

Localized Modes in Open One-Dimensional Dissipative Random Systems

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We consider, both theoretically and experimentally, the excitation and detection of the localized quasimodes (resonances) in an open dissipative 1D random system. We show that, even though the amplitude of transmission drops dramatically so that it cannot be observed in the presence of small losses, resonances are still clearly exhibited in reflection. Surprisingly, small losses essentially improve conditions for the detection of resonances in reflection as compared with the lossless case. An algorithm is proposed and tested to retrieve sample parameters and resonance characteristics inside the random system exclusively from reflection measurements.

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Random stratified media are found in numerous geological and biological settings as well as in fabricated materials. Wave interactions in such systems determine, for example, reflection and transmission from multilayer dielectric stacks used as optical reflectors, filters, and lasers and propagation of seismic waves in the Earth's crust, microwave radiation in sand layers, and sonic waves in the oceans. In addition, though considerable effort has been expended to develop highly periodic structures, deviations from periodicity can significantly modify the characteristics of optical and microwave tunable photonic crystals. At the same time, it may be possible to utilize highly disordered samples, for many applications. For example, tunable switches or narrow line laser sources [1] can be created in randomly stacked systems or in disordered fibers. Since the location, width, and intensity of resonance are random, these can be found only by direct measurement of the field inside the sample, which is generally not feasible. Although strong localization of waves and resonances in one-dimensional random media has been extensively studied theoretically [2–4], most of the analytical results were obtained for lossless systems and for values averaged over ensembles of random realizations. But results for an ensemble do not provide the intensity spectrum within a particular sample, which is often essential.

In this Letter, we consider samples in which dissipation is weak in the sense that the condition

$$\frac{\Gamma_t}{v_g} l_{\text{loc}} \equiv \Gamma l_{\text{loc}} \ll \Gamma L \ll 1 \quad (1)$$

is satisfied. Here Γ_t and Γ , are, respectively, the temporal and spatial decrements of the wave energy due to loss, v_g is the local group velocity of the wave, L is the sample length, and l_{loc} is the localization length.

The nature of wave propagation in absorbing random systems is illustrated by measurements in a single-mode

microwave rectangular waveguide filled with random elements. Samples are composed of random mixtures of 31 randomly oriented elements which are 8 mm in length and are comprised of a solid ceramic first half and a second half which is milled to create an air space between two thin ceramic walls of thickness 0.8 mm, five 4 mm-long, high dielectric material slabs, identical to the first half of the binary element, and five 4 mm-long, low dielectric Styrofoam slabs. Only with the introduction of the 4 mm elements are states introduced deep within the band gap. The samples are prepared by choosing a random order of the elements and a random orientation of the binary element (ceramic part facing either input or output) in the waveguide. Comparison of measurements of the frequencies of the first mode at the two band edges of the band gap in a periodic structure of binary elements with a one-dimensional scattering matrix simulation which includes waveguide dispersion gives $n_1 \sim 1.67$ and $n_2 \sim 1.08$ for the refractive indices of the first and second portions of these elements, respectively. The field inside the waveguide is detected along a 2 mm-wide slot along the length of the top of the waveguide. The field is weakly coupled to an antenna so that it is not affected by the measurement. The core of the coaxial adapter without a protruding antenna picks up the field inside an enclosure coupled to the waveguide through a 2 mm hole which is pressed against the slot. The detector is translated in 1 mm steps along the slot. The in and out of phase components of the field were collected at each position with use of an HP8720C vector network analyzer. Spectra over 6 GHz were collected in 3.5 MHz steps. The cutoff frequency of the empty waveguide is 13.5 GHz. The frequency range covered is slightly larger than the stop band of the periodic structure.

