Seeing an Invisible Axion in the Supersymmetric Particle Spectrum

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I describe how under favorable circumstances, the existence of an invisible axion could correlate with a distinctive CERN Large Hadron Collider sparticle spectrum, in particular, through a gluino $\sim \ln(M_P/m_{3/2})$ times heavier than other gauginos.

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The invisible axion is the best-motivated solution to the strong CP problem. The QCD Lagrangian in principle could contain a CP-violating θ angle,

$$S = -\frac{1}{4g^2}\int d^4x F^a_{\mu\nu}F^{a,\mu\nu} + \ldots + \frac{\theta}{32\pi^2} \times \int d^4x F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}. \quad (1)$$

The presence of such a θ-angle would give rise to observable CP violation in strong interactions and, in particular, to an electric dipole moment for the neutron. Experiment constrains $|\theta| \lesssim 10^{-10}$ [1,2]. As θ is periodic with an allowed range $-\pi < \theta < \pi$, this is unnatural. The problem of why $|\theta|$ is so small is the strong CP problem.

In the Peccei-Quinn solution [3], θ is promoted to a dynamical field $\theta(x)$.

$$\mathcal{L}_\theta = \mathcal{L}_{\rm QCD} - \frac{1}{2}f_a^2\partial_\mu\theta\partial^\mu\theta + \frac{\theta}{32\pi^2}F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}. \quad (2)$$

The dimensionful quantity f_a is known as the axion decay constant. The action for the canonically normalized field $a \equiv f_a\theta$ is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu a\partial^\mu a + \frac{a}{32\pi^2 f_a}F^a_{\mu\nu}\tilde{F}^{a,\mu\nu}. \quad (3)$$

QCD instanton effects generate a potential for θ. This potential may be computed using the pion Lagrangian (see, e.g., section 23.6 of [4]), giving

$$V(a) \sim m_\pi^2 f_\pi^2\left[1 - \cos\left(\frac{a}{f_a}\right)\right], \quad (4)$$

where $f_\pi \sim 90$ MeV is the pion decay constant. The potential (4) dynamically minimizes θ at zero, thus solving the strong CP problem.

a is known as the (invisible) axion. It is extremely light and has mass

$$m_a \sim \frac{m_\pi^2 f_\pi^2}{f_a^2} \sim \left(\frac{10^9\ \text{GeV}}{f_a}\right)10^{-2}\ \text{eV}. \quad (5)$$

Astrophysical and cosmological constraints imply that if a exists, the decay constant f_a lies within a narrow window 10^9 GeV $\lesssim f_a \lesssim 3 \times 10^{11}$ GeV [5]. The lower bound is hard and comes from supernova cooling. The upper bound assumes a standard cosmology and comes from the requirement that the energy density in the axion field due to primordial misalignment does not exceed the current dark matter density. This may however be relaxed with nonstandard cosmologies, for example, involving late-time inflation.

Although a solution of the strong CP problem only requires the axion to couple to QCD, in most axion models the axion also couples to QED. In this case, axions can convert to photons in a background magnetic field through the Primakoff effect. This $a\gamma\gamma$ coupling is the basis for direct axion searches such as axion helioscopes [6], cavity experiments [7], or laser-based searches [8]. There is no unambiguous detection of an axion in any of these experiments. There does exist a possible signal in the PVLAS experiment [8] which is however in tension with existing astrophysical bounds.

The purpose of this Letter is to point out that under favorable assumptions, the invisible axion may also indirectly manifest its existence in particle colliders such as the LHC, through the sparticle spectrum and, in particular, through the existence of a gluino very much heavier than other gauginos by a factor $\sim \ln(M_P/m_{3/2}) \sim 30$. This will follow from considerations of moduli stabilization in string and supergravity scenarios, and its relation to the existence of an axionic shift symmetry.

We first recall why axions in string theory are both natural and abundant. For example, in braneworld scenarios, the action for a Dp-brane contains a term [9,10]

$$\frac{2\pi}{l_s^{p+1}}\int e^{2\pi\alpha' F} \wedge \sum C_q \in S_{\rm brane}, \quad (6)$$

with $l_s = 2\pi\sqrt{\alpha'}$ the string length. The sum in (6) is over higher-dimensional antisymmetric Ramond-Ramond form fields, while F is the gauge field strength on the brane. Expanding (6) gives a term

$$\frac{1}{2(2\pi)l_s^{p-3}}\int_{\mathbb{M}_4\times\Sigma} F \wedge F \wedge C_{p-3}.$$

Σ is the $(p-3)$-dimensional cycle wrapped by the brane.

The dimensionally reduction of C_{p-3} then gives a four-dimensional axion for the gauge group on the brane.

