Horsley and Babiker Reply: We partly agree and partly disagree with the Comment [1] by Spavieri and Rodriguez (SR) on the issues raised by us in [2]. Specifically, we agree with SR's assertion that a particle endowed with an electric dipole moment in our magnetic field experiences a null *mechanical force*, but we disagree, on grounds of gauge invariance, with their argument that the Röntgren interaction is, in general, insufficient for the description of the quantum phase. More significantly, SR do not explain why the use of the nonvanishing force in their Eq. (1) leads, surprisingly, to an apparent total agreement between the semiclassical treatment and quantum theory. Such an explanation, as we show here, should illuminate the subtle connection between the classical and quantum descriptions and highlight nonlocality as a unique property of the Aharonov-Bohm–type quantum phase that cannot be explained from a classical mechanical force perspective. We have recently completed a comprehensive analysis of the quantum phase problem using Feynman's functional integral approach [3] and are also now in a position to explain why a semiclassical treatment of the Röntgren phase based on SR's Eq. (1) leads, intriguingly, to an apparent complete agreement with quantum theory.

Our starting point here is the Lagrangian for a particle of mass M, position vector $\mathbf{R}$, endowed with an electric dipole moment $\mathbf{d}$ and subject to a magnetic field $\mathbf{B}$. It is $L = \frac{1}{2}M\dot{\mathbf{R}}^2 + \mathbf{d}\cdot\dot{\mathbf{R}}\times\mathbf{B}$. This gauge-invariant Lagrangian, in the dipole approximation, arises naturally from a procedure involving a gauge transformation applied to the conventional Lagrangian of an electrically neutral two-particle system forming an electric dipole in the presence of electromagnetic fields [4]. The corresponding Hamiltonian arises from a Power-Zienau-Woolley canonical transformation [5] applied to the conventional Hamiltonian. In contrast to SR's assertions in the last paragraph in [1], we have only the Röntgren interaction term. There are no other terms of the same order, of the kind arising from the conventional gauge-dependent Lagrangian, which is the focus of SR's treatment. The appearance of the vector potential in [6,7], rather than just the magnetic field, is indicative of an unphysical gauge-dependent interaction.

The canonical momentum conjugate to the position vector $\mathbf{R}$ arising from our Lagrangian is $\mathbf{P}_c = \partial L/\partial\dot{\mathbf{R}} = M\dot{\mathbf{R}} - \mathbf{d}\times\mathbf{B}$. We now introduce the new concept of a *canonical* force $\mathbf{F}_c$ as the rate of change of canonical momentum. We have $\mathbf{F}_c \equiv \dot{\mathbf{P}}_c = \mathbf{F}_M - d/dt\{\mathbf{d}\times\mathbf{B}\}$, where $\mathbf{F}_M = M\ddot{\mathbf{R}}$ is the classical *mechanical* force. As SR emphasize in their Comment [1], one finds $\mathbf{F}_M = 0$, a result that was also pointed out by Wilkens [8]. Hence the canonical force becomes $\mathbf{F}_c = -d/dt\{\mathbf{d}\times\mathbf{B}\}$. For a time-independent dipole moment vector (i.e., $\dot{\mathbf{d}} = 0$) one finds $\mathbf{F}_c = -\dot{\mathbf{R}}\cdot\boldsymbol{\nabla}\{\mathbf{d}\times\mathbf{B}\}$. With $\mathbf{E} = \dot{\mathbf{R}}\times\mathbf{B}$, as in Ref. [2], the canonical force can be written as

$$\mathbf{F}_c = \nabla\{\mathbf{d}\cdot\mathbf{E}\}. \tag{1}$$

We have therefore identified the force $\mathbf{F}_{\mathrm{HB}}$ in Eq. (1) of SR [also Eq. (2) in Ref. [2]] as the canonical, not the mechanical, force. It follows that the canonical force is the one needed in the semiclassical treatment, as shown in [2], for full agreement with quantum theory. This revelation begs the obvious question as to whether, in general, semiclassical treatments of quantum phase in which the canonical force is used provide a reliable theoretical framework. The answer is indeed yes; it is easy to show further that the Aharonov-Bohm phase [9] as well as the Aharonov-Casher phase [10] conform with this expectation. It is clear in all three cases that, although there may be change in the canonical momentum of the entire particle plus field system during an interference experiment, this is not evident as any classical acceleration of the particle throughout its motion.

The main conclusions that can be drawn from this are that, first, Aharanov-Bohm–type phase phenomena are purely quantum mechanical and that, second, they are truly nonlocal in the sense that they arise even in the absence of a local mechanical force. We consider that the explanation above settles the controversy of quantum phase, first initiated by Boyer [11,12] and which we have highlighted in our Letter [2]. There are no truly classical, or semiclassical, bases involving mechanical force that would lead to a reliable description of quantum phase phenomena of the Aharonov-Bohm type.

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Received 5 September 2006; published 19 December 2006
DOI: 10.1103/PhysRevLett.97.258902
PACS numbers: 03.65.Vf, 03.65.Sq, 03.65.Ta

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