# Comparing Contact and Dipolar Interactions in a Bose-Einstein Condensate

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We have measured the relative strength $\varepsilon_{dd}$ of the magnetic dipole-dipole interaction compared with the contact interaction in a dipolar chromium Bose-Einstein condensate. We analyze the asymptotic velocities of expansion of the condensate with different orientations of the atomic magnetic moments. By comparing the experimental results with numerical solutions of the hydrodynamic equations for dipolar condensates, we obtain $\varepsilon_{dd} = 0.159 \pm 0.034$. We use this result to determine the $s$-wave scattering length $a = (5.08 \pm 1.06 \times 10^{-9})\,\mathrm{m} = (96 \pm 20)\,a_0$ of $^{52}$Cr. This is fully consistent with our previous measurements on the basis of Feshbach resonances and therefore confirms the validity of the theoretical approach used to describe the dipolar Bose-Einstein condensate.

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Gaseous Bose-Einstein condensates (BEC) with dipole-dipole interaction (DDI) have become a fast growing field of theoretical and experimental interest. Many new exciting phenomena are expected. Because of the anisotropic character of the DDI, most of these phenomena depend strongly on the symmetry of the trap. The expected phenomena range from modifications of the ground state wave function [1,2], the expansion [3–5], the excitation spectrum [6–9], and stability criteria [1,7,10] to the occurrence of new quantum phases in optical lattices [11] and dramatic influence on the formation of vortices and vortex lattices [12,13]. Dipolar BECs are now also discussed in the context of spinor condensates [14,15], where the combination of large spin and magnetic moment leads to new effects such as the conversion of spin into angular momentum [14,16]. For all of these phenomena, the relative strength of the DDI compared with the contact interaction is a very important parameter. In two recent publications, we have reported on the generation of a BEC of chromium atoms ($^{52}$Cr) [17] and the observation of magnetic dipole-dipole interaction (MDDI) in the BEC [18]. In the latter case, we have shown that depending on the orientation of the magnetic moments of the condensed chromium atoms with respect to the long axis of our optical dipole trap, the expansion dynamics of the BEC is modified. The dynamic behavior of the condensate aspect ratio after release from an anisotropic trap was studied experimentally and compared with numerical calculations based on the description of the chromium BEC by superfluid hydrodynamic theory including dipole-dipole interaction [5,8,19]. The observed behavior showed an excellent qualitative agreement with the theoretical prediction. In this Letter, we discuss a method that allows us to determine the strength of the MDDI compared with the contact interaction with high accuracy. The excellent agreement between the results of our measurements and theoretical predictions confirms the validity of the theoretical methods that are being developed to describe BECs with dipole-dipole interactions and thus marks an important step toward a deeper understanding of Bose-Einstein condensation under these conditions.

Because the absolute strength of the MDDI can be calculated if the dipole moment of the atoms is known, one can use a measurement of the relative strength of the MDDI to determine the $s$-wave interaction strength, which is proportional to the $s$-wave scattering length $a$. Many different techniques have been used to determine the $s$-wave scattering lengths of ultracold atoms and acquire remarkable accuracy on the few per cent level or even better. Examples are the use of Feshbach resonance measurements or photoassociation spectroscopy [20–22]. But these methods require detailed knowledge of the molecular potentials that are involved. In cases where this knowledge is missing or where the experimental effort for using one of the aforementioned methods is too high, one has to rely on other techniques. The drawback of most of these methods is that they come with large error bars, often because the number of atoms enters the measurement. Examples are the $^{23}$Na scattering length in $|F = 1, m_F = -1\rangle$ of $a_{\mathrm{Na}} = (65 \pm 30)\,a_0$ [23] and the scattering length of metastable helium $a_{\mathrm{He}^*}$ [24], both from measurements of the mean-field energy. Our first determination of the chromium scattering length yielded $a_{\mathrm{Cr}} = (170 \pm 39)\,a_0$ [25]. It was based on cross-dimensional thermalization [26] and had a large systematic error due to an uncertainty in the density and atom-number determination. We will show in this Letter that a measurement of the relative strength of the magnetic dipole-dipole interaction in a BEC can be used to obtain precise values for $a$ without such a strong dependence on the determination of the number of atoms.

