B_s-$\bar{B}_s$ Mixing in Grand Unified Models

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We study B_s-$\bar{B}_s$ mixing in supersymmetry grand unified SO(10), SU(5) models where the mixings among the second and third generation squarks arise due to the existence of flavor violating sources in the Dirac and Majorana couplings which are responsible for neutrino mixings. We find that when the branching ratio of $\tau \to \mu\gamma$ decay is enhanced to be around the current experimental bound, B_s-$\bar{B}_s$ mixing may also contain large contribution from supersymmetry in the SO(10) boundary condition. Consequently, the phase of B_s-$\bar{B}_s$ mixing is large (especially for small $\tan\beta$ and large scalar mass m_0) and can be tested by measuring CP asymmetries of B_s decay modes.

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Flavor changing processes are important not only to test the Kobayashi-Maskawa theory [1] and to determine the parameters in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, but also to examine new physics. Recent measurement of B_s-$\bar{B}_s$ mass difference,

$$\Delta M_s = 17.77 \pm 0.10 \pm 0.07\ \mathrm{ps}^{-1}, \tag{1}$$

by the D0 and Collider Detector at Fermilab (CDF) Collaborations [2] can impact on the flavor structure of new physics beyond the standard model (SM) [3,4]. The experimental constraints for new physics are not very severe yet because deviations from the SM prediction can be buried in the errors of CKM parameters and lattice calculation.

Supersymmetry (SUSY) is one of the most promising candidates of new physics. SUSY can provide a natural prospect to have a large hierarchy in the theories, and in the minimal SUSY standard model (MSSM), gauge forces can unify at a high scale, which leads to a successful realization of grand unified theories (GUTs). However, the flavor sector has not yet been well accepted in the MSSM due to the fact that SUSY breaking terms can induce large flavor changing neutral currents (FCNCs). Actually, the experimental constraints of FCNCs introduce flavor degeneracies of the SUSY particles, especially in first and second generations, if SUSY particles are lighter than around 2–3 TeV [5].

In order to suppress the SUSY FCNCs, the squarks and sleptons are assumed to be degenerate at the GUT scale as a nature of the SUSY breaking. Even though the degeneracy is realized at a scale, the renormalization group equation (RGE) flow induces flavor violation for squarks and sleptons at low energy. In this scenario, the flavor violating effects in SUSY breaking terms are small at the weak scale and satisfy the current FCNC constraints. The small flavor violations originate from mixings in the Yukawa couplings characterized by the CKM mixings as well as the neutrino mixings. In the MSSM, the induced FCNCs in the quark sector are not large since the CKM mixings are small. In the lepton sector, on the other hand, sizable FCNC effects can be generated and a testable amount of flavor violating lepton decay can be obtained [6], which is related to the large mixings for the neutrino oscillations.

In the GUT models, the flavor violation at the weak scale can be related to the GUT scale physics. As a consequence of the quark-lepton unification, the large neutrino mixings not only introduce flavor violations in the lepton sector, but also in the quark sector. The relation of flavor violation in the quark and the lepton sectors depends on unification of matters and ways to obtain light neutrino masses. Therefore, investigating the FCNC effects, we may obtain a footprint of the GUT models.

The new result on B_s-$\bar{B}_s$ mixing can restrict the flavor violation in the quark sector involving b and s quarks. In GUT models, the B_s-$\bar{B}_s$ oscillation can be correlated to the $\tau \to \mu\gamma$ decay. Because $\mathrm{Br}(\tau \to \mu\gamma)$ is being measured at the B factories, the future results will be able to probe new contributions from the GUT models. In this Letter, we calculate B_s-$\bar{B}_s$ mixing and $\mathrm{Br}(\tau \to \mu\gamma)$ in SU(5) and SO(10) GUT models, and study the implication of the correlation between B_s-$\bar{B}_s$ mixing and $\mathrm{Br}(\tau \to \mu\gamma)$ decay in the CP asymmetry of B_s-$\bar{B}_s$ mixing (under experimental investigation) to decipher GUT models.

