Lambda_0^2} + \frac{1}{2\Lambda_2^2}\right)\right] \quad (11)$$

$$f_K = f\left[1 + M_K^2\left(\frac{1}{\Lambda_0^2} + \frac{1}{2\Lambda_2^2}\right)\right] \quad (12)$$

if isospin symmetry is assumed, $q = u$ or d. As a result, we obtain the following set of density matrix elements:

$$\langle 0|\bar{q}\gamma_5 q|\pi^0\rangle = \pm i\frac{f_\pi}{2\sqrt{2}}\frac{M_\pi^2}{m_q} \quad (13)$$

$$\langle 0|\bar{q}\gamma_5 s|K\rangle = if_K\frac{M_K^2}{m_s + m_q} \quad (14)$$

$$\langle 0|\bar{q}\gamma_5 q|\eta\rangle = i\frac{f_\pi}{2\sqrt{6}}\frac{M_\pi^2}{m_q}(c\theta - \sqrt{2}s\theta)\left[1 + \frac{M_\eta^2 - M_\pi^2}{\Lambda_0^2}\right] \quad (15)$$

$$\langle 0|\bar{q}\gamma_5 q|\eta'\rangle = i\frac{f_\pi}{2\sqrt{6}}\frac{M_\pi^2}{m_q}(\sqrt{2}c\theta + s\theta)\left[1 + \frac{M_{\eta'}^2 - M_\pi^2}{\Lambda_0^2}\right] \quad (16)$$

$$\langle 0|\bar{s}\gamma_5 s|\eta\rangle = -i\frac{f_K}{\sqrt{3}}\frac{M_K^2}{m_s + m_q}(\sqrt{2}c\theta + s\theta) \times\left[1 + \frac{M_\eta^2 - M_K^2}{\Lambda_0^2} + 2(M_K^2 - M_\pi^2) \times\left(\frac{1}{\Lambda_1^2} - \frac{1}{4\Lambda_2^2}\right)\right] \quad (17)$$

$$\langle 0|\bar{s}\gamma_5 s|\eta'\rangle = i\frac{f_K}{\sqrt{3}}\frac{M_K^2}{m_s + m_q}(c\theta - \sqrt{2}s\theta) \times\left[1 + \frac{M_{\eta'}^2 - M_K^2}{\Lambda_0^2} + 2(M_K^2 - M_\pi^2) \times\left(\frac{1}{\Lambda_1^2} - \frac{1}{4\Lambda_2^2}\right)\right] \quad (18)$$

where we introduce the abbreviation $(c\theta, s\theta)$ for $(\cos\theta, \sin\theta)$. On the basis of Eq. (5), we neglect $\mathcal{O}(1/\Lambda_i^4)$ corrections. This parametrization of the density matrix elements for π^0, η and η' follows from a $U(3)_L \times U(3)_R$ invariant theory and is thus fully consistent with the $U(1)_A$ and $SU(3)_V$ symmetry requirements on the pseudoscalar mixing and masses in the isospin limit.

Equations (13) and (14) are the well-known hadronic matrix elements already derived in the large-N_c limit using chiral perturbation theory [12]. These hadronic matrix elements feel the usual effects of $SU(3)_V$ violation on the decay constants and masses in the pseudoscalar flavor-octet.

Equations (15) and (16) display the highly nontrivial effect of the $U(1)_A$ breaking: in the absence of the axial anomaly, the ideal masses [see Eq. (3)] and mixing [see Eq. (7)] would consistently imply $\langle 0|\bar{q}\gamma_5 q|\eta\rangle = 0$ and $\langle 0|\bar{q}\gamma_5 q|\eta'\rangle = \pm\langle 0|\bar{q}\gamma_5 q|\pi^0\rangle$. They nicely confirm our original guess expressed in Eq. (4). Moreover, a global fit of the pseudoscalar masses, mixing, and decay constants has already been undertaken in our previous work [9]. The resulting values for the $\Lambda_{0,1,2}$ cut-offs

$$\Lambda_0 \simeq 1.2 \text{ GeV}, \qquad \Lambda_1 \simeq 1.2 \text{ GeV}, \qquad \Lambda_2 \simeq 1.3 \text{ GeV} \quad (19)$$

are indeed all around 1 GeV, as anticipated.