The interaction energy of two magnetic dipoles separated by the distance $\vec{r}$ is given by

$$U_{dd}(\vec{r}) = \frac{\mu_0 \mu_m^2}{4\pi r^3}\left(1 - \frac{3(\vec{e}_\mu \vec{r})^2}{r^2}\right) \qquad (1)$$

where $\mu_m$ is the magnetic moment of the atoms and the orientation $\vec{e}_\mu$ of these dipoles is parallel to an external

 

magnetic field $\vec{B}$. For bosonic chromium atoms in the $^7S_3$ ground state, the magnetic moment is $\mu_m = 6\mu_B$. The strength of the dipole-dipole interaction is given by the prefactor of $U_{dd}$ and can be compared with the coupling constant $g$ of the $s$-wave interaction

$$U_{sw}(\vec{r}) = g\delta(\vec{r}) = \frac{4\pi\hbar^2 a}{m}\delta(\vec{r}), \tag{2}$$

where $m$ is the mass of the atoms. The relative strength of the dipole-dipole interaction is given by the dimensionless parameter

$$\varepsilon_{dd} = \frac{\mu_0\mu_m^2 m}{12\pi\hbar^2 a}. \tag{3}$$

It is chosen such that a homogeneous condensate is unstable if $\varepsilon_{dd} > 1$ in a static magnetic field [27].

In contrast to the $s$-wave interaction which can be understood as a local, contactlike interaction (2), the dipole-dipole interaction is long-range and anisotropic. In a condensate with density distribution $n(\vec{r}) = |\phi(\vec{r})|^2$, it gives rise to the mean-field potential [1,8]

$$\Phi_{dd}(\vec{r}) = \int U_{dd}(\vec{r} - \vec{r}')|\phi(\vec{r}')|^2 d^3r'. \tag{4}$$

The integral in (4) reflects the nonlocal character of the interaction. If this interaction in addition to the contact interaction is taken into account, the well known Gross-Pitaevskii equation can be written in the form

$$i\hbar\frac{\partial}{\partial t}\phi(\vec{r}, t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + U_{\text{ext}}(\vec{r}) + g|\phi(\vec{r}, t)|^2 + \int U_{dd}(\vec{r} - \vec{r}')|\phi(\vec{r}', t)|^2 d^3r'\right)\phi(\vec{r}, t). \tag{5}$$

O'Dell *et al.* have shown in [8] that even under the influence of the dipole-dipole mean-field potential $\Phi_{dd}(\vec{r})$, the density distribution has the shape of an inverted parabola in the Thomas-Fermi limit. As in the case of pure contact interaction, a wave function of the form

$$|\phi(\vec{r})|^2 = n_{c,0}\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right) \tag{6}$$

is a self-consistent solution of the superfluid hydrodynamic equations [28] derived from the Gross-Pitaevskii equation (5), even in the presence of the dipole-dipole interaction [8,19]. $R_x$, $R_y$, and $R_z$ are the Thomas-Fermi radii of the condensate. The anisotropy of the dipole-dipole interaction manifests itself in a modification of the aspect ratio of the trapped condensate [19,27]. This anisotropy is also revealed during the expansion of a dipolar condensate [3,18].