Figure 1 shows the intensity $I(x, \nu)$ inside two random configurations normalized to the intensity of the incident wave as a function of coordinate and frequency. Distinct

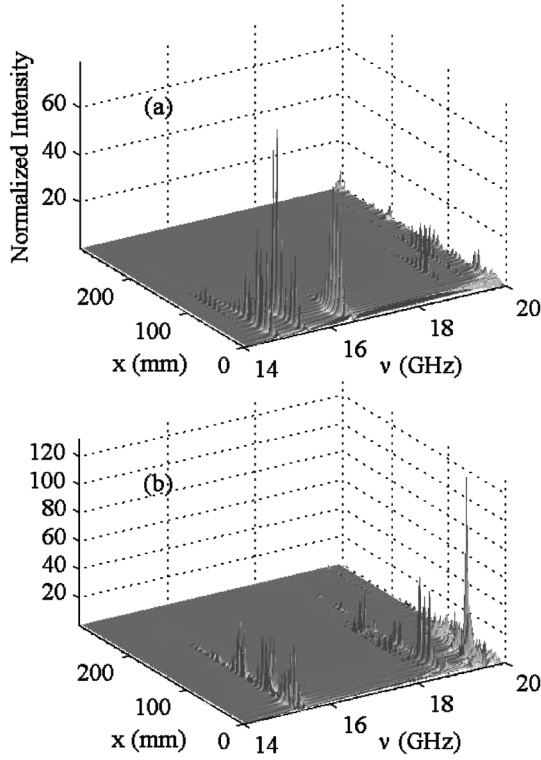


FIG. 1. Intensity vs frequency and position inside the sample for two different samples.

localized states with high intensity are seen in Fig. 1(a). In addition, multiple peaks in space such as those shown in Fig. 1(b) are found in other configurations. This field distribution has been discussed theoretically by Lifshits and Kirpichenkov and by Pendry [5]. Such a distribution has been termed a “necklace state,” though they correspond to several spectrally overlapping states rather than to a single state [6]. These states are associated with non-Lorentzian spectra, which have been observed in optical transmission through random layered media [6,7].

The reflectance spectrum $R(\nu)$ for the distributions in Fig. 1(a) is shown in Fig. 2(a). The instrumental error of the measurements of the reflection coefficient was about 5%. The transmittance (intensity at the output) $T(\nu) \equiv I(L, \nu)$ is below the experimental noise level [Fig. 2(b)]. Sharp minima are observed in $R(\nu)$ that are not accompanied by an essential increase of $T(\nu)$ [note the difference in the scales in Figs. 2(a) and 2(b)]. This is in contrast to the lossless case where R and T are connected by energy conservation, $R + T = 1$. Dips in $R(\nu)$ arise because the energy lost at each point within the sample is proportional to the product of the intensity of the field at that point and spatial absorption coefficient. Energy conservation requires

$$1 - T - R = I_{\text{loss}} = \Gamma \int_0^L \frac{v_g}{v_{g0}} I(x, \nu) \varepsilon(x) dx, \quad (2)$$

where I_{loss} is the rate of energy dissipated in the sample, $\varepsilon(x)$ is the dielectric constant, and v_{g0} is the group velocity

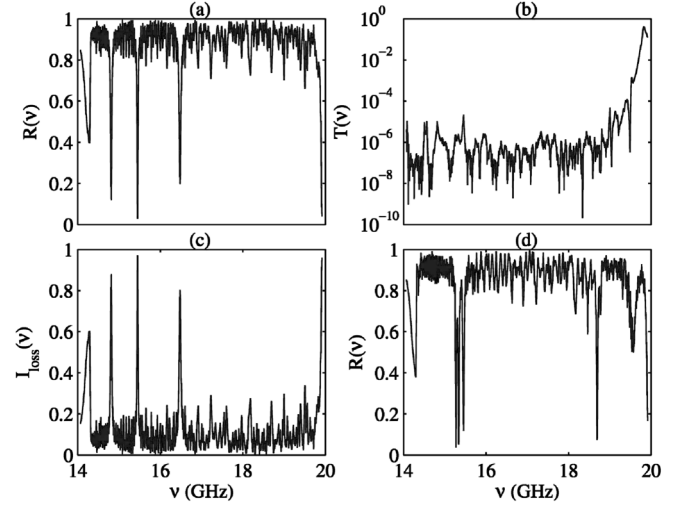


FIG. 2. Spectra of (a) reflection, (b) transmission, and (c) normalized energy loss $I_{\text{loss}}(\nu)$ for Fig. 1(a). The reflection spectrum of the sample with the distribution shown in Fig. 1(b) is shown in (d).

in the empty waveguide. For the following estimations, permittivity $\varepsilon(x)$ is replaced by its mean value $\bar{\varepsilon}$, whereas $v_{g0} \approx c$ and $v_g \approx c/\sqrt{\bar{\varepsilon}}$. Then the rate of energy dissipation can be written as $I_{\text{loss}} \approx \Gamma \sqrt{\bar{\varepsilon}} \int_0^L I(x, \nu) dx$.

