Spin Hall Effects for Cold Atoms in a Light-Induced Gauge Potential

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We propose an experimental scheme to observe spin Hall effects with cold atoms in a light-induced gauge potential. Under an appropriate configuration, the cold atoms moving in a spatially varying laser field experience an effective spin-dependent gauge potential. Through numerical simulation, we demonstrate that such a gauge field leads to observable spin Hall currents under realistic conditions. We also discuss the quantum spin Hall state in an optical lattice.

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The spin Hall effect has recently attracted strong interest in condensed matter physics because of its connection to quantum Hall physics [1,2] and its potential applications in spintronics [3–9]. In analogy to the conventional Hall effect related to charge currents, the spin Hall effect refers to the generation of a spin current transverse to an applied electric field. It has been proposed to occur in certain solid-state systems with some primitive experimental demonstration [3]. An essential requirement for observation of the spin Hall effect is the generation of an effective spin-dependent gauge potential either in momentum space [5–7] or in real space [8].

It has been widely acknowledged that ultracold atomic gases provide an ideal playground to experimentally investigate some fundamental phenomena originally connected with condensed matter systems [10]. The remarkable controllability in these systems allows a clean study of much complicated physics in a controllable fashion. The generation of an effective gauge potential in atomic systems has raised significant interest, from the earlier implementation of rotating traps [11] to the more recent work on light-induced gauge fields [12–18]. Although most previous work focuses on the study of scalar gauge fields, it is natural to ask whether it is possible to study the *spin* Hall effect in an atomic system.

In this paper, we propose an experimental scheme for observation of the spin Hall effects in a cold atomic gas. We show that for atoms with a simple Λ-type three-level configuration moving in a spatially varying laser field, a spin-dependent gauge potential in the real space naturally arises in connection with the Berry phase associated with the atomic motion. Under an applied effective "electric" field, which can be generated from gravity for instance, the atoms will follow a spin-dependent trajectory, which leads to a net spin current in the direction perpendicular to the electric and the gauge field while the mass current is zero. Furthermore, we show it is easy to generate different forms of the gauge field in this system, with a strong periodic gauge field as an example. The diverse configurations of the gauge field, in combination with the tunable interaction and the controllable potentials for the atomic gas, may allow us to study various kinds of interesting Hall physics in this system. With this gauge field, we also discuss the associated quantum spin Hall effect for fermionic atoms in an optical lattice.

We consider an atomic gas with each atom having an Λ-type level configuration as shown in Fig. 1(a). The ground states $|1\rangle$ and $|2\rangle$ are coupled to an excited state $|3\rangle$ through spatially varying laser fields, with the corresponding Rabi frequencies Ω_1 and Ω_2, respectively. Different from the previous work [15,18], we assume here off-resonant couplings for the single-photon transi-

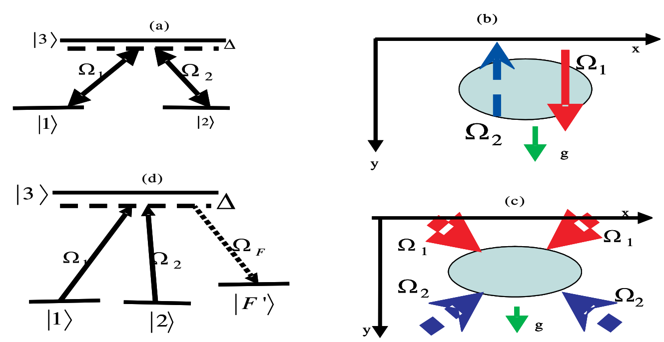

FIG. 1 (color online). Schematic representation of the light-atom interaction for generation of effective spin-dependent gauge fields. (a) Three-level Λ-type atoms interacting with laser beams characterized by the Rabi frequencies Ω_1, Ω_2 through the Raman-type coupling with a large single-photon detuning Δ. (b) The configurations of the Raman laser beams. Configuration I: two counterpropagating and overlapping laser beams with shifted spatial profiles (see also Ref. [18]). (c) Configuration II: A periodic gauge field can be created by four overlapping laser beams propagating along the shown directions. The upper two form the Raman beam Ω_1 while the lower two form Ω_2. (d) A Raman configuration to transfer the bright state to a different hyperfine level $|F'\rangle$ for detection. The $|1\rangle$ and $|2\rangle$ are assumed to be different Zeeman states on the same hyperfine level $|F\rangle$.

tions with the same large detuning Δ, and we will use the bright state as well as the dark state to realize a spin-dependent gauge field for the atoms.

