Enhanced Polarization of the Cosmic Microwave Background Radiation from Thermal Gravitational Waves

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If inflation was preceded by a radiation era, then at the time of inflation there will exist a decoupled thermal distribution of gravitons. Gravitational waves generated during inflation will be amplified by the process of stimulated emission into the existing thermal distribution of gravitons. Consequently, the usual zero temperature scale invariant tensor spectrum is modified by a temperature dependent factor. This thermal correction factor amplifies the *B*-mode polarization of the cosmic microwave background radiation by an order of magnitude at large angles, which may now be in the range of observability of the Wilkinson Microwave Anisotropy Probe.

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Inflation [1], in addition to solving the horizon and flatness problems of the standard hot big-bang model, generates a nearly scale invariant density perturbation, which has been tested in the observations of the cosmic microwave background radiation (CMBR) angular spectrum. One prediction of inflationary models which has not yet been tested is the existence of a nearly scale invariant spectrum of gravitational waves [2,3]. The definitive test of the existence of these cosmological gravitational waves would be the observation of B mode polarization in the CMB. The recent Wilkinson Microwave Anisotropy Probe (WMAP) 3 yr results [4] give only an upper bound on the B mode polarization. $\frac{(l+1)l}{2\pi} C^{BB}_{l=(2-6)} < 0.05(\mu\mathrm{K})^2$.

In this Letter we show that if inflation was preceded by a radiation era, then there would be a thermal background of gravitons at the time of inflation. This thermal distribution of gravitons would have decoupled close to Planck era. The generation of tensor perturbation during inflation would be by *stimulated emission* [5] into this existing thermal background of gravitational waves. This process changes the power spectrum of tensor modes by an extra temperature dependent factor $\coth(k/2T)$. At large angular scales ($l \leq 30$) the power spectrum $P_T = A_T k^{n_T}$ of gravitational waves generated during inflation would have a spectral index $n_T = -1 - 2\epsilon$, instead of the standard slow roll inflation prediction $n_T = -2\epsilon$ which implies that $C^{BB}_{l=3} \simeq 10 \times C^{BB}_{l=30}$. If a thermal enhancement of low l BB modes exists it should be observable with WMAP or in upcoming the Planck [6] experiment. In the conclusions, we discuss the implications on inflation models from the observations or nonobservation of this low l thermal enhancement.

The tensor perturbations have two independent degrees of freedom which can be chosen as h^+ and $h^\times$ polarization modes. To compute the spectrum of gravitational waves $h(\mathbf{x}, \tau)$ during inflation, we express $h^{(+)}$ and $h^{(\times)}$ in terms of the creation- annihilation operator,

$$h^{(i)}(\mathbf{x}, \tau) = \frac{\sqrt{16\pi}}{a(\tau)M_p} \int \frac{d^3k}{(2\pi)^{3/2}} [a_{\mathbf{k}} f_k(\tau) + a^\dagger_{-\mathbf{k}} f^*_k(\tau)] e^{i\mathbf{k}\cdot\mathbf{x}} \equiv \int \frac{d^3k}{(2\pi)^{3/2}} h_{\mathbf{k}}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{1}$$

where $a(\tau)$ is the scale factor, $\mathbf{k}$ is the comoving wave number, $k = |\mathbf{k}|$, and $M_p = 1.22 \times 10^{19}$ GeV is the Planck mass and $i = +, \times$. The power spectrum of the tensor perturbations is defined as

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle \equiv \frac{2\pi^2}{k^3} P_T \delta^3(\mathbf{k} - \mathbf{k}'). \tag{2}$$

