Alpha Channeling in Mirror Machines

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The injection of radio frequency waves can cool charged particles trapped in a magnetic mirror. This cooling effect relies upon waves with azimuthal and axial phase velocities resonating with ions in different axial locations. The ions are then forced to diffuse along highly constrained orbits, such that they can only exit the magnetic trap at low energy. This cooling effect may have application to magnetic fusion mirror machines, where the free energy of the fusion by-products, the α particles, might be channeled into the waves that effect the cooling, thereby both extracting the fusion ash quickly and making that energy available in a convenient form for more useful purposes.

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Because of their engineering simplicity, high-β, and steady-state operation, mirror machines and related open-trap machines are attractive concepts for controlling nuclear fusion. The confinement occurs by magnetic mirroring, without the magnetic field lines closing upon themselves within the region of confinement. Unfortunately, the prospects for fusion based on these concepts are dimmed by their low Q factor, the ratio of fusion power produced to auxiliary power required [1,2]. Nonetheless, numerous improvements have been proposed to enhance the reactor prospects of mirror fusion, including those through uses of rf heating. Coupling rf power into the mirror tends to pump-out plasma [3,4], but it might also be used to inject plasma [5,6]. I suggest here a rf ejection effect, but concomitant with a cooling effect, similar to the alpha channeling effect in tokamaks [7], where rf waves diffuse energetic α particles [the charged by-products of deuterium-tritium (DT) fusion] along paths highly constrained in joint energy-position space, such that α particles born inside the mirror exit necessarily cold, with their energy transferred to the waves.

In a tokamak, by channeling the wave energy to fuel ions, the effective fusion reactivity might be more than doubled with the fusion ash removed quickly [8]. The α particle energy might also be diverted to other useful purposes, like providing the current necessary for steady-state operation [9]. These effects would lower significantly the cost of electricity by tokamak fusion [10]. Similarly, this cooling and channeling effect might be used to advantage in open field line machines. Although the open geometry of the mirror makes fueling and ash removal much easier, the heating of ions and the quick removal of ash may be more critical in improving the Q of the mirror reactor. In a tokamak, α particles take up valuable plasma pressure; in a mirror, the fusion ash takes up valuable electric potential, so the quick removal of the energetic ash makes room for fuel ions. In mirror geometry, using the channeled energy to maintain ion and electron temperature disparities may also be important.

The α-channeling effect in tokamaks exploits the higher population of high-energy α particles in the tokamak interior compared to that of low-energy α particles at the periphery. Because of the population inversion, upon injecting waves that diffuse resonant particles along diffusion paths connecting these regions, hot α particles diffuse to the periphery and cool at the same time [7]. However, the population inversion for fuel deuterium ions is opposite to that of the α particles. There are no MeV fuel ions in the center, but there are many relatively cold fuel ions near the periphery. Thus, in a tokamak, the same wave that taps α particle energy, while rejecting the α particles to the periphery, also can fuel the plasma by sucking in fresh fuel ions and heating them. Similar possibilities might be expected in mirror machines.

Since both mirrors and tokamaks have a symmetry direction, the diffusion paths can be written similarly. For interaction by means of the resonance $\omega - k_\parallel v_\parallel = n\Omega$, where ω is the wave frequency, Ω is the cyclotron frequency, $k_\parallel$ is the parallel wave number, n is the harmonic of the resonance, and $v_\parallel$ is the wave parallel velocity, the diffusion paths obey

$$dP_\phi/dW = n_\phi/\omega, \tag{1}$$

$$d\mu/dW = qn/m\omega, \tag{2}$$

where $\mu = mv_\perp^2/2B$ is the ion magnetic moment, q is the ion charge, and $W = \mu B + mv_\parallel^2$ is the kinetic energy. The canonical angular momentum, $P_\phi = r(mv_\phi + qA_\phi)$ and the radius r are defined with respect to the symmetry direction, namely, toroidal in the tokamak and azimuthal in the mirror. While the α-channeling effects in both tokamaks and mirrors employ diffusion paths defined by Eqs. (1) and (2), the α-channeling effects in a mirror exploit the very different periphery of the mirror machine. Particles exit a tokamak by diffusing across magnetic field lines past the last closed magnetic surface. In mirror machines, the open geometry defines a very different periphery in the phase space that includes both configuration space and velocity space. Particles exit not only radially across field lines, but also axially along open field lines.