Equations (17) and (18) consistently combine the effects of $U(1)_A$ and $SU(3)_V$ violations. Our results for the scale-independent density matrix elements $2m_s\langle 0|\bar{s}\gamma_5 s|\eta^{(\prime)}\rangle$ are compared with previous works in Table I. The second column includes flavor $SU(3)$ breaking with a realistic octet-singlet mixing angle but with the ideal mass relations for $\eta^{(\prime)}$ [see Eq. (3)]. We can see an excellent numerical agreement in this limit which is rather peculiar since any realistic $\eta-\eta'$ mixing excludes ideal $\eta-\eta'$ masses from the viewpoint of the $U(1)_A$ symmetry. The first column includes full $U(1)_A$ and $SU(3)_V$ breaking effects. The magnitude of the η' hadronic matrix element increases then by about 60% while the one for η only decreases by about 10%. Theoretical uncertainties associated with higher order $SU(3)_V$, and $U(1)_A$ corrections are expected to be $M_K^4/\Lambda^4 \simeq 5\%$ and $M_K^2 M_{\eta^{(\prime)}}^2/\Lambda^4 \simeq 15\%$, respectively.

Equations (13)–(18) turn out to be crucial for an estimate of the $B^0 \to K^0\eta^{(\prime)}$ decay amplitudes. Indeed, their typical M^2/m chiral enhancement, at the basis of the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decays [14], is such that the $Q_6 \equiv \sum_q (\bar{b}_L q_R)(\bar{q}_R s_L)$ weak operator provides the main contribution for the $b \to s\bar{q}q$ penguin-induced hadronic processes involving two pseudoscalars in the final state. (Let us emphasize that Q_6 does not contribute to vector-pseudoscalar final states and, in particular, to $B \to K^*\eta^{(\prime)}$). In the limit where, diagrammatically, $B^0 \to K^0\eta^{(\prime)}$ come from $b \to s\bar{d}d$ and $b \to s\bar{s}s$ penguin processes, while $B^0 \to K^0\pi^0$ comes only from $b \to s\bar{d}d$, we may thus write

$$\frac{A(B^0 \to K^0\eta^{(\prime)})}{A(B^0 \to K^0\pi^0)}\bigg|_{Q_6} = \frac{\langle K^0|\bar{b}_L s_R|B^0\rangle}{\langle \pi^0|\bar{b}_L d_R|B^0\rangle}\left[\frac{\langle \eta^{(\prime)}|\bar{b}_L d_R|B^0\rangle}{\langle K^0|\bar{b}_L s_R|B^0\rangle} + \frac{\langle \eta^{(\prime)}|\bar{s}_R s_L|0\rangle}{\langle K^0|\bar{d}_R s_L|0\rangle}\right]. \quad (20)$$

We already notice that in the absence of the axial anomaly, the η' wave-function has no $s\bar{s}$ component and the $b \to s\bar{s}s$ penguin process contribution to $B^0 \to K^0\eta'$ vanishes. In that ideal limit, the η'/π^0 ratio in Eq. (20) is equal to -1 and thus $Br(B^0 \to K^0\eta') = Br(B^0 \to K^0\pi^0)$!.

TABLE I. Comparison of numerical results for the $\bar{s}\gamma_5 s$ density matrix elements in GeV^3 units. The second column includes flavor $SU(3)$ breaking with a realistic octet-singlet mixing angle, $\theta = -22°$, but with the ideal mass relations for $\eta^{(\prime)}$ [see Eq. (3)]. Our result agrees with the previous works in this peculiar limit. The first column includes full $U(1)_A$ and $SU(3)_V$ breaking effects. We find that the magnitude of the η' hadronic matrix element increases by about 60% while the one for η only decreases by about 10%.

	this work		previous works	
$\theta = -22°$	$U(1)_A \times SU(3)_V$	$SU(3)_V$	AG [13]	BN [4]
$2im_s\langle 0|\bar{s}\gamma_5 s|\eta\rangle$	$+0.053 \pm 0.008$	$+0.058$	$+0.057$	$+0.055$
$2im_s\langle 0|\bar{s}\gamma_5 s|\eta'\rangle$	-0.109 ± 0.016	-0.069	-0.071	-0.068