The full quantum state of the atoms $|\Phi(\mathbf{r})\rangle$ (including both the internal and the motional degrees of freedom) can be expanded as $|\Phi(\mathbf{r})\rangle = \sum_{j=1}^{3} \phi_j(\mathbf{r})|j\rangle$, where $\mathbf{r}$ denotes the atomic position. The Hamiltonian of the atom has the form $H = \frac{\mathbf{P}^2}{2m} + V(\mathbf{r}) + H_{\rm int}$, where m is the atomic mass, $V(\mathbf{r})$ denotes the external trapping potential which we assume to be diagonal in the internal states $|j\rangle$ with the form $V(\mathbf{r}) = \sum_j V_j(\mathbf{r})|j\rangle\langle j|$, and $H_{\rm int}$ is the laser-atom interaction Hamiltonian, given by

$$H_{\rm int} = \begin{pmatrix} 0 & 0 & \Omega_1 \\ 0 & 0 & \Omega_2 \\ \Omega_1^* & \Omega_2^* & 2\Delta \end{pmatrix} \quad (1)$$

in the basis $\{|1\rangle, |2\rangle, |3\rangle\}$. We parametrize the Rabi frequencies through $\Omega_1 = \Omega \sin\theta e^{i\varphi}$ and $\Omega_2 = \Omega\cos\theta$ with $\Omega = \sqrt{|\Omega_1|^2 + |\Omega_2|^2}$ (θ and φ are in general spatially varying). The eigenvectors (the dressed states) $|\chi\rangle = (|\chi_1\rangle, |\chi_2\rangle, |\chi_3\rangle)^{\rm Tr}$ of the Hamiltonian $H_{\rm int}$ are specified by $|\chi\rangle = U(|1\rangle, |2\rangle, |3\rangle)^{\rm Tr}$ (Tr denotes the transposition), where

$$U = \begin{pmatrix} \cos\theta & -\sin\theta e^{-i\varphi} & 0 \\ \sin\theta\cos\gamma e^{i\varphi} & \cos\theta\cos\gamma & -\sin\gamma \\ \sin\theta\sin\gamma e^{i\varphi} & \cos\theta\sin\gamma & \cos\gamma \end{pmatrix}, \quad (2)$$

and γ is given by $\tan\gamma = (\sqrt{\Delta^2 + \Omega^2} - \Delta)/\Omega$, with the corresponding eigenvalues $\lambda = (0, \Delta - \sqrt{\Delta^2 + \Omega^2}, \Delta + \sqrt{\Delta^2 + \Omega^2})^{\rm Tr}$. In the new basis $|\chi\rangle$, the full quantum state of the atom $|\Phi(\mathbf{r})\rangle$ is written as $|\Phi(\mathbf{r})\rangle = \sum_j \Psi_j(\mathbf{r})|\chi_j(\mathbf{r})\rangle$, where the wave functions $\Psi = (\Psi_1, \Psi_2, \Psi_3)^{\rm Tr}$ obey the Schrödinger equation $i\hbar\partial_t\Psi = \tilde{H}\Psi$, with the effective Hamiltonian $\tilde{H}$ taking the form

$$\tilde{H} = \frac{1}{2m}(-i\hbar\nabla - \tilde{\mathbf{A}})^2 + \tilde{V}(\mathbf{r}), \quad (3)$$

where $\tilde{\mathbf{A}} = i\hbar U\nabla U^\dagger$ and $\tilde{V}(\mathbf{r}) = \lambda I + UV(\mathbf{r})U^\dagger$ (I is the 3×3 unit matrix) [19]. From Eq. (3), one can see that in the new basis the atoms can be considered as moving in a gauge potential $\tilde{\mathbf{A}}$ and a scalar potential $\tilde{V}(\mathbf{r})$.