The low-energy limit of string theory is 4-dimensional $\mathcal{N} = 1$ supergravity. In this limit, axions a_i appear as the imaginary parts of scalar components of moduli multiplets,

$$a_i = \mathrm{Im}(T_i), \qquad T_i = \tau_i + i a_i, \tag{7}$$

where the real parts τ_i typically parametrize the geometry of the compactification manifold and determine the coupling for the gauge theory associated with the axion. Axions are $U(1)$-valued and thus have an exact symmetry $a_i \to a_i + 2\pi$. The continuous shift symmetry $a_i \to a_i + \epsilon$ is valid perturbatively, implying that up to nonperturbative effects, neither the Kähler potential nor superpotential can depend on the axions. If Φ_j denote the superfields with no axionic components, this implies that perturbatively

$$W(T_i, \Phi_j) = W(\Phi_j), \qquad \mathcal{K}(T_i, \Phi_j) = \mathcal{K}(T_i + \bar{T}_i, \Phi_j). \tag{8}$$

We suppose such a stringy invisible axion does indeed exist and solves the strong CP problem. In that case,

$$a = \mathrm{Im}(T_{\mathrm{QCD}}), \tag{9}$$

where $T_{\mathrm{QCD}} = \tau_Q + ia$ is whatever modulus is the QCD gauge kinetic function. This follows from the supergravity couplings [11] (we use h_a to denote the gauge kinetic function),

$$\mathcal{L} \sim -\frac{\mathrm{Re}(h_a)}{4} F^a_{\mu\nu} F^{a,\mu\nu} + i\frac{\mathrm{Im}(h_a)}{8} F^a_{\mu\nu} \tilde{F}^{a,\mu\nu} + \ldots. \tag{10}$$

Depending on exactly how the moduli are stabilized, the physical axion may be an admixture of (9) with other moduli. The saxion τ_Q is an ordinary scalar field and thus will have nonderivative couplings to matter. If τ_Q were massless, these couplings would generate long-range Yukawa forces. The nonobservation of such fifth forces implies this cannot hold—the saxion must be massive and a potential must be generated for it.

The generation of moduli potentials in string theory—i.e., moduli stabilization—has received much recent attention [12,13]. The supergravity F-term potential is determined by both the Kähler potential and superpotential,

$$V_F = e^{\mathcal{K}}(\mathcal{K}^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2). \tag{11}$$

As noted above, the axion shift symmetry implies the modulus T_{QCD} cannot appear perturbatively in the superpotential. T_{QCD} can appear nonperturbatively, and nonperturbative superpotentials represent a popular approach to moduli stabilization. For example, in the Kachru-Kallosh-Linde-Trivedi scenario [13], the Kähler and superpotential are given by

$$\mathcal{K} = -2\ln[\mathcal{V}(T_i + \bar{T}_i)], \quad W = W_0 + \sum_i A_i e^{-a_i T_i}. \tag{12}$$

The constant W_0 comes from 3-form fluxes while $\mathcal{V}$ is the volume of the internal space. The moduli are stabilized by solving $D_i W = 0$ for all Kähler moduli. A further strong justification for the presence of nonperturbative superpotentials is that they can naturally generate the weak scale or Planck scale hierarchy, as in, e.g., the racetrack scenario [14] (see [15] for some recent work) or the exponentially large volume models of [16,17].

However, for the axion a to solve the strong CP problem, its saxion partner τ_Q should not be stabilized through a nonperturbative superpotential. The axionic solution to the strong CP problem requires that QCD instantons dominate the axion potential. These generate a potential (4) of magnitude $\sim\Lambda^4_{\mathrm{QCD}} \sim 10^{-75} M_P^4$. However, a superpotential term

$$W = W_0 + \ldots + A_Q e^{-aT_{\mathrm{QCD}}} + \ldots, \tag{13}$$

as may appear in (12), depends on the axion a through the phase of the exponent and thus generates an axion mass through the potential (11). The typical size of this potential is $\sim e^{\mathcal{K}} W\bar{W} \sim m^2_{3/2} M_P^2$, which for a TeV-scale gravitino mass is $\sim(10^{11}\ \mathrm{GeV})^4$. The putative QCD axion a obtains a mass at a scale comparable to τ_Q, $m_a \sim m_{\tau_Q} \sim m_{3/2} \sim 1$ TeV. This is much larger than the range (5) 10^{-6} eV $\lesssim m_a \lesssim 10^{-3}$ eV associated with QCD effects in the allowed f_a window. The term (13) washes out the effect of QCD instantons and the minimum for a becomes uncorrelated with the vanishing of the θ-angle, leaving the strong CP problem no longer solved. Consequently, τ_Q must instead be stabilized perturbatively through the structure of the Kähler potential—as $\mathcal{K} = \mathcal{K}(T + \bar{T})$, this does not generate a potential for the axion.