The existence of second-third generation (23) mixing elements in squarks and slepton mass matrices generate B_s mixing and $\tau \to \mu\gamma$ decay, respectively. We first investigate the arise of 23 elements in squark and slepton mass matrices in the grand unified theories.

SU(5) *model.*—In a SU(5) grand unified model, the superpotential involving the Yukawa couplings is as follows: $W_Y = Y_{uij}/4\ \mathbf{10}_i\mathbf{10}_j H_{\mathbf{5}} + \sqrt{2}Y_{dij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{H}_{\bar{\mathbf{5}}} + Y_{\nu ij}\bar{\mathbf{5}}_i N_j H_{\mathbf{5}} + M_{\nu ij}/2\ N_i N_j$, where $\bar{\mathbf{5}}$ contains the right-handed down-type quarks (D^c) and left-handed lepton doublets (L), and i, j denote the generation indices. The left-handed quark doublets (Q), right-handed up-type quarks (U^c), and right-handed charged-leptons (E^c) are unified in $\mathbf{10}$ multiplet, and N is the right-handed neutrino. Because two large neutrino mixings have been observed in nature, the Y_ν coupling is expected to have large off-diagonal elements, which will generate off-diagonal terms

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in the SUSY breaking scalar mass matrices for the scalar $\bar{\mathbf{5}}$ multiplet via $\bar{H}_{\bar{5}}$ and N loops [6]. One, thus, expects a large 23 element in the SUSY breaking scalar mass matrices for $\tilde{D}^c$ and $\tilde{L}$ due to the large atmospheric mixing and possibly large ν_τ Dirac Yukawa coupling. Therefore, the SUSY contribution for the amplitude of B_s-$\bar{B}_s$ mixing can be enhanced along with the branching ratio of $\tau \to \mu\gamma$.

Because our purpose is to investigate the flavor violation in the 23 sector of squark and sleptons and to probe how it relates the B_s-$\bar{B}_s$ mixing and flavor violating τ decay, we are only discussing the 23 element. For this purpose, we consider the following simplified SU(5) boundary condition for the SUSY breaking scalar mass matrices at the GUT scale where SU(5) is broken to the SM:

$$M^2_{\mathbf{10}} = M^2_{\tilde{Q}} = M^2_{\tilde{U}^c} = M^2_{\tilde{E}^c} = m_0^2 \mathbf{1}, \tag{2}$$

$$M^2_{\bar{\mathbf{5}}} = M^2_{\tilde{D}^c} = M^2_{\tilde{L}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & \delta^* & 1 \end{pmatrix} m_0^2. \tag{3}$$

The δ denotes the flavor mixing term arising from the neutrino Yukawa couplings discussed above. We assume the above boundary condition in the basis where the down-type quark Yukawa matrix is diagonal (GUT scale): $Y_d = Y_d^{\rm diag}$, $Y_u = V_{\rm CKM}^{\rm T} Y_u^{\rm diag} P_u V_{uR}$, $Y_e = V_{eL} Y_e^{\rm diag} P_e V_{eR}^\dagger$, where the up- and down-type quarks and the charged-lepton Yukawa couplings $Y_{u,d,e}^{\rm diag}$ are real (positive) diagonal matrices and $P_{u,e}$ are diagonal phase matrices. In a minimal SU(5) GUT, only $H_{\mathbf{5}}$ and $\bar{H}_{\bar{\mathbf{5}}}$ couple to matter fields, we have $V_{uR} = V_{\rm CKM}$, $V_{eL} = V_{eR} = \mathbf{1}$, and $Y_d^{\rm diag} = Y_e^{\rm diag}$. We do not assume the minimal choice of Higgs fields, but assume $V_{eL} \simeq \mathbf{1}$ to keep the relation of flavor violation between the quark and the lepton sectors.

We note that one can also have first-second and first-third generations mixings in squarks and sleptons. However, they can be small by the choice of 13 mixing in Y_ν and the hierarchical pattern of the neutrino coupling. If these elements are large, $\mu \to e\gamma$ will become large.