We are interested in the subspace spanned by the two lower internal eigenstates $\{|\chi_1\rangle, |\chi_2\rangle\}$ (called, respectively, the dark and the bright state). This gives an effective spin-1/2 system, and in the spin language we also denote $|\chi_\uparrow\rangle \equiv |\chi_1\rangle$ and $|\chi_\downarrow\rangle \equiv |\chi_2\rangle$. In the case of a large detuning ($\Delta \gg \Omega$), both states $|\chi_\uparrow\rangle$ and $|\chi_\downarrow\rangle$ have negligible contribution from the initial excited state $|3\rangle$, so they are stable under atomic spontaneous emission. Furthermore, we assume the adiabatic condition, which requires the off-diagonal elements of the matrices $\tilde{\mathbf{A}}$ and $\tilde{V}$ are much smaller than the eigenenergy differences $|\lambda_i - \lambda_j|$ ($i, j = 1, 2, 3$) of the states $|\chi_i\rangle$. This gives the quantitative condition $F \ll \Omega^2/2\Delta$, where $F = \cos^2\theta|\mathbf{v}\cdot\nabla(\tan\theta e^{i\varphi})|$ ($\mathbf{v}$ is the typical velocity of the atom) represents the two-photon Doppler detuning [18]. Under this adiabatic condition, the Schrödinger equation for the wave function Ψ becomes diagonal in the basis $\{|\chi_i\rangle\}$, and in the lower subspace spanned by $\{|\chi_\uparrow\rangle, |\chi_\downarrow\rangle\}$, the effective Hamiltonian takes the form

$$H_{\rm eff} = \begin{pmatrix} H_\uparrow & 0 \\ 0 & H_\downarrow \end{pmatrix}, \quad (4)$$

where $H_\sigma = \frac{1}{2m}(-i\hbar\nabla - \mathbf{A}_\sigma)^2 + V_\sigma(\mathbf{r})$, ($\sigma = \uparrow, \downarrow$). The gauge and the scalar potentials $\mathbf{A}_\sigma$ and V_σ for the spin-σ component are given by $\mathbf{A}_\sigma = i\hbar\langle\chi_\sigma|\nabla|\chi_\sigma\rangle$ and $V_\sigma(\mathbf{r}) = \lambda_\sigma + \langle\chi_\sigma|V|\chi_\sigma\rangle + \frac{\hbar^2}{2m}[\langle\nabla\chi_\sigma|\nabla\chi_\sigma\rangle + |\langle\chi_\sigma|\nabla\chi_\sigma\rangle|^2]$, respectively. Through Eq. (2), one can find out that $\mathbf{A}_\uparrow = -\mathbf{A}_\downarrow = -\hbar\sin^2\theta\nabla\varphi$ and the related gauge field

$$\mathbf{B}_\sigma = \nabla\times\mathbf{A}_\sigma = -\eta_\sigma\hbar\sin(2\theta)\nabla\theta\times\nabla\varphi, \quad (5)$$

where $\eta_\uparrow = -\eta_\downarrow = 1$. We get exactly a spin-dependent gauge field from the above configuration of the laser-atom coupling, which is critical for the spin Hall effect.

We consider two specific configurations of the laser beams, which generate different spatial variations of the gauge field. First, two counterpropagating Gaussian laser beams with shifted centers generate a spatially slowly varying gauge field [18]. The spatial profiles of the corresponding Rabi frequencies Ω_j have the form $\Omega_j = \Omega_0\exp[-(x - x_j)^2/\sigma_0^2]\exp(-ik_jy)$, ($j = 1, 2$), where the propagating wave vectors $k_1 = -k_2 = k/2$ and the center positions $x_1 = -x_2 = \Delta x/2$ [see Fig. 1(b)]. Under these two laser beams, the gauge field is given by

$$\mathbf{A}_\sigma^I = \frac{-\eta_\sigma\hbar k}{1 + e^{-x/d}}\mathbf{e}_y, \qquad \mathbf{B}_\sigma^I = \frac{\eta_\sigma\hbar k}{4d\cosh^2(x/2d)}\mathbf{e}_z, \quad (6)$$

where $d = \sigma_0^2/(4\Delta x)$. Second, through overlapping of two standing-wave laser beams as shown in Fig. 1(c) with the corresponding Rabi frequencies $\Omega_1 = \Omega_0\cos(kx\sin\alpha)e^{iky\cos\alpha}$ and $\Omega_2 = \Omega_0\sin(kx\sin\alpha)e^{-iky\cos\alpha}$ (α is the angle of the propagating laser beams to the y axis), we generate a spatially periodic gauge field, given by

$$\mathbf{A}_\sigma^{\rm II} = 2\eta_\sigma\hbar k'\sin^2(k'x)\mathbf{e}_y, \qquad \mathbf{B}_\sigma^{\rm II} = 2\eta_\sigma\hbar k'^2\sin(2k'x)\mathbf{e}_z \quad (7)$$

with $k' = k\sin\alpha$. Under a slowly varying gauge field $\mathbf{B}_\sigma^{\rm I}$, one can have local Landau levels, and with a spatially periodic gauge field $\mathbf{B}_\sigma^{\rm II}$, we expect to have Bloch-type wave functions, similar to the case of particles in a periodic potential.