In the following we will determine the dipole-dipole strength parameter $\varepsilon_{dd}$ by analyzing the dynamic behavior of the Thomas-Fermi radii $R(t)$ of expanding dipolar condensates. The experimental apparatus and techniques that are used are described in detail in [5,29]. By applying a small homogeneous external field (~11.5 G), oriented either along the $y$- or $z$-axis, shortly (~7 ms) before releasing the condensate from the trap, we obtained two sets of measured radii of a ballistically expanding condensate with different alignment of the atomic magnetic moments. The trap from which the condensate was released was a crossed optical dipole trap that was elongated in $z$ direction with trap parameters of $\omega_x = 2\pi\,942$ Hz, $\omega_y = 2\pi\,712$ Hz, and $\omega_z = 2\pi\,128$ Hz. After the initial phase of the expansion during which the mean-field energy is converted entirely into kinetic energy, the radii of the condensate grow linearly with time. We parametrize their evolution as

$$R_i(t) = R_i^* + v_i^* t, \tag{7}$$

with $i = [x, y, z]$. Because this parametrization neglects the initial acceleration, the $R_i(0) = R_i^*$ are smaller than the Thomas-Fermi radii. Similar to the case without dipole-dipole interactions, the radii as well as the asymptotic velocities $v_i^*$ of the expansion scale with the number of atoms $N$ and scattering length $a$ as $(Na)^{1/5}$ (see also [5]). In particular,

$$v_i^* = C_i(Na)^{1/5} \tag{8}$$

with constants of proportionality $C_i$ that only depend on known or measured quantities: i.e., the trap parameters $\omega_x$, $\omega_y$, $\omega_z$, the atomic mass $m$, and to a small but relevant and measurable extent on $\varepsilon_{dd}$ and the magnetic field direction. Thus the $C_i$ are independent of $N$ (and $a$) and are calculated using the hydrodynamic theory of an expanding dipolar condensate [5]. Table I shows the expected asymptotic velocities $v_y^* = C_y(30000 \times 103a_0)^{1/5}$ in $y$ direction and the corresponding values for $C_y$, calculated for pure contact interaction ($\varepsilon_{dd} = 0$), $y$ polarization, and $z$ polarization. The numbers are calculated for the measured trap parameters, 30 000 atoms, and a scattering length of $a = 103a_0$ [30]. The scattering length of $103\,a_0$ corresponds to a dipole-dipole strength parameter of $\varepsilon_{dd} = 0.148$.

To determine the asymptotic velocity, we use the condensate radii $R(t)$ measured in a time-of-flight series of absorption images. Because the number of camera pixels that are covered by the condensate is much larger in the

TABLE I. Asymptotic velocity in the $y$ direction and corresponding proportionality constant $C_y$ calculated numerically for the case of vanishing dipole-dipole interaction $\varepsilon_{dd} = 0$, and for $\varepsilon_{dd} = 0.148$ and polarization along $\hat{y}$ and $\hat{z}$. Velocities calculated for 30 000 atoms, and $a = 103a_0$.

| Polarization | $v_y^*[10^{-3}$ m/s] | $C_y[m^{4/5}]$ |
|---|---|---|
| No dipoles | 8.528 | 0.0488 |
| $y$ polarization | 9.085 | 0.0519 |
| $z$ polarization | 8.283 | 0.0474 |

direction of fast expansion ($y$ direction in our setup), one can expect the most accurate results when considering $R_y(t)$. Figure 1 shows the dependence of the condensate radius $R_y(t)$ with polarization along $\hat{y}$ (left figure) and $\hat{z}$ (right figure) on the time-of-flight for 67 different expansion times. Because the number of atoms is fluctuating during such a series of experiments, the radii fluctuate due to their dependence on $N$. To eliminate these fluctuations, we divide each measured radius $R_y$ by the fifth root of the number of condensed atoms $N$ in the corresponding experiment and multiply them with the mean value of the fifth root of the atom-numbers $\langle N^{1/5}\rangle$ in all experiments.