SO(10) *model.*—In a SO(10) model, the flavor violations can be more enhanced compared to the SU(5) case since all matters are unified in spinor representation and couple to $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs fields [7], e.g., $W_Y = \frac{1}{2} h_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10} + \frac{1}{2} f_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}$. The mixings in the neutrino Dirac Yukawa coupling may be small since right-handed neutrinos are also unified with other quarks and leptons. In a SO(10) model, however, there could be sources for large flavor violations in Majorana couplings for both left- and right-handed neutrinos in the type II seesaw scenario [8]. The Majorana couplings are unified to the $\mathbf{16} \times \mathbf{16} \times \overline{\mathbf{126}}$ coupling, and also affect the quark fields. The couplings will give rise to observable amount of flavor violations to the sparticle mass matrices via the GUT particle loops. Based on the above discussions, the following mass terms can arise in a SO(10) model at the GUT scale:

$$M^2_{\mathbf{16}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta \\ 0 & \delta^* & 1 \end{pmatrix} m_0^2.$$

When the matters couple to only $\mathbf{10}$ and $\overline{\mathbf{126}}$ Higgs fields, the Yukawa matrices are symmetric and the boundary condition is $Y_d = Y_d^{\rm diag} P_d$, $Y_u = V_{\rm CKM}^{\rm T} Y_u^{\rm diag} P_u V_{\rm CKM}$, $Y_e = V_{ql} Y_e^{\rm diag} P_e V_{ql}^{\rm T}$. We do not assume the minimal choice of Higgs fields but assume $V_{ql} \simeq \mathbf{1}$. It needs to be noted that the diagonal phase matrices $P_{u,d,e}$, which enter into our calculations, can not be rotated away for this boundary condition.

It appears that in order to suppress the proton decay and to obtain the correct fit of fermion masses one needs to extend the minimal SO(10) model. The new superpotential includes a $\mathbf{120}$ Higgs field: $W_Y = \frac{1}{2} h_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10} + \frac{1}{2} f_{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}} + \frac{1}{2} h'_{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{120}$. In this case, the symmetric nature of the Yukawa matrices is lost since $\mathbf{120}$ Higgs coupling h' is antisymmetric. In this context, Hermitian Yukawa matrices can be considered with a $\mathbf{120}$ Higgs field and a parity symmetry to reduce the number of parameters and to solve SUSY *CP* problem [9]. The SO(10) symmetry is broken in the basis where Y_d is diagonal, since the left- and right-handed fields are rotated by conjugated unitary matrices, e.g., $Q \to VQ$ and $U^c \to V^* U^c$. In the original SO(10) basis, the SUSY breaking mass matrices are real by the parity symmetry. As a result, in the basis where the down-type quark Yukawa matrix is diagonal, the squarks and slepton masses at the GUT scale are related by the following relation: $M^2_{\tilde{Q}} = M^{2*}_{\tilde{U}^c} = M^{2*}_{\tilde{D}^c} = M^2_{\tilde{L}} = M^{2*}_{\tilde{E}^c}$. So we see that two mass relations are possible in a SO(10) model at the GUT scale. We call one of them symmetric (since the Yukawa couplings are symmetric) and the other one Hermitian (since the Yukawa couplings are Hermitian).

We use the above mass matrices for the boundary condition at the GUT scale and then calculate the masses at the weak scale by using RGEs to calculate the mixing of B_s-$\bar{B}_s$ and ${\rm Br}(\tau \to \mu\gamma)$. In calculating the mass differences of mesons, one encounters nonperturbative factors originating from strong interaction. In the ratio of B_s and B_d mass differences, many common factors cancel, and the ratio can be calculated more accurately rather than the respective mass differences. We have $\frac{\Delta M_s^{\rm SM}}{\Delta M_d^{\rm SM}} = \frac{M_{B_s}}{M_{B_d}} \xi^2 |\frac{V_{ts}}{V_{td}}|^2$, where $\xi \equiv \sqrt{B_{B_s}} f_{B_s} / (\sqrt{B_{B_d}} f_{B_d}) = 1.23 \pm 0.06$ [10] is a ratio of decay constants $f_{B_{s(d)}}$ and bag parameters $B_{B_{s(d)}}$ for $B_{s(d)}$ mesons.