Off resonance, the ensemble average intensity falls exponentially with decay length l_{loc} . The energy loss can therefore be estimated as $I_{\text{loss}} \sim \Gamma l_{\text{loc}} \sqrt{\bar{\varepsilon}} \ll 1$. Since the off-resonance transmittance $T \sim T_{\text{typ}} \equiv \exp(-L/l_{\text{loc}}) \ll 1$, the reflectivity R is close to unity, as it would be in the lossless case, but it saturates at $R \sim R_{\text{typ}} = 1 - \Gamma l_{\text{loc}} \sqrt{\bar{\varepsilon}} - T_{\text{typ}}$.

On resonance, loss can be of order unity due to the high intensity in localized states, even when Eq. (1) holds [Fig. 2(c)]. This suppresses both the reflected and transmitted fluxes. In Fig. 2, weak loss is seen to suppress the transmitted wave on resonance below experimental noise, while the suppression of the reflected wave is easily observed. The necklace shown in Fig. 1(b) is also easily detected in reflection as a group of closely spaced peaks with the number of peaks equal to that of the coupled states underlying the necklace [Fig. 2(d)].

To interpret the experimental data presented above, we extended the approach developed in Ref. [8] for modeling lossless random systems by incorporating small absorption in the model. The approach is based on associating each localized state at a frequency ν with a structure comprised of an essentially transparent segment (“well”) of small length $l \ll L$, surrounded by essentially nontransparent segments (“barriers”) with small transmittance $T_{1,2} \ll 1$ characteristic of “typical” nonresonant configurations. The configuration forms a high- Q resonator, and wave propagation through it can be treated as a particular case of the general problem of the transmission through an open resonant system, regular or random, whether it is a quantum-mechanical potential well, an optical or micro-

wave resonator, or a 1D random medium. The distinguishing feature of the random medium is that there are no regular walls, and the separate transmission coefficients of the isolated left and right segments of the sample, T_1 and T_2 , respectively, are exponentially small as a result of Anderson localization.

A necklace state exists in a random configuration when at some frequency there are several effective cavities separated by typical nontransparent segments. Generalization of the theory to this case amounts to the solution of a bit more complicated problem of the resonant propagation through a set of spatially separated potential wells (coupled resonators) whose number is equal to the number of dips in the measured frequency dependence of the reflection coefficient [Fig. 1(b)]. It is beyond the scope of this Letter and will be considered in a future publication.

Calculations of the resonant transmittance and reflectance of an open resonator with the absorption rate Γ and of the intensity inside the effective cavity yield, respectively,

$$T_{\text{res}} = \frac{4T_1T_2}{(\Gamma l\sqrt{\bar{\epsilon}} + T_1 + T_2)^2}, \quad (3)$$

$$R_{\text{res}} = 1 - T_{\text{res}} - \Gamma l\sqrt{\bar{\epsilon}}T_{\text{res}}/T_2, \quad (4)$$

$$I_{\text{res}} = T_{\text{res}}/T_2. \quad (5)$$

The half-width of the resonant peaks in $R(\nu)$ is given by

$$\delta\nu = \frac{c}{4\pi l\bar{\epsilon}}(\Gamma l\sqrt{\bar{\epsilon}} + T_1 + T_2). \quad (6)$$

In the case of Anderson localization, coefficients $T_{1,2}$ are approximately given by $T_{1,2} \simeq \exp[-(L/2 \pm d)/l_{\text{loc}}]$, where d is the coordinate of the resonance relative to the center of the sample, so that $T_1T_2 \simeq T_{\text{typ}}$. Equations (3)–(5) satisfy the conservation law Eq. (2) if $I_{\text{loss}} \simeq I_{\text{res}}\Gamma l\sqrt{\bar{\epsilon}}$. The reflection coefficient R_{res} and the rescaled intensity at the resonance $I_{\text{res}}^* = T_{\text{typ}}^{1/2}I_{\text{res}}$ depend on the position of the cavity and on the absorption through universal functions of two dimensionless parameters $a = \exp(-d/l_{\text{loc}})$ and $b = 2\Gamma l\sqrt{\bar{\epsilon}}T_{\text{typ}}^{-1/2}$:

$$R_{\text{res}} = \left(\frac{b + a^{-1} - a}{b + a^{-1} + a}\right)^2, \quad I_{\text{res}}^* = \frac{8a}{(b + a^{-1} + a)^2}. \quad (7)$$