However, the above argument does not apply to the moduli $T_{SU(2)}$ and $T_{U(1)}$ controlling the $SU(2) \times U(1)$ gauge couplings—here the instanton amplitudes are so small the θ angle is irrelevant. These moduli must be stabilized to avoid fifth forces, but there is no restriction on how this is achieved. The favorable assumption we make is that at least one of these moduli is stabilized nonperturbatively, while T_{QCD} is stabilized perturbatively.

This assumption requires that the $SU(2)$ and $SU(3)$ gauge couplings come from different moduli, which are stabilized through different mechanisms. There is then no obvious link between the different gauge couplings. This is not compatible with Grand Unified Theory (GUT) phenomenology, in which at high energies, there is a single gauge group whose coupling is controlled by a single modulus. It instead fits better with intersecting-brane approaches to realizing the Standard Model (for a review see [18]), in which the different Standard Model gauge group lives on different stacks of branes. As generally holds for intersecting-brane models, the GUT-scale unification of the Minimal Supersymmetric Standard Model (MSSM) coupling constants should then be regarded as accidental.

We now consider how different stabilization mechanisms affect the pattern of soft terms. Very generally, the pattern of the MSSM soft terms is determined by how supersymmetry is broken. We assume gravity-mediation—in this case, the soft terms are determined by the structure of the hidden sector—i.e., moduli—potential. For a gauge group with gauge kinetic function h_a, the gaugino masses are given by

$$M_a = \frac{1}{2}\frac{1}{\mathrm{Re}h_a}\sum_\alpha F^\alpha \partial_\alpha h_a. \quad (14)$$

The F-terms F^α are defined by

$$\begin{aligned} F^\alpha &= e^{\mathcal{K}/2}\sum_{\bar\beta}\mathcal{K}^{\alpha\bar\beta}D_{\bar\beta}\bar W \\ &= e^{\mathcal{K}/2}\sum_{\bar\beta}\mathcal{K}^{\alpha\bar\beta}\partial_{\bar\beta}\bar W + e^{\mathcal{K}/2}\sum_{\bar\beta}\mathcal{K}^{\alpha\bar\beta}(\partial_{\bar\beta}\mathcal{K})\bar W. \quad (15)\end{aligned}$$

In the case that $h_a = T_a$, the associated gaugino mass is

$$M_a = \frac{F^a}{2\tau_a}. \quad (16)$$

Note the mass in (16) is a Lagrangian parameter and applies at the compactification scale; to determine the physical mass, we must run this down to the TeV scale.

The point of the favorable assumption is that if a modulus T_a is stabilized through nonperturbative superpotential corrections, the associated F-term is generically suppressed. To be explicit, in an expansion in $\frac{1}{\ln(M_P/m_{3/2})}$, the two contributions to (15) cancel to leading order [19] (see also [20]). The magnitude of the resulting F-term is then

$$F^a \sim \frac{2\tau_a m_{3/2}}{\ln(M_P/m_{3/2})}, \quad (17)$$

and the gaugino associated to T_a has a mass suppression,

$$M_a \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})}. \quad (18)$$

This logarithmic suppression can be shown to be entirely a feature of the nonperturbative stabilization. In particular, it does not depend on whether the stabilization is approximately supersymmetric or not.

We argued above that $\tau_Q = \mathrm{Re}(T_{\mathrm{QCD}})$ should be stabilized perturbatively through the structure of the Kähler potential. The Kähler potential is nonholomorphic and thus hard to compute, and so it is difficult to be explicit. However, the only fact we need is that for this case the logarithmic suppression of (18) does not hold. Instead we obtain the generic behavior

$$M_a \sim m_{3/2}. \quad (19)$$

Such behavior is indeed found in explicit models of perturbative stabilization (e.g. see [21]).

We now come to the point of this Letter. What the Peccei-Quinn solution to the strong CP problem tells us is that whatever the modulus controlling the QCD gauge coupling is, it should be stabilized perturbatively through the Kähler potential rather than nonperturbatively through the superpotential. This is to avoid generating a potential for the QCD axion. In this case, the associated gaugino—i.e. the gluino—will not have a suppressed mass, and we expect $m_{\tilde g} \sim m_{3/2}$ at the compactification scale.

However, under the favorable circumstances above, at least one of the moduli $T_{SU(2)}$ and $T_{U(1)}$ is stabilized nonperturbatively, and its associated gaugino mass will be suppressed. Consequently, either $m_{\tilde B}$ or $m_{\tilde W}$ will have a suppressed mass,

$$M_a \sim \frac{m_{3/2}}{\ln(M_P/m_{3/2})}. \quad (20)$$

If (20) holds, anomaly-mediated contributions are also relevant for the exact gaugino masses: it is a numerical coincidence that $\ln(M_P/1\ \mathrm{TeV}) \sim 0.4(8\pi^2)$.