It is convenient to parametrize the SUSY contribution by two real parameters C_{B_s} and ϕ_{B_s} in model-independent way as [11]

$$C_{B_s} e^{2i\phi_{B_s}} \equiv \frac{M_{12}(B_s)}{M_{12}(B_s)^{\rm SM}}, \tag{4}$$

where $M_{12}(B_s) = M_{12}(B_s)^{\rm SM} + M_{12}(B_s)^{\rm SUSY}$ denotes the off-diagonal element of B_s-$\bar{B}_s$ mass matrix. Superscript

SM (SUSY) stands for SM (SUSY) contribution. The mass difference is given as $\Delta M_s = 2|M_{12}(B_s)|$.

We now discuss the SUSY contributions in B_s-$\bar{B}_s$ mixing. When the flavor degeneracy is assumed in the MSSM, the chargino diagram dominates the SUSY contributions of $M_{12}(B_{s,d})$. In this case, $\phi_{B_s} \simeq 0$ in Eq. (4), and the ratio of mass differences in the MSSM is almost same as in the SM. In the general parameter space for soft SUSY breaking terms, the gluino box diagram dominates the SUSY contribution. The gluino ($\tilde{g}$) contribution can be written in the following mass insertion form

$$\frac{M_{12}^{\tilde{g}}}{M_{12}^{\rm SM}} \simeq a[(\delta_{LL}^d)_{32}^2 + (\delta_{RR}^d)_{32}^2] - b(\delta_{LL}^d)_{32}(\delta_{RR}^d)_{32}, \quad (5)$$

where a and b depend on squark and gluino masses, and $\delta^d_{LL,RR} = (M^2_{\tilde{d}})_{LL,RR}/\tilde{m}^2$ ($\tilde{m}$ is an averaged squark mass). The matrix $M^2_{\tilde{d}}$ is a down-type squark mass matrix $(\tilde{Q}, \tilde{D}^{c\dagger})M^2_{\tilde{d}}(\tilde{Q}^\dagger, \tilde{D}^c)^T$ in the basis where down-type quark mass matrix is real (positive) diagonal. When squark and gluino masses are less than 1 TeV, $a \sim O(1)$ and $b \sim O(100)$. We also have contributions from δ^d_{LR}, but we neglect them since they are suppressed by $(m_b/m_{\rm SUSY})^2$. It is worth noting that the SO(10) boundary condition gives much larger SUSY contribution to $M_{12}(B_s)$ compared to the SU(5) case since both off-diagonal elements for LL and RR are large and $b \gg a$ in the formula, Eq. (5).

For the calculation of observables at weak scale, we need to use the basis where $Y_{d,e}$ are real (positive) diagonal matrix. In this basis, the boundary condition is given as $M^2_{\tilde{Q}} = P_d M^2_{\tilde{D}^c} P^\dagger_d$, $M^2_{\tilde{L}} \simeq P_e M^2_{\tilde{E}^c} P^\dagger_e$. Because the decay width is proportional to the squared absolute values of decay amplitudes, the phase of δ in the SUSY breaking mass matrix at the GUT scale and the phase of P_e in the Yukawa couplings are less important for $\text{Br}(\tau \to \mu\gamma)$. On the other hand, the phases of δ and P_d are important for $M_{12}(B_s)^{\rm SUSY}$. Because of those phases, the argument of $M_{12}(B_s)^{\rm SUSY}$ can be completely free.

In Fig. 1, we plot the maximal and minimal values of the ratio of mass differences versus branching ratio of $\tau \to \mu\gamma$ in the case of $\tan\beta = 10$ where $\tan\beta$ is a ratio of up- and down-type Higgs vacuum expectation values. In the plot, we use $M_{1/2} = 300$ GeV and $A_0 = 0$ for the universal gaugino mass and the universal trilinear scalar coupling coefficient at GUT scale. The universal scalar mass at the GUT scale is $m_0 = 200$ for the SU(5) plot, and $m_0 = 200$, 400 GeV for the SO(10) plots. We use $|V_{\rm td}/V_{\rm ts}| = 0.192 \pm 0.009$ which is obtained by the global CKM parameter fit without using experimental data for ΔM_s [11]. The ratio of mass differences is proportional to $\xi^2/|V_{\rm td}|^2$. The $\text{Br}(\tau \to \mu\gamma)$ is almost proportional to $\tan^2\beta$, while $\Delta M_s/\Delta M_d$ does not depend on $\tan\beta$ much. We find that $\text{Br}(\tau \to \mu\gamma)$ does not have much dependence on m_0 (for $m_0 \lesssim 500$ GeV), while the SUSY contribution of ΔM_s depends on m_0.