I now show how an efficient α-channeling effect can be produced in mirror geometry using multiple regions of rf power. Although it is not by itself a viable fusion design, for simplicity, consider the "simple mirror", with vanishing plasma potential. These ideas will pertain more generally to mirror-type, open-ended configurations as well. Suppose further that mirror-trapped ions or α particles are wave heated at just one axial position $z = z_{\rm rf}$, where z is the axial direction along the mirror. The trapped particles can be described by their perpendicular and parallel energies, $W_{\perp 0} \equiv W_\perp(z = 0)$ and $W_{\parallel 0} \equiv W_\parallel(z = 0)$, as they cross the mirror midplane at $z = 0$. The particles affected by the rf waves lie between the rays shown in Fig. 1. The lower ray represents the trapped-untrapped boundary for mirror ratio $R_M \equiv B_{\max}/B_0$, where B_0 is the magnetic field minimum at the midplane and $B_{\max}$ is magnetic field maximum. The upper ray represents the boundary for particles reaching the region of rf power at $z = z_{\rm rf}$, where the magnetic field is $B_{\rm rf}$, with $B_0 < B_{\rm rf} < B_{\max}$. The upper ray is determined by the mirror ratio $R_{\rm rf} \equiv B_{\rm rf}/B_0$. Particles with higher perpendicular energy than the upper ray are mirror reflected before reaching the region of rf, whereas particles with a lower perpendicular energy than the lower ray are not mirror confined at all. Were a plasma potential Φ between the midplane and z_{rf} included, the slope for mirroring at z_{rf} would remain $1/(R_{\rm rf} - 1)$, but the $W_{\perp 0}$ intercept would now be $q\Phi/(R_{\rm rf} - 1)$. Such an energy offset of a few tens of keV will not affect significantly α particles, born at 3.5 MeV.

Upon interaction with the rf field at $z = z_{\rm rf}$, the perpendicular energy $W_\perp(z_{\rm rf})$ of the particle at $z = z_{rf}$ changes so that $W_\perp(z_{\rm rf}) \rightarrow W_\perp(z_{\rm rf}) + \Delta W_\perp$. The parallel energy changes by $W_\parallel(z_{\rm rf}) \rightarrow W_\parallel(z_{\rm rf}) + \Delta W_\parallel$. Since the wave-particle interaction is a stochastic process, $\Delta W_\perp$ and $\Delta W_\parallel$ can be positive or negative, resulting in energy diffusion. The magnitude of the diffusion coefficient [11], so long as it is high enough, is less essential here than the diffusion path. For interaction by means of the resonance $\omega - k_\parallel v_\parallel = n\Omega$, the diffusion path can be written from Eq. (2) as $\Delta W_\perp = \Delta W_\parallel n\Omega/(\omega - n\Omega) = \Delta W_\parallel n\Omega/(k_\parallel v_\parallel)$. For example, for cyclotron interactions, we have $n = 1$; in the limit then of $k_\parallel v_\parallel \rightarrow 0$, we have $\Delta W_\parallel = 0$, so that only perpendicular diffusion occurs.

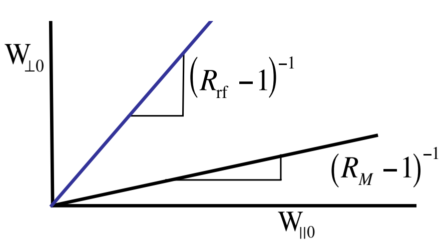

FIG. 1 (color online). Region of rf diffusion in midplane energy coordinates.