The usual quantization condition between the fields and their canonical momenta yields $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}')$ and the vacuum satisfies $a_{\mathbf{k}}|0\rangle = 0$. If the graviton field had zero occupation prior to inflation then $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = \delta^3(\mathbf{k} - \mathbf{k}')$ and the vacuum satisfies $a_{\mathbf{k}}|0\rangle = 0$. If the graviton field had zero occupation prior to inflation then $\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}} \rangle = 0$ and we would obtain a correlation function $\sim |f_k(\tau)|^2$. However, if the graviton field was in thermal equilibrium at some earlier epoch it will retain its thermal distribution even after decoupling from the other radiation fields and its occupation number will be given by

$$\langle a^\dagger_{\mathbf{k}} a_{\mathbf{k}'} \rangle = \left(\frac{1}{e^{k/T} - 1}\right) \delta^3(\mathbf{k} - \mathbf{k}'). \tag{3}$$

Using Eqs. (1) and (3) it can be seen that,

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = \frac{16\pi |f_k(\tau)|^2}{a^2(\tau) M_p^2} \left(1 + \frac{2}{e^{\frac{k}{T}} - 1}\right) \delta^3(\mathbf{k} - \mathbf{k}')$$
$$= \frac{16\pi |f_k(\tau)|^2}{a^2(\tau) M_p^2} \coth\left[\frac{k}{2T}\right] \delta^3(\mathbf{k} - \mathbf{k}'). \tag{4}$$

From the defining relation, Eq. (2), for the tensor power spectrum and Eq. (4) we find that the power spectrum for the thermal inflatons can be expressed in terms of the mode functions $f_k(\tau)$ as

$$P_T(k) = \frac{8k^3}{\pi M_p^2} \frac{|f_k|^2}{a^2(\tau)} \coth\left[\frac{k}{2T}\right]. \quad (5)$$

The mode functions $f_k(\tau)$ obey the minimally coupled Klein-Gordon equation:

$$f_k'' + \left(k^2 - \frac{a''}{a}\right) f_k = 0. \quad (6)$$

In a quasi de Sitter universe during inflation, conformal time τ ($d\tau \equiv dt/a$) and the scale factor during inflation $a(\tau)$ are related by $a(\tau) = -1/H\tau(1-\epsilon)$ where $\epsilon = \frac{M_P^2}{16\pi}(\frac{V'}{V})^2$ and V is the potential for the inflaton field.

For constant ϵ the mode functions $f_k(\tau)$ obey the minimally coupled Klein-Gordon equation [7],

$$f_k'' + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right] f_k = 0, \quad (7)$$

where $k = |\mathbf{k}|$ and, for small ϵ and δ, $\nu = \frac{3}{2} + \epsilon$. Equation (7) has the general solution given by,

$$f_k(\tau) = \sqrt{-\tau}[c_1(k) H_\nu^{(1)}(-k\tau) + c_2(k) H_\nu^{(2)}(-k\tau)]. \quad (8)$$

When the modes are well within the horizon they can be approximated by flat spacetime solutions $f_k^0(\tau) = (1/\sqrt{2k})e^{-ik\tau}$, ($k \gg aH$). Matching the general solution in Eq. (8) with the solution in the high frequency (flat spacetime) limit gives the value of the constants of integration $c_1(k) = (\sqrt{\pi}/2)e^{i(\nu+1/2)(\pi/2)}$ and $c_2(k) = 0$. Equation (8) then implies that for $-k\tau \gg 1$ or $k \ll aH$,

$$f_k(\tau) = e^{i(\nu-1/2)(\pi/2)} 2^{\nu-3/2} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} \frac{1}{\sqrt{2k}} (-k\tau)^{1/2-\nu}. \quad (9)$$

Substituting the solution as given in Eq. (9) for the super-horizon modes in ($k \ll aH$) in Eq. (5) for the tensor power spectrum, we obtain

$$P_T(k) = \frac{16\pi}{M_P^2}\left(\frac{H}{2\pi}\right)^2 \left(\frac{k}{aH}\right)^{n_T} \coth\left[\frac{k}{2T}\right], \quad (10)$$

with $n_T = 3 - 2\nu = -2\epsilon$. We can now rewrite the power spectrum as

$$P_T(k) = A_T(k_0)\left(\frac{k}{k_0}\right)^{n_T} \coth\left[\frac{k}{2T}\right], \quad (11)$$

where k_0 is referred to as the pivot point and $A(k_0)$ is the normalization constant $A_T(k_0) = \frac{16\pi}{M_P^2}(\frac{H_{k_0}}{2\pi})^2$, where H_{k_0} is the Hubble parameter evaluated when $aH = k_0$ during inflation.