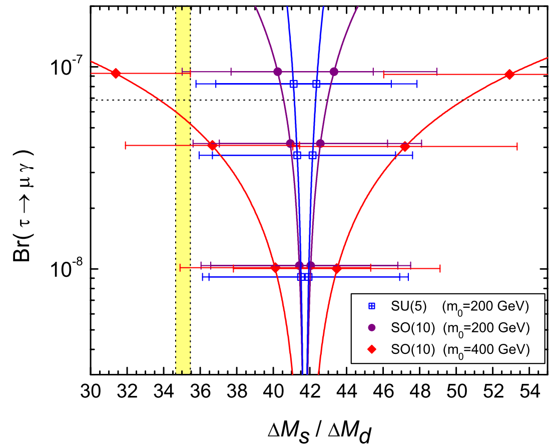

FIG. 1 (color online). Minimal and maximal values for ratio of mass differences versus $\text{Br}(\tau \to \mu\gamma)$ under SU(5) and SO(10) boundary conditions. We show $|\delta| = 0.05$, 0.1, 0.15 points. The dotted lines show 90% C.L. region of the experimental data.

In order to illustrate that any phase is possible for $M_{12}(B_s)^{\rm SUSY}$, we plot the real and the imaginary part of $M_{12}(B_s)$ in the case where $\text{Br}(\tau \to \mu\gamma) = 6.8 \times 10^{-8}$ [12] for $\tan\beta = 10$ in Fig. 2. We use the same values for $M_{1/2}$ and A_0 as in Fig. 1 and $m_0 = 500$ GeV. Using SU(5) and SO(10) (Hermitian) boundary mass values, we vary the phase of δ. With SO(10) symmetric boundary conditions, we fix δ to be real (positive) and vary the phase in the diagonal phase matrix P_d. In the case of SO(10) with Hermitian boundary conditions, the plot is a double circle due to the gluino contribution as shown in Eq. (5). Because of the RGE effect, the double circle does not overlap completely. We note that even if the radius of circle becomes large, $\Delta M_s = 2|M_{12}(B_s)|$ has experimentally allowed solutions, as long as $|M_{12}(B_s)^{\rm SUSY}| \lesssim 2|M_{12}(B_s)^{\rm SM}|$, though one needs to adjust the phases in boundary conditions. It is

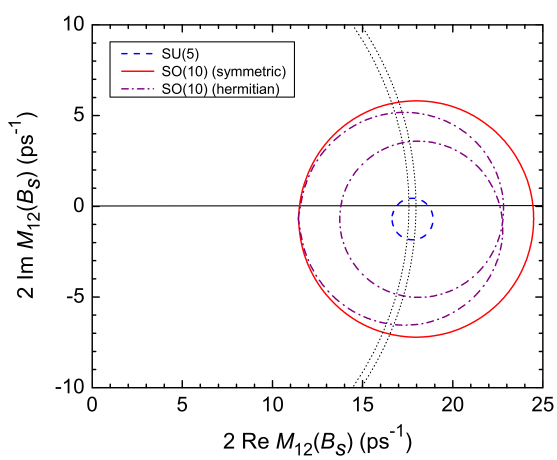

FIG. 2 (color online). Re-Im plot for $2M_{12}(B_s)$ when $\text{Br}(\tau \to \mu\gamma)$ saturates experimental bound. We use $\sqrt{B_{B_s}} f_{B_s} = 262$ MeV [10]. The dotted lines show 90% C.L. region of the experimental data.