The spin Hall effect can be demonstrated by observing a spin Hall current. In the following, first we propose an experiment to detect the spin Hall current in an atomic gas with the above configuration of the light-atom coupling, and then we discuss the quantum spin Hall effect in an optical lattice. For observation of the spin Hall effect, we need an effective electric field $\mathbf{E}$ which drives atoms in one direction, and a spin current should be observed in a direction perpendicular both to the electric field $\mathbf{E}$ and the gauge field $\mathbf{B}_\sigma$. The electric field can be conveniently provided through gravity on the neutral atoms. We assume the internal state of the atoms is in superposition of the spin $\uparrow$ and $\downarrow$ components under the laser beams shown in Figs. 1(b) and 1(c). The external atomic trap is turned off at time $t = 0$, and the atoms fall off due to gravity with an acceleration $g = 9.8\ \mathrm{m/s^2}$ (along the direction $\mathbf{e}_y$). Under the effective gauge field, the equations of motion are

$$\dot{x}_\sigma = p_\sigma^x/m, \qquad \dot{p}_\sigma^x = [(\partial_x A_\sigma)p_\sigma^y - A_\sigma \partial_x A_\sigma]/m - \partial_x V_\sigma, \tag{8}$$

$$\dot{y}_\sigma = [p_\sigma^y - A_\sigma]/m, \qquad \dot{p}_\sigma^y = mg, \tag{9}$$

where the gauge potential A_σ is either A_σ^{I} or A_σ^{II}, and V_σ is the corresponding scalar potential induced by the same laser beams. The coordinates and the momenta x_σ, y_σ, p_σ^x, p_σ^y are understood as variables (operators) in the classical (quantum) cases, respectively.

To have some intuitive idea, in Figs. 2(a) and 2(b) we show the typical classical trajectories of the atoms under the gauge fields B_σ^{I} or B_σ^{II}. One can clearly see that the trajectory of the atom depends on its spin state σ, and such a dependence leads to the spin Hall current in the horizontal direction for the case of many particles. For the gauge field B_σ^{I}, the trajectory is a parabola, while for B_σ^{II} it is an oscillation around the nearest stable point. The spin-dependent stable points for B_σ^{II} are determined by the zeros of the corresponding Lorentz force, which are given by $x_n = (n + 1/2)\pi/k\sin\alpha$ ($x_n = n\pi/k\sin\alpha$) with an integer n for the spin-$\uparrow$ ($\downarrow$) component, respectively.

The trajectory of a single atom is hard to detect, and it is much easier in experiments to measure the density evolution of an ensemble of noninteracting atoms. We assume at $t = 0$ (the moment when the trap is turned off), the number density and the velocity distribution of the atomic gas are both described by Gaussian functions with $\rho_r(x, y) = (2\pi\sigma_r^2)^{-1}e^{-(x^2+y^2)/2\sigma_r^2}$ and $\rho_v(v_x, v_y) = (2\pi\sigma_v)^{-1}e^{-(v_x^2+v_y^2)/2\sigma_v^2}$, respectively, where σ_r and σ_v characterize the corresponding spatial and velocity variances. These variances include contributions from both quantum uncertainties of the atomic motion and classical broadening due to the finite temperature effect. The evolution of the density profile of the atomic gas is simulated numerically by solving Eqs. (8) and (9), and the results are shown in Fig. 2(c) and 2(d) for the gauge fields B_σ^{I} and B_σ^{II} respectively. Under B_σ^{I}, the ensemble splits into the spin-up and spin-down clusters, which is a manifestation of the spin Hall current along the x direction. Under B_σ^{II}, the atoms form periodic patterns with micro-separation of the different spin components.