The angular power spectrum of the BB polarization modes generated by the gravitational waves is given by [8],

$$\frac{C_l^{BB}}{(4\pi)^2} = \int dk k^2 P_T(k) \times \left| \int_0^{\eta_0} d\eta\, g(\eta) h_k(\eta) \left[2j_l'(x) + \frac{4j_l(x)}{x}\right]\right|^2, \quad (12)$$

where $g(\eta) = \dot{\kappa} e^{-\kappa}$ is the visibility function, $\dot{\kappa}$ is the differential optical depth for Thomson scattering, and $x = k(\eta_0 - \eta)$. The EE polarization also gets a contribution from the tensor perturbations but it is dominated by the scalar perturbations. So the best signal for gravitational waves is the BB polarization angular spectrum which is generated by the primordial tensor perturbations only.

The temperature dependent factor becomes important when the ratio $k/(2T)$ is less than unity. The comoving wave-number k and the comoving temperature T can be related to the physical parameters at the time of inflation as follows. Taking the largest measurable perturbation scale $k_{\rm now}/a_{\rm now} \simeq R_h^{-1}$ (where $R_h = 4000$ Mpc is the size of the present horizon), and assuming that perturbations of the present horizon scale were just leaving the inflationary horizon H^{-1} at the beginning of inflation, we see that the temperature at the beginning of inflation T_i/a_i must be

$$\frac{k}{2T} = \frac{Ha_i}{2T_i} < 1, \quad (13)$$

in order to have a significant effect on the tensor power spectrum. $T_i/a_i \sim (30V/g_* \pi^2)^{1/4}$, V being the inflaton potential which is related to the curvature at the time of inflation, $H = (8\pi/3)^{1/2} V^{1/2}/M_P$ and $g_* \sim 100$ is the effective number of relativistic particles. Therefore inflation is expected to start at a temperature $T_i/a_i = 0.24(HM_P)^{1/2}$. Actually the gravitons which are decoupled will have a temperature slightly below the radiation temperature because of the particles (such as the inflaton itself) which have annihilated into radiation prior to inflation. But as the effective number of particles $g_* \sim 100$ is large, this difference of temperature is not significant. So for inflation

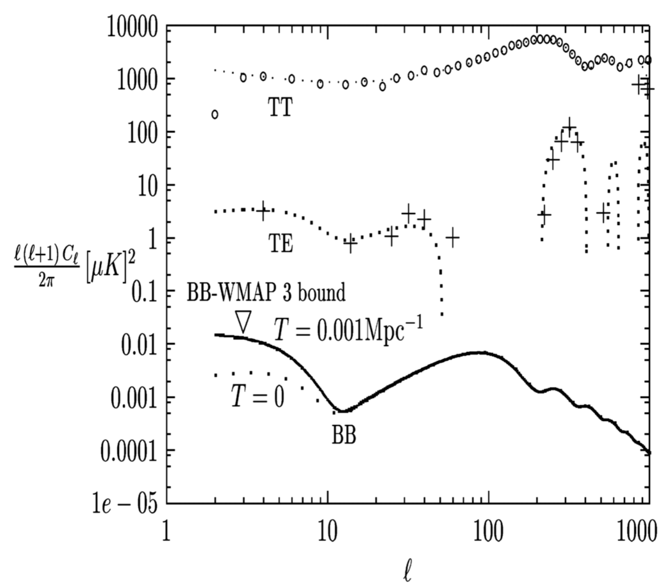

FIG. 1. The TT, TE, and the BB correlations with thermal gravitons at temperature $T = 0.001$ Mpc^{-1}, with the WMAP three years data [4]. For comparison we have plotted the BB angular correlations at $T = 0$. We see that with a temperature $T = 0.001$ Mpc^{-1} the BB correlations are amplified at $l < 30$.