There is then a small hierarchy between the masses of the gluino and the lightest gaugino set by $\ln(M_P/m_{3/2})$. The axionic solution to the strong CP problem therefore suggests a distinctive gaugino spectrum, with for example

$$m_{\tilde g} \sim 30 m_{\tilde W}. \quad (21)$$

As the gluino mass tends to increase under Renormalization Group flow, this hierarchy will not be diluted by the running to low energy. Such a hierarchy may be unnatural from the viewpoint of low-energy field theory. However, the relations (20) and (21) are another reminder that naturalness in string theory and naturalness in effective field theory are cognate but nonidentical concepts.

The spectrum of (20) is very distinctive and can be easily distinguished from minimal supergravity models, where the gaugino mass ratios are universal at the compactification scale and the physical masses have $m_{\tilde g} \sim 5 m_{\tilde W}$. This justifies our original claim: the existence of an invisible axion can under favorable circumstances correlate with a very distinctive gaugino spectrum at particle colliders such as the LHC.

Let us compare the above approach to ‘‘seeing‘‘ the axion to more conventional routes. One advantage is that it implies that the existence of a QCD axion could have consequences for areas such as collider physics very different from the astrophysical topics normally considered. The above gaugino spectrum is also very distinctive, involving a sharp hierarchy between the lightest and heaviest gauginos.

Another advantage is that the arguments above are insensitive to the value of the decay constant f_a and whether or not the axion couples to photons. While decay constants $f_a \gtrsim 10^{12}$ GeV are disfavored by standard cosmology, this upper bound is not solid and may be evaded by nonstandard cosmologies or small initial axion misalignment, $|\theta_i| \ll 1$. Furthermore, ‘‘generic’’ string compactifications give $f_a \sim 10^{16}$ GeV—for recent discussions of f_a in string compactifications, see [22–24]. In the case that $f_a \gg 10^{12}$ GeV or $g_{a\gamma\gamma} = 0$, detection by direct search experiments would be extremely difficult. However, the above arguments are unaltered, and the gluino hierarchy may still be seen. Other than the vanishing of $\theta_{\rm QCD}$, the relation (21) could then be the only way the existence of the axion affects observable physics.

An obvious disadvantage of the above approach is that it is indirect: even if seen, the gaugino hierarchy of (21) would be at best an indication of an axion rather than a detection. It also requires a favorable assumption. In principle, all moduli could be stabilized perturbatively, in which case there exists quasiuniversal behavior $M_a \sim m_{3/2}$ with no distinct gluino hierarchy: a failure to observe the hierarchy (21) would not rule out the invisible axion.

We also note it would be straining the LHC to actually see both the light gaugino and the hierarchically heavier gluino. Doing so would require a favorable sparticle spectrum, with a very light gaugino at ~100 GeV near the LEP bounds and a gluino at the extreme reach of the LHC with $m_{\tilde{g}} \sim 2$ TeV. As $m_{\tilde{g}} \sim m_{3/2}$, this would also suggest a generally heavy scalar spectrum with TeV scalars having $m_i \sim m_{3/2}$. In this case, the only sparticles accessible at the LHC may be winos or binos.

Before finishing, let us make a comment on the axino, the fermionic partner of the axion. In global supersymmetry, axinos receive a mass $m_{\tilde{a}} \sim m_{\rm susy}^2/f_{\rm PQ}$, where $m_{\rm susy}$ represents the susy-breaking scale, and can be relatively light in the keV range. However, in supergravity, axinos generically receive a mass $m_{\tilde{a}} \sim m_{3/2}$ from the Lagrangian [25], although the exact value depends on the details of the Kähler and superpotentials. This expectation of $\mathcal{O}(m_{3/2})$ axinos is not affected by the considerations in this Letter.

Let us conclude by restating the assumptions and result. We assume gravity-mediated supersymmetry breaking. To avoid generating a potential for the QCD axion, the modulus $\tau_{\rm QCD}$ controlling the QCD gauge coupling must be stabilized perturbatively. We also assume one of the moduli $\tau_{SU(2)}$ or $\tau_{U(1)}$ is stabilized through a nonperturbative superpotential. In this case, there is a generic hierarchy of $\ln(M_P/m_{3/2})$ between the gluino and the lightest gaugino.

We find it both interesting and amusing that the invisible axion could give a hint of its existence as above in the perhaps unlikely spot of the sparticle spectrum. This may be particularly important if the properties of the axion are such as to make its direct detection impossible.

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