As a first order approximation, the effect of the $U(1)_A$ and $SU(3)_V$ violations on the pseudoscalar masses can be safely neglected compared to the B-mass scale. So, we simply express the B-to-light meson hadronic matrix elements in terms of CG and focus on the axial $U(1)$ and flavor $SU(3)$ breaking corrections for the vacuum-to-light meson transitions. In this approximation, we easily obtain

$$\frac{A(B^0 \to K^0\eta)}{A(B^0 \to K^0\pi^0)} = -\sqrt{2}\left[\left(\sqrt{\frac{1}{6}}c\theta - \sqrt{\frac{1}{3}}s\theta\right) + \left(-\frac{2}{\sqrt{6}}c\theta - \sqrt{\frac{1}{3}}s\theta\right)\zeta\right] \quad (21)$$

$$\frac{A(B^0 \to K^0\eta')}{A(B^0 \to K^0\pi^0)} = -\sqrt{2}\left[\left(\sqrt{\frac{1}{6}}s\theta + \sqrt{\frac{1}{3}}c\theta\right) + \left(-\frac{2}{\sqrt{6}}s\theta + \sqrt{\frac{1}{3}}c\theta\right)\zeta'\right] \quad (22)$$

where the $U(1)_A$ and $SU(3)_V$ breaking effects associated with the pseudoscalar masses are fully encoded in the parameters

$$\zeta^{(\prime)} \equiv 1 + \frac{M_{\eta^{(\prime)}}^2 - M_K^2}{\Lambda_0^2} + 2(M_K^2 - M_\pi^2)\left(\frac{1}{\Lambda_1^2} - \frac{1}{4\Lambda_2^2}\right). \quad (23)$$

For a sensible value of the mixing angle, $\theta = -19.5°$ (i.e., $\cos\theta = 2\sqrt{2}/3$ and $\sin\theta = -1/3$), we have $\eta = (u\bar{u} + d\bar{d} - s\bar{s})/\sqrt{3}$ and $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, such that Eqs. (21) and (22) simply reduce to

$$\frac{A(B^0 \to K^0\eta)}{A(B^0 \to K^0\pi^0)}\bigg|_{\theta=-19.5°} = -\sqrt{\frac{2}{3}}[1 - \zeta] \quad (24)$$

$$\frac{A(B^0 \to K^0\eta')}{A(B^0 \to K^0\pi^0)}\bigg|_{\theta=-19.5°} = -\sqrt{\frac{1}{3}}[1 + 2\zeta']. \quad (25)$$

For the physical mixing angle $\theta = -(22 \pm 1)°$, corrections to Eqs. (24) and (25) are less than 5%. In the absence of the axial anomaly, the ideal relations given in Eq. (3) for $M_{\eta^{(\prime)}}$ imply $\zeta^{(\prime)} = 1.41(1.09)$. If in addition we assume $M_K = M_\pi$, then $\zeta = \zeta' = 1$, and we recover the well-known flavor $SU(3)$ relations between the $B^0 \to K^0[0^{-+}]$ branching ratios (η':η:π^0 = 3:0:1) [15]. But again, the $U(1)_A$ symmetry requires physical $\eta^{(\prime)}$ masses for a physical $\eta - \eta'$ mixing angle. Hence we have

$$\zeta = 1.29 \pm 0.19, \qquad \zeta' = 1.72 \pm 0.26. \quad (26)$$

Here, the main uncertainty comes from the higher order $SU(3)_V$ and $U(1)_A$ corrections to the $\eta^{(\prime)}$ hadronic matrix elements given in Table I. Theoretical uncertainties on the Λ_i scales are subdominant [9]. The impact of the $U(1)$ anomaly on the $B^0 \to K^0\eta'$ branching ratio is displayed in Fig. 1 for a mixing angle $\theta = -22°$. Varying θ around its