To experimentally detect the spin current (or spin separation) as shown in Fig. 2, right before the imaging one can transfer the dressed bright state $|\chi_\downarrow\rangle$ to a different hyperfine level $|F'\rangle$ by turning on a laser pulse (with a Rabi frequency Ω_F) that couples the excited state $|3\rangle$ to $|F'\rangle$ [see Fig. 1(d)]. This pulse, together with the original laser beams Ω_1 and

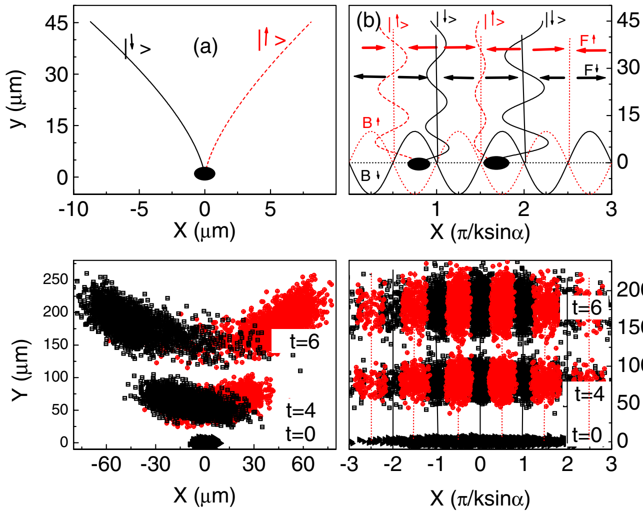

FIG. 2 (color online). Spin-dependent trajectories of a single atom (a),(b) and spin-dependent evolution of the density profiles of an ensemble of atomic gas (c),(d) under gravity (which provides an effective electric filed) and a light-induced gauge potential. The spin current along the x direction is a manifestation of the spin Hall effect. The gauge potentials in (a),(c) and (b),(d) are generated by the laser configurations I and II, respectively. The sinusoids in (b) denote the effective gauge fields B_σ. The directions of the Lorentz forces F_σ change periodically in this case and are shown by the arrows there. The dotted (solid) vertical lines in (b) and (d) denote the stable equilibrium positions for spin-up (spin-down) atoms. In (c) and (d), the density profiles of the atomic gas are shown at time $t = 0, 4, 6$ ms. For calculations in (a)–(d), we take the following typical experimental parameters with $\sigma_0 = 10\ \mu\mathrm{m}$, $\Delta x = 2.5\ \mu\mathrm{m}$, $k = 10^7\ \mathrm{m}^{-1}$ for the laser configuration I, and $k\sin\alpha = 5\times 10^5\ \mathrm{m}^{-1}$ for the laser configuration II. In both configurations, $\Omega_0^2/\Delta = 10^6$ Hz. In (a),(b), the initial atomic velocity is assumed to be zero, and the initial positions $x = 0$, $y = 0$ for (a) and ($x = 2.5$, $5\ \mu\mathrm{m}$, $y = 0$) for (b). The parameters for the atomic ensemble in (c),(d) are given by $\sigma_r = 2.0\ \mu\mathrm{m}$ and $\sigma_v = 0.5$ cm/s. The atomic mass is taken to be the one for ^{87}Rb. With the above parameters, we have checked the adiabatic condition is well satisfied during the evolution.

Ω_2, make a Raman transition with an effective Hamiltonian $H_R = (\Omega_F^*\Omega/\Delta)|\chi_\downarrow\rangle\langle F'| + \mathrm{H.c.}$ (note that the dark state $|\chi_\uparrow\rangle$ is still decoupled because of the phase relation between Ω_1 and Ω_2). Although the form of the bright state $|\chi_\downarrow\rangle$ is spatially varying, the Rabi frequency Ω (and thus also $\Omega_F^*\Omega/\Delta$) is spatially constant (for the laser configuration II) or almost constant (in the overlap region for the laser configuration I). We can thus choose the pulse duration so that it makes a complete Raman transition (π pulse), and the atomic motion can be neglected during such a short duration. After this Raman π pulse, the initial different dressed spin states are mapped to different hyperfine levels, and the populations in different atomic hyperfine levels can be separately imaged with the known experimental techniques.