Figures 3(a) and 3(b) depict the dependencies of I_{res}^* and R_{res} on parameters a and b . The resonances can be detected by peaks in the reflection spectrum $R(\nu)$. The bright area in Fig. 3(a) corresponds to well-excited localized states with high values of the intensity I_{res} , whereas the bright area in Fig. 3(b) indicates easily detectable resonances with pronounced minima in $R(\nu)$.

If dissipation is extremely weak so that $b \ll 1$, the influence of dissipation on the resonances is negligible, $I_{\text{loss}} \ll 1$, and strongly excited modes are located near the center and in the first, input half of the sample [Fig. 3(a)] as is in a lossless system [8]. It follows from Fig. 3(b) that for $b \ll 1$, easily detected modes are excited only in a relatively small region near the center of the sample. Thus,

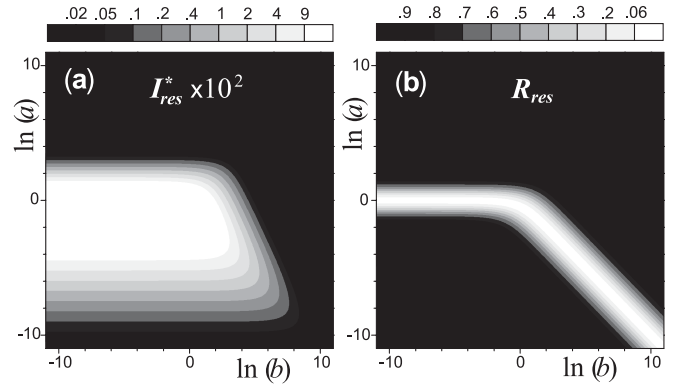


FIG. 3. (a) The rescaled resonant intensity and (b) reflection coefficient via parameters a and b .

only a relatively small part of excited resonances can be detected via $R(\nu)$ or $T(\nu)$ measurements.

If $b \gg 1$ [but Eq. (1) still holds], $I_{\text{loss}} \sim 1$ and small losses strongly affect the resonance characteristics. The absorption substantially reduces resonance transmission $T_{\text{res}} \ll 1$ and also gives rise to the nonmonotonic intensity distribution inside the first (close to the input) effective barrier. The intensity of the resonances decreases with increasing b [Fig. 3(a)], and the region, where well-excited resonances are located, becomes narrower and closer to the input. At the same time, the region in which resonant states can be detected becomes *larger* and also shifts towards the input [Fig. 3(b)]. For $b \gg 1$, almost all well-excited resonances are detectable via $R(\nu)$ measurements. Thus, the “detectability” of resonances is *improved* when small dissipation is present. It can be shown from Eqs. (7) that the number of resonances that can be detected in reflection is twice as large when $b \gg 1$ than in the opposite case ($b \ll 1$) for any given minimum in the discernible value of R . Moreover, at different values of b (or Γ), resonances in different regions of the sample are excited and detectable, which can provide for the scan of the sample through variations of losses. Thus, surprisingly, the presence of loss can make it easier to find the characteristics of the eigenmodes of a random sample.

It is important to notice that for $b \gg 1$ the reflection coefficient at resonances is a nonmonotonic function of dissipation [Fig. 3(b)] and reaches zero when

$$b = a - a^{-1}. \quad (8)$$

This effect is well known in optics and microwave electronics as critical coupling [9]. Since no energy is reflected from the sample at critical coupling, it corresponds to the resonance with the highest intensity I_{res} . Equation (8) determines the position $x_c = L/2 + d$ of the most powerful resonance in a sample with a given Γ :

$$x_c \simeq -l_{\text{loc}} \ln(\Gamma l\sqrt{\bar{\epsilon}}). \quad (9)$$

Unlike the lossless case, the disappearance of reflection is caused not by the high transparency of the sample but

rather by strong absorption in the effective cavity. The distance x_c as well as the maximal intensity do not depend on the total length L of the sample, and the sample can be considered as a half-infinite random medium.