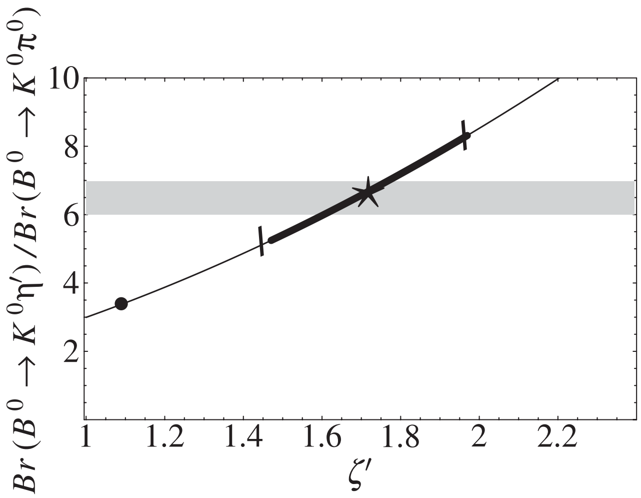

FIG. 1. The $B^0 \to K^0\eta'$ to $B^0 \to K^0\pi^0$ ratio of branching ratios as a function of the η' mass-dependent parameter ζ', for a fixed value of the mixing angle $\theta = -22°$. The usual flavor $SU(3)$ relation, $Br(B^0 \to K^0\eta')/Br(B^0 \to K^0\pi^0) \simeq 3$, is obtained at $\zeta' = 1$. The dot corresponds to $SU(3)_V$ but no $U(1)_A$ corrections to ζ', i.e., $\zeta' = 1.09$. Finally, the star represents full $SU(3)_V$ and $U(1)_A$ corrections to ζ', i.e., $\zeta' = 1.72 \pm 0.26$. The shaded area displays the current experimental range.

physical value would only induce a few percent correction. The result with only $SU(3)_V$ breaking corrections (a dot in Fig. 1) is obtained for $\zeta' = 1.09$. As we have shown in Table I, this peculiar limit reproduces the numerical values of the hadronic matrix elements used in the previous works. Our result, which consistently includes the effect of the $U(1)_A$ violation on the masses and mixing (a star in Fig. 1), is based on Eq. (26) for ζ'. We observe a strong increase for the $B^0 \to K^0\eta'$ branching ratio, in agreement with the data. On the other hand, the relatively small η mass induces a small $U(1)_A$ correction to ζ, which does deviate from one mainly through the $SU(3)_V$ violation. As a result, the $B^0 \to K^0\eta$ branching ratio stays well below the experimental bound due to the efficient cancellation in Eq. (24) but is, at the same time, extremely sensitive to the uncertainties on ζ and θ. For $\zeta = 1.29 \pm 0.19$ and $\theta = -(22 \pm 1)°$, its value runs between 0.01×10^{-6} and 1.10×10^{-6}. At this level, contributions from the other penguin operators might be non-negligible.

Now that we understand the $B \to K\eta^{(\prime)}$ branching ratios, it would be interesting to estimate the other channels involving $\eta^{(\prime)}$ as well as the $B \to \eta' X_s$ inclusive spectrum. However, we should already emphasize that the anomalous enhancement which we have advocated in this Letter only applies to density hadronic matrix elements, not to current ones. Thus, no similar enhancement occurs in $B \to K^*\eta'$ where the dominant contribution comes from the current-current penguin operators.

In conclusion, we reconsidered the puzzle of large $B \to K\eta'$ branching ratio and pointed out a missing $U(1)_A$ breaking correction to the penguin hadronic matrix element. We estimated this correction in the framework of a low-energy effective theory of QCD in the large-N_c limit. We provided the expression for all the density matrix elements relevant to the hadronic B decays, in terms of physical masses and decay constants. We found a rather large increase (60%) of $|\langle 0|\bar{s}\gamma_5 s|\eta'\rangle|$ and a moderate decrease (10%) of $|\langle 0|\bar{s}\gamma_5 s|\eta\rangle|$ compared to the previous works. The sizable corrections for the η' should not come as a surprise since its hadronic matrix element vanishes in the absence of the axial anomaly. This correction may explain why the $B^0 \to K^0\eta'$ branching ratio is about 6 times larger than the $B^0 \to K^0\pi^0$ one.

This work was supported by the Belgian Federal Office for Scientific, Technical and Cultural Affairs through the Interuniversity Attraction Pole $P5/27$.

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