We now consider quantum spin Hall effect with fermionic atoms in an optical lattice. In this case, the scalar potential $V_\sigma(\mathbf{r})$ is spatially periodic. We assume the optical lattice has a higher intensity along the vertical direction so that the tunneling rate along the z axis is negligible. One then has an effective 2D system in the x-y plane. The gauge field $\mathbf{B}_\sigma(\mathbf{r})$ (along the z axis) is assumed to be nearly constant or spatially periodic in the lattice (which corresponds to the above laser configurations I and II, respectively). The wave function in this case can still be written as $\Psi_\sigma(\mathbf{r}) = \sum_{n\mathbf{k}} u_n^\sigma(\mathbf{k},\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}}$, where $\mathbf{k}$ is the Bloch wave vector and the nth band wave function $u_{n\sigma}(\mathbf{k},\mathbf{r})$ satisfies the Schrödinger equation with the effective Hamiltonian [20]

$$H_\mathbf{k}^\sigma = \frac{\hbar^2}{2m}\{(-i\partial_x + k_x)^2 + [-i\partial_y + k_y - A_\sigma(x)]^2\} + V_\sigma(x,y) \tag{10}$$

Under an effective electric field $\mathbf{E}$ along the y direction ($E_y = mg$ through acceleration g), the Hall current along the x direction is given by $J_x^\sigma \equiv \langle \rho_n^\sigma v_x^\sigma \rangle = \sigma_{xy}^\sigma E_y$ for the spin-σ component with the linear response theory. The Hall conductivity then has the expression $\sigma_{xy}^\sigma = (1/2\pi\hbar)\sum_{n,\mathbf{k}} \rho_\sigma(\epsilon_n^\sigma(\mathbf{k}))(\partial_{k_x} a_{ny}^\sigma - \partial_{k_y} a_{nx}^\sigma)$, where $a_{n\mu}^\sigma(\mathbf{k}) \equiv i\hbar\langle u_n^\sigma(\mathbf{k})|\partial_{k_\mu} u_n^\sigma(\mathbf{k})\rangle$ ($\mu = x, y$) and $\rho^\sigma(\epsilon_n^\sigma(\mathbf{k}))$ denote the density of states of the nth band with the band energy $\epsilon_n^\sigma(\mathbf{k})$. The mass and the spin currents are defined by $J_x^m = J_x^\uparrow + J_x^\downarrow$ and $J_x^s = J_x^\uparrow - J_x^\downarrow$, respectively. With $A_\uparrow = -A_\downarrow$ and V_σ nearly independent of the spin σ, we have $J_x^m = 0$ and $J_x^s = 2J_x^\uparrow$. So, there is a net spin current, as a characteristic feature of the spin Hall effect. The spin Hall current is quantized if the chemical potential of the system (controlled by the atom number density) is inside a band gap. In this case, $\sigma_{xy}^\sigma = (1/2\pi\hbar)C_{xy}^\sigma$, where $C_{xy}^\sigma = i/2\pi \int dk_x dk_y (\langle \partial_{k_x} u_n^\sigma | \partial_{k_y} u_n^\sigma\rangle - \langle \partial_{k_y} u_n^\sigma | \partial_{k_x} u_n^\sigma \rangle)$ is the Chern number which takes only integer values [20]. The spin current is then an integer multiple of $1/(\pi\hbar)$.

Finally, we briefly discuss detection of the quantum spin Hall effect. With a nearly constant gauge field (such as the one provided by the laser configuration I in Fig. 1), the cyclotron length scale is estimated by $l \sim \sqrt{\hbar/B_I} \sim \sigma_0/\sqrt{k\Delta x}$. The region of the gauge field is characterized by the laser profile with an area about $S \approx 4d \times k\sigma_0^2$. The degeneracy of each Landau level is then estimated by $S/(\pi l^2) \sim 3.2 \times 10^3$. With about 10^6 atoms in a three-dimensional optical lattice which separates the atomic gas into about 10^2 independent layers, the atom number of each layer is of the order of 10^4, so only a few lowest Landau levels will be occupied. As the filling number is of the order of unity, the quantum effect should become important at low temperature. To detect the quantum spin Hall effect, one can apply an effective electric field by tilting the lattice along one direction (through gravity or through lattice acceleration induced with a nonlinear frequency chirp on the laser fields that form the optical lattice), and then detect the spin current accumulation along the vertical direction through separate imaging of the two different spin components as described before.

In summary, we have proposed an experimental scheme to realize effective spin-dependent gauge potentials on cold atoms with laser beams and to observe the spin Hall effect with an ensemble of atomic gas.

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Note added in proof.—During publication of this work, we became aware that the atomic spin Hall effect was also discussed by Liu *et al.* in Ref. [21] under a different atomic configuration.

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