$$\bar{R}_y = \frac{R_y}{N^{1/5}}\langle N^{1/5}\rangle. \tag{9}$$

In this way we get a series of time dependent radii which are now independent of the atom number. Open circles in Fig. 1 represent the measured $R_y(t)$, crosses with error bars mark the rescaled $\bar{R}(t)$ which show much fewer fluctuations. A linear fit to the rescaled data for times larger than 3 ms to focus only on the asymptotic behavior, yields $v_y^* = 9.56 \pm 0.14$ mm/s for $y$ polarization. For $z$ polarization, we get $v_y^* = 8.78 \pm 0.12$ mm/s. By using the above rescaling, the errors $\Delta v_y^* = \pm 0.14$ mm/s and $\Delta v_y^* = \pm 0.12$ mm/s in the fitted slope $v_y^*$ for $y$- and $z$-polarization, respectively, do not contain fluctuations of the atom number anymore.

If we consider only the case of one polarization, invert Eq. (8), and insert the fitted velocity $v_y^*$ and the constant $C_y$ from Table I, we get

$$a = \frac{1}{\langle N\rangle}\left(\frac{v_y^*}{C_y}\right)^5 \tag{10}$$

for the scattering length. The error consists of two contributions: first an error $\Delta v^*$ asymptotic velocity due to the noise in the data that is not correlated with the number of atoms. Second, an error in the mean number of atoms, mainly due to an uncertainty in the detuning of the probe beam. Because a detuning from resonance can lead only to an underestimation of the number of atoms, the error in the scattering length $a$ caused by this uncertainty is only toward smaller values of $a$. We estimate a maximum detuning of $\Delta\delta_{\text{probe}} = \pm 0.25\Gamma$ which leads to an estimated error in the number of atoms of $\Delta N/N = -0.25$. The relative error in $a$ for $y$ polarization is then $\frac{\Delta a}{a} = \frac{\Delta\langle N\rangle}{\langle N\rangle} + 5\frac{\Delta v^*}{v^*} = -0.25 \pm 0.073$. Hence, the scattering length of $^{52}$Cr determined for $y$ polarization with this method is $a = (138^{+10}_{-45})\, a_0$. For $z$ polarization, we get a consistent value of $a = (133^{+9}_{-42})\, a_0$. Because of the relatively large systematic error in the number of atoms, this way to determine the scattering length yields only quite inaccurate values. In the following, we will use the full set of data from both polarizations to determine the scattering length with much higher accuracy and independent of the number of condensed atoms.

We use the two rescaled asymptotic velocities

$$\tilde{v}_y^* = \frac{v_y^*}{\langle N^{1/5}\rangle} \tag{11}$$

$\tilde{v}_y^*(\vec{B} \parallel \hat{y})$ and $\tilde{v}_y^*(\vec{B} \parallel \hat{z})$ for polarization along $\hat{y}$ and $\hat{z}$, respectively, to determine $\varepsilon_{dd}$ by analyzing their ratio. To