at the GUT scale, $V^{1/4} \sim 10^{15}$ GeV, we have $H \sim 10^{11}$ GeV and the temperature at the start of inflation $T_i/a_i \sim 10^{14-15}$ GeV. So the enhancement of the graviton power spectrum by the factor $\coth(\frac{k}{2T}) = \coth(\frac{Ha_i}{2T_i})$ could be by as large as a factor of 10^{4-5} at low k due to thermal gravitons.

In Fig. 1 we show the angular correlations of CMBR temperature and polarization assuming a thermal graviton spectrum (along with the WMAP three years data [4]). The plots for TT, TE, and BB correspond to comoving graviton temperature $T = 0.001$ Mpc^{-1}. For comparison we have plotted the BB angular correlations at $T = 0$. We see that with a temperature $T = 0.001$ Mpc^{-1} the BB correlations are amplified at $l < 30$. We see that only the BB correlation is enhanced by the correction to the tensor power spectrum as expected. The contribution of tensors to the TT angular spectrum is comparable at low l to the contribution from the scalars and there exists the possibility that this large tensor contribution at low l may be detected from the analysis of the TT angular spectrum alone.

We have added the unlensed scalar and tensor contributions to generate the TT, EE, TE, and BB correlations. The plots were obtained by running CMBFAST [9], with the following parameters: $\Omega_b = 0.05$, $\Omega_c = 0.25$, and $\Omega_v = 0.70$. Optical depth $\tau = 0.08$ and Hubble parameter $h = 0.7$. The value of scalar spectral index $n_s = 0.97$ and the value of tensor spectral index is taken $n_T = -0.01$. Tensor to scalar ratio is taken to be $r(k_0) = 0.1$ at $k_0 = 0.002$ Mpc^{-1}. The output of the CMBFAST was normalized to the WMAP values at $k = 0.002$ Mpc^{-1} (i.e., $l = 30$). For the curves shown in Fig. 1 the tensor power spectra is modified due to thermal effects with $\frac{k}{2T} = 500k$. At $k = 0.0002$ Mpc^{-1}, $\coth(500k) = 10$ so there is a large enhancement of the BB polarization at $l = 2$–6, whereas at $k_0 = 0.002$ Mpc^{-1}, $\coth(500k_0) \sim 1.3$ and there is hardly any enhancement of the BB signal [or in the value of $r(k_0)$ in keeping with the observational constraints from WMAP + SDSS [10]]. The magnitude of the comoving graviton temperature needed to produce this effect is $T/a_{\rm now} = 10^{-3}$ Mpc^{-1}. This corresponds to a temperature of $T_i/a_i \simeq 4 \times R_h^{-1} \times a_{\rm now}/a_i = 4H$ (where $R_h \sim 4000$ Mpc is the size of the present horizon). As we have seen inflation can start as soon as the temperature T_i/a_i falls below $V^{1/4} \sim 10^4 H$. Consequently, a temperature larger than $4H$ at the beginning of inflation is not unreasonably high.