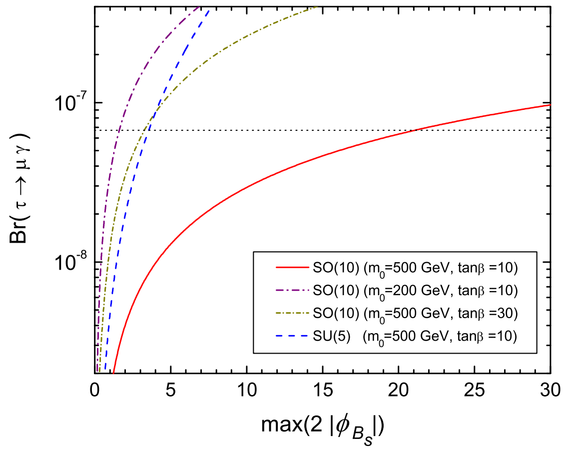

FIG. 3 (color online). $\max(2|\phi_{B_s}|)$ (degree) versus $\mathrm{Br}(\tau \to \mu\gamma)$.

worth emphasizing that the phases of $M_{12}(B_s)$ are large in such solutions.

In order to see the maximum allowed phase in the B_s-$\bar{B}_s$ mixing, we plot the maximal value of $2|\phi_{B_s}|$ versus $\mathrm{Br}(\tau \to \mu\gamma)$ in Fig. 3. We use the same values for $M_{1/2}$ and A_0 as in Fig. 1. When δm_0^2 in the boundary condition becomes large, the radius of the circle in Fig. 2 becomes large, and $|\phi_{B_s}|$ can be large consequently. One can approximately obtain $\max(\sin 2|\phi_{B_s}|) \simeq |M_{12}^{\tilde{g}}/M_{12}^{\mathrm{SM}}|$. When the $|\phi_{B_s}|$ is maximized, we find $C_{B_s} \simeq \cos\max(2|\phi_{B_s}|)$. The model-independent constraints for C_{B_s} and ϕ_{B_s} are $C_{B_s} = 0.97 \pm 0.27$ and $2\phi_{B_s} = (-4 \pm 30)^\circ \cup (186 \pm 30)^\circ$ [11]. The phase ϕ_{B_s} can be measured by CP asymmetry of the decay $B_s \to J/\psi\phi$ and the semileptonic decay $B_s \to l^- X$. The phase ϕ_{B_s} in the SO(10) case can be larger than in the SU(5). As shown in Fig. 3, $2\phi_{B_s}$ can be around 20° in the SO(10) case before the parameter space gets ruled out by the $\mathrm{Br}(\tau \to \mu\gamma)$. In order to obtain a large phase, large m_0 and small $\tan\beta$ are needed, which leads to an important implication. The Higgs boson mass bounds for MSSM restricts the lower values of $\tan\beta$ and $M_{1/2}$. In the minimal supergravity model, the scalar mass m_0 is restricted to be less than around 200 GeV (for $m_0 <$ 1 TeV) [13] for $\tan\beta = 10$ by the WMAP data [14], and as a result, the phase $|\phi_{B_s}|$ cannot be very large. Interestingly, the large m_0 solution ($m_0 > 1$ TeV) for dark matter content [15] may generate large $\phi_{B_s} \sim 90^\circ$ which is allowed by the experimental data. However, the muon $g-2$ [16] (using the e^+e^- data) restricts $m_0 \lesssim 500$ GeV at the 2 sigma level when $\tan\beta = 10$.

In conclusion, we have studied the correlation between $\mathrm{Br}(\tau \to \mu\gamma)$ and B_s-$\bar{B}_s$ mixing using SU(5) and SO(10) models. The SO(10) models can have larger effects on B_s-$\bar{B}_s$ mixing compared with the SU(5) models. We find that when $\mathrm{Br}(\tau \to \mu\gamma)$ is enhanced, B_s-$\bar{B}_s$ mixing may also contain large contribution from SUSY. Consequently, the phase of $M_{12}(B_s)$ is large for a SO(10) model and can be tested by measuring CP asymmetries of the B_s decay modes. It is interesting to note that the phase of B_s-$\bar{B}_s$ mixing can be large for smaller $\tan\beta$ and large scalar mass m_0 at the GUT scale.

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