Equations (3)–(6) enable one to formulate an algorithm for retrieval of the internal characteristics of localized states via external measurements. Indeed, quantities R_{res} , T_{res} , $\delta\nu$, $R_{\text{typ}} = 1 - \Gamma l_{\text{loc}} \sqrt{\bar{\epsilon}} - T_{\text{typ}}$, and $T_{\text{typ}} = \exp(-L/l_{\text{loc}})$ can, in principle, be measured outside the sample. Then Eqs. (3)–(6) constitute a system of equations, from which two internal parameters of the sample Γ and l_{loc} and three parameters of the resonances l , d , and I_{res} can be found. However, even small dissipation can make transmittance nonmeasurable (as it was in our experiment), and only the reflected signal can be analyzed. Nonetheless, the developed model still allows determination of parameters of the sample and resonances. When $b \gg 1$, the sample can be treated as a half-infinite, and one can put $T_2 = 0$ in Eqs. (3)–(6) and obtain the following sample parameters:

$$\Gamma = \frac{2\pi\delta\nu\sqrt{\bar{\epsilon}}}{c}(1 - \sqrt{R_{\text{res}}}), \quad l_{\text{loc}} \simeq \frac{1 - R_{\text{typ}}}{\Gamma\sqrt{\bar{\epsilon}}}. \quad (10)$$

The absorption rate Γ and the localization length l_{loc} are expressed via readily measured quantities R_{res} , $\delta\nu$, and R_{typ} . The values of Γ and l_{loc} calculated by means of Eq. (10) with $\delta\nu$ and R_{res} measured for all configurations are in agreement within $\Gamma = 3.76 \times 10^{-3} \text{ cm}^{-1} \pm 5\%$ and $l_{\text{loc}} \simeq 1.26 \text{ cm}$. The central position x_c of a resonance and its peak intensity I_{res} are given by

$$I_{\text{res}} = \frac{1 - R_{\text{res}}}{\Gamma l \sqrt{\bar{\epsilon}}}, \quad (11)$$

$$x_c = -l_{\text{loc}} \ln \left[\frac{2\pi\delta\nu l \bar{\epsilon}}{c} (1 + \sqrt{R_{\text{res}}}) \right]. \quad (12)$$

To check the algorithm of the retrieval of the internal parameters of resonances from the outside measurements of the reflection coefficient, we have compared the coordinates x_c and the normalized intensities I_{res} of the resonances calculated by Eq. (11) (setting the characteristic dimension of the cavity as $l \simeq 2l_{\text{loc}}$) with the corresponding values measured immediately inside the waveguide. As examples, the calculated (first row) and measured (second row) values of x_c and I_{res} for three resonances depicted in Fig. 1(a) ($\nu = 14.8, 15.5, \text{ and } 16.5 \text{ GHz}$) are given in the following table:

	x_c (cm)	I_{res}
Calc.	4.0, 4.8, 3.6	190, 150, 123
Meas.	4.2, 5.5, 3.6	86, 201, 96

Note that most of the detected resonances were in vicinity of the critical coupling regime [white area in Fig. 3(b)].

To conclude, localized states in a 1D lossy random system have been studied experimentally and analytically. The theoretical analysis is based on a model which describes isolated disorder-induced resonances in the framework of the general theory of resonant systems. It is shown that, when the dissipation length is much larger than the total length of the sample, the effect of dissipation off resonance is negligible. However, on resonance, the effect is dramatic and vanishes only when the dissipation length is exponentially large as compared to the sample length. Otherwise, the energy at a resonant frequency dissipates substantially due to the exponentially high intensity of the field inside the effective resonant cavity, resulting in the exponential small resonant transmission. The reflection coefficient depends nonmonotonically on the absorption and drops to zero at some values of the dissipation length (critical coupling effect). An algorithm has been developed for retrieval of the decrement of absorption and the localization length as well as the peak intensities and the positions of the localized modes inside a sample from the measurements of the reflection coefficient. Surprisingly, losses improve the detectability of resonances and change the conditions for their excitation. By changing the loss, one can scan the sample detecting modes excited at different points of the sample. The algorithm has been tested by comparison to the direct measurements of the microwave field inside a disordered single-mode waveguide.

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