In standard inflation models the vacuum fluctuations of the inflaton field give the density perturbations. The inflaton is assumed to be decoupled from the radiation at the onset of inflation, however, if there was a radiation era prior to inflation then the scalar curvature power spectrum is modified by the same temperature dependent factor [11] as the tensor power spectrum in Eq. (10),

$$P_{\mathcal{R}}(k) = \frac{H^4}{4\pi^2\dot{\phi}^2}\left(\frac{k}{aH}\right)^{n_s-1}\coth\left[\frac{k}{2T}\right]. \tag{14}$$

The extra temperature dependent term implies that there should be an upturn of the TT anisotropy spectrum at low l. This expected upturn in $l(l+1)C_l$ is not seen in the WMAP one-year TT spectrum [11]. This means that there is no significant number density of background density of inflatons at the time when the modes which are currently entering our horizon, were exiting the horizon during inflation. This could happen for two reasons. The background density of inflatons may have decayed or annihilated into lighter particles by this time or the inflaton was cooled from the expected temperature of $0.24(HM_P)^{1/2}$ to below H by the time the modes corresponding to our present horizon were leaving the de Sitter horizon. This implies that there were an extra $\Delta N = \ln[0.24(M_P/H)^{1/2}]$ e-foldings (which has the value $\Delta N \sim 10$ for GUT scale inflation) than what is needed to solve the horizon problem. In the case of gravitons the first condition does not apply as they decouple at the Planck scale and if the expected upturn in the BB mode spectrum is not seen that would imply that the duration of inflation was longer than what is needed to solve the horizon problem.

In warm inflation models [12] where the inflaton is in thermal equilibrium with the radiation bath and the scalar curvature perturbations are generated by thermal fluctuations instead of by quantum fluctuations there is no $\coth(k/2T)$ correction in the inflaton power spectrum due to stimulated emission. However, this correction factor will be present in the graviton spectrum since gravitons are still produced by quantum fluctuations. The scalar curvature perturbation in warm inflation is [13]

$$P_{\mathcal{R}}^{(\rm warm)} = \left(\frac{\pi}{4}\right)^{1/2}\frac{H^{5/2}\Gamma^{1/2}T_r}{\dot{\phi}^2}, \tag{15}$$

where Γ designates the decay width of the inflaton field and T_r is the temperature of the radiation bath.

There are observational constraints on the tensor scalar ratio

$$r(k_0) = \frac{P_T(k_0)}{P_{\mathcal{R}}(k_0)}. \tag{16}$$

From the combination of WMAP 3 yr data [10] and Sloan digital sky survey (SDSS) large scale structure surveys [14] we have the bound $r(k_0 = 0.002\ \text{Mpc}^{-1}) < 0.28$(95% C.L.) where $k_0 = 0.002$ Mpc^{-1} corresponds to $l = \tau_0 k_0 \simeq 30$ with the distance to the decoupling surface $\tau_0 = 14\,400$ Mpc. SDSS measures galaxy distributions at redshifts $a \sim 0.1$ and probes k in the range $0.016h$ Mpc$^{-1} < k < 0.11h$ Mpc^{-1}. From the expressions of $P_{\mathcal{R}}$ in warm inflation, Eq. (15), and P_T we see that the scalar-tensor ratio in warm inflation models (assuming a nearly scale invariant tensor power spectrum) has a scale dependence at large angles given by

$$r(k) \simeq r(k_0)\frac{\coth[\frac{k}{2T}]}{\coth[\frac{k_0}{2T}]} \simeq r(k_0)\left(\frac{k_0}{k}\right). \tag{17}$$

We see that $r(k)$ has a spectral index $n_T \sim -1$ for large scale perturbations. If we consider $k \sim 0.0002\ \mathrm{Mpc}^{-1}$, which corresponds to $l \sim 3$, then the value of $r(k) = 10r(k_0)$. So even with $r(k_0) \sim 0.1$ as constrained by galaxy surveys, we can have $r(k) \simeq 1$ at the quadrupole anisotropy. The B mode polarization at $l = 3$ is enhanced from its value at $l = 30$ by a corresponding factor of 10. This is true as long as the temperature $T_i/a_i \leq 10^4 H$, which as we have seen in the earlier discussion is expected if there is a thermal era prior to inflation.