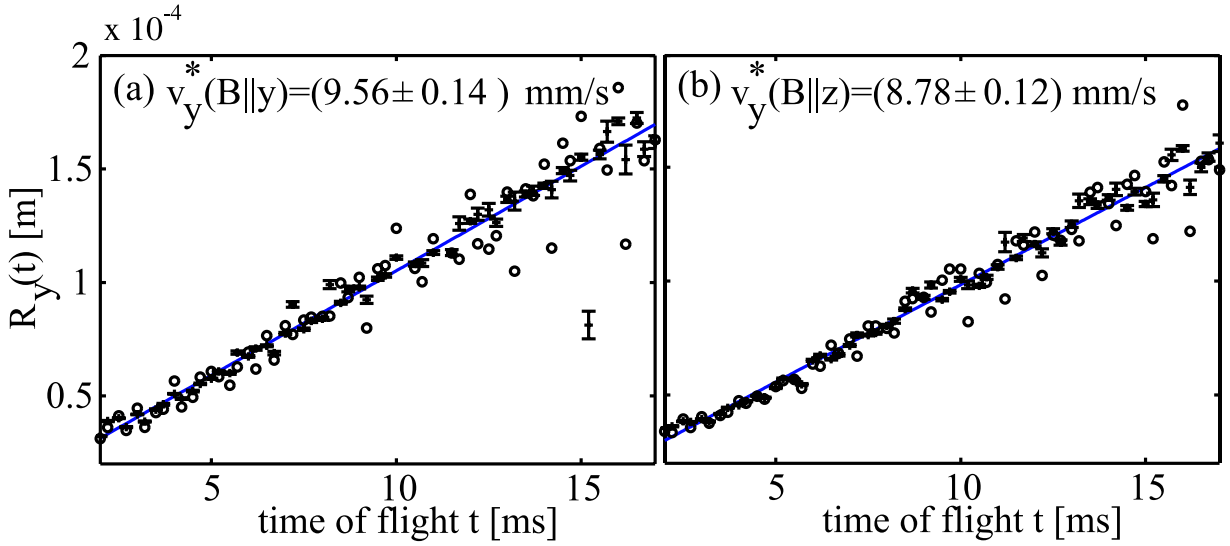


FIG. 1 (color online). Measured dependence of the condensate radius $R_y(t)$ on the time of flight. (a) Dipoles aligned along the $y$ axis, average number of atoms was 29 000; (b) dipoles aligned along the $z$ axis, average number of atoms was 31 000. Open circles: measured radii; crosses: measured radii rescaled using (9); solid black line: linear fit to the rescaled radii for times of flight $t_{\text{tof}} > 3$ ms. The error bars on the rescaled radii stem from uncertainties in the condensate radii and atom numbers, both determined from 2D fits of absorption images.

first order in $\varepsilon_{dd}$ (in the expected range of $\varepsilon_{dd}$, higher orders are negligible), the ratio has the form

$$\frac{\tilde{v}_y^*(\vec{B} \parallel \hat{y})}{\tilde{v}_y^*(\vec{B} \parallel \hat{z})} = 1 + D\varepsilon_{dd}. \quad (12)$$

It depends only on the asymmetry introduced by the dipole-dipole interaction because the contribution of the $s$-wave scattering to the total energy is independent of the polarization. $D$ is again a numerical constant. If we use the measured asymptotic velocities, we obtain $\varepsilon_{dd} = 0.159 \pm 0.034$ in very good agreement with the value of $\varepsilon_{dd} = 0.148 \pm 0.019$ that one would expect for $a = (103 \pm 13)\, a_0$.

In turn, because the dipole moment and mass of the atoms that contribute in (3) are known, the measured $\varepsilon_{dd}$ can be used to determine the scattering length $a = \frac{\mu_0 \mu_m^2 m}{12\pi\hbar^2 \varepsilon_{dd}} = (96 \pm 20)\, a_0$. This result is in excellent agreement with the value of $(103 \pm 13)\, a_0$ that has been obtained by comparing the measured positions of Feshbach resonances in chromium collisions with multichannel calculations [30].

In conclusion, we have measured the relative strength of the magnetic dipole-dipole interaction in a chromium BEC by analyzing its expansion with different polarization after release from an anisotropic trapping potential. We obtain the relative strength parameter $\varepsilon_{dd} = 0.159 \pm 0.034$. This result was used to determine the $s$-wave scattering length of $^{52}$Cr $a = (96 \pm 20)\, a_0$ in excellent agreement with the results of theoretical analysis of measured Feshbach resonances. In contrast to many other methods that are commonly used to determine the $s$-wave scattering length, this method does not depend on the accuracy of the atom-number determination. Furthermore, it does not require knowledge of any details of the molecular potentials. We expect it to be well suited to determine the scattering length close to a Feshbach resonance with high accuracy, especially for small scattering lengths where the dipole-dipole interaction becomes as important as the contact interaction. The excellent agreement between experimental results and theory constitutes a confirmation of the theoretical approach that is used to describe the dipolar BEC.

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