For example, taking the inflaton potential to be $V = (1/2)m^2\phi^2$, we have the scalar power

$$P_{\mathcal{R}}^{(\mathrm{warm})}(k_0) = 5.3\frac{\Gamma^{5/2}\phi_0^{1/2}T_r}{M_p^{5/2}m^{3/2}}, \tag{18}$$

and the tensor power

$$P_T(k_0) = \frac{128\pi}{9}\frac{m^2\phi_0^2}{M_P^4}\coth\left[\frac{k_0}{2T}\right], \tag{19}$$

and the scalar-tensor ratio

$$r(k_0) = 8.413\left(\frac{m^{7/2}\phi_0^{3/2}}{M_P^{3/2}}\right)\frac{1}{\Gamma^{5/2}T_r}\coth\left[\frac{k_0}{2T}\right], \tag{20}$$

where ϕ_0 is the value of the inflaton field when the scale $k_0 = 0.002\ \mathrm{Mpc}^{-1}$ was leaving the inflaton horizon. By choosing the parameters $m = 1.4 \times 10^{12}$ GeV, $\Gamma = 0.5 \times 10^{13}$ GeV, $T \simeq T_r = 0.24 \times 10^{16}$ GeV, $\phi_0 \simeq 0.8 \times 10^{19}$ GeV we have $P_R \simeq 2.3 \times 10^{-9}$ as required by WMAP 3 yr data and $r(k_0) = 0.095$. The value of r is larger at $k = 0.0002\ \mathrm{Mpc}^{-1}$ by a factor of $\sim$10 and the B modes are magnified at $l = 3$ compared to their value at $l = 30$ by a factor 10, also in warm inflation scenarios.

Direct observation of gravitational waves would nail the last still unconfirmed prediction of inflation. The amplitude of gravitational waves gives the Hubble curvature during inflation and would tell us the value of the inflation potential [15,16]. In addition gravitational waves produced during inflation can have several applications such as leptogenesis by the gravitational spin-coupling to neutrinos [17] or by a gravitational Chern-Simon coupling of the lepton number current [18]. Observation of the B-mode polarization in the CMB would confirm the existence of primordial superhorizon gravitational waves. Observationally, the 3 yr WMAP data only give an upper bound on C_l^{BB} with $l = (2 - 6)$ [4]. The error bars on the C_l^{BB} are presently a factor of 5 larger than the predictions from standard inflation theory with scalar-tensor ratio as large as 0.3, which is close to the observational upper bound $r_{0.002} < 0.28$ (95% C.L.). In this Letter we show that due to thermal gravitons, the C_l^{BB} at low $l \simeq (2\text{–}6)$ could be larger by a factor of 10 compared to what would be expected from the observational constraint on r and could be within the range of observability of WMAP. The upcoming Planck experiment [6] will measure $C_{l=(1-10)}^{BB}$ at the level of $10^{-4}(\mu\mathrm{K})^2$. Ground based polarization experiments [19] such as QUaD, QUIET, Clover, and PolarBear measure anisotropies at small angular scales only (at $l > 100$ where thermal effects discussed in this Letter are negligible) and can observe C_l^{BB} at the level $10^{-2}(\mu\mathrm{K})^2$. These experiments can probe r in the range 0.05–0.1 independent of thermal effects. A combination of data from WMAP or Planck at large angles and ground based polarization experiments at small angles will therefore either observe or definitely rule out the thermal enhancement effect.

If WMAP or Planck rule out a spectral index of $n_T \sim -1$ at low l, which is the prediction from thermal gravitons, then for the standard inflationary models it would mean that the duration of inflation has to be longer by $\Delta N = \ln(0.24(M_P/H)^{1/2})$ e-foldings than what is needed to solve the horizon problem. Warm inflation models [12,13] cannot evade this constraint by supercooling during inflation. If B modes are observed and the tensor spectral index at low l is not close to -1, then warm inflation models can be ruled out.

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