Assuming the adiabatic invariance of the magnetic moment outside the resonance, the energy kicks $\Delta W_\perp$ and $\Delta W_\parallel$ at $z = z_{rf}$ result in the midplane change

$$W_{\perp 0} \rightarrow W_{\perp 0} + \Delta W_\perp/R_{\rm rf}, \tag{3}$$

$$W_{\parallel 0} \rightarrow W_{\parallel 0} + \Delta W_\parallel + \Delta W_\perp(1 - R_{\rm rf}^{-1}), \tag{4}$$

$$r_0 \rightarrow r_0 + R_{\rm rf}^{1/2}\Delta W_\perp(qB\omega/k_\theta)^{-1}. \tag{5}$$

For azimuthal mode number $n_\phi \neq 0$, the linked diffusion in perpendicular energy and radius at the axial rf location z_{rf} gives the radial kick $\Delta r_{\rm rf} = \Delta W_\perp/(qB\omega/k_\phi)$ [7]. Equation (5) then follows, since, from flux conservation, the radial kick at the midplane obeys $\Delta r_0 = R_{\rm rf}^{1/2}\Delta r_{\rm rf}$.

Now note the two necessary conditions for the channeling effect: First, the diffusion paths must connect high-energy particles in the interior with low-energy particles in the periphery of a confinement device, where the periphery is defined as a point of marginal confinement and the interior is defined as a point where particles are well-confined. In a mirror, one periphery is the trapped-passing boundary, or the lower ray in Fig. 1. Second, the diffusion to high energy must be limited. Although the more general case may offer other possibilites, I will show that both of these conditions can be satisfied in a mirror geometry through the case of perpendicular diffusion only, or $\Delta W_\parallel = 0$, which is realizable through $k_\parallel v_{\parallel \rm res} \rightarrow 0$, where $v_{\parallel \rm res}$ is the resonant parallel velocity.

To construct the channeling effect, first note that from Eqs. (3) and (4), for $\Delta W_\parallel = 0$, the slope of the energy change in midplane coordinates, $\Delta W_{\perp 0}/\Delta W_{\parallel 0} = (R_{rf} - 1 + Rk_\parallel v_\parallel/\Omega)^{-1}$, which, for $k_\parallel v_\parallel/\Omega$ small, approaches the same slope as the rf interaction boundary. This means that particles diffuse in energy parallel to that boundary. However, only particles resonant with the wave are affected. The resonance condition, $\omega - k_\parallel v_\parallel = n\Omega$, selects a parallel energy $W_{\parallel \rm res}$ at $z = z_{rf}$, which is a function of the local wave and magnetic field parameters. The resonant region in midplane coordinates then obeys for $\Delta W_\parallel \rightarrow 0$

$$W_{\perp 0} = (W_{\parallel 0} - W_{\parallel \rm res})/(R_{\rm rf} - 1). \tag{6}$$

Thus, particles kicked by the rf wave at $z = z_{rf}$ diffuse along the trajectory indicated by Eq. (6). Since the slope of the diffusion path is to order $k_\parallel v_\parallel/\Omega$ the same as the slope of the resonance condition, particles effectively remain in resonance as they are diffused.

Along the diffusion path, α particles that gain energy remain mirror trapped, but particles that lose energy eventually encounter the trapped-passing boundary and are lost. The energies upon exit are

$$W_{\perp \rm exit} = W_{\parallel \rm res}/(R_M - R_{\rm rf}) \tag{7}$$

$$W_{\parallel \rm exit} = W_{\parallel \rm res}(R_M - 1)/(R_M - R_{\rm rf}). \tag{8}$$

Thus, by picking $W_{\|\mathrm{res}}$ small, the energy lost at the boundary can be made small.

The α channeling effect is now guaranteed by meeting the second necessary condition, namely, by limiting the diffusion to high energy. Velocity-space only mechanisms that might naturally limit the diffusion include superadiabatic [12,13] or other adiabatic effects [14]. However, these mechanisms are less robust than connecting diffusion in space to diffusion in energy through Eqs. (5). The azimuthal phase velocity, ω/k_ϕ, can be chosen of one sign so that the α particles are forced to the mirror axis when gaining energy and to larger radius when losing energy. Particles that temporarily gain energy are forced radially closer to the magnetic axis; ultimately the energy gain is limited by the initial distance from the magnetic axis. Although any energy gained is ultimately recovered when the particle exits, a large energy gain would allow nonideal effects, like collisions, to leak away energy. The optimal radial excursion $\Delta r_{\max}$ is thus about a plasma radius, so that particles not also exit radially before losing their birth energy. The energy gain is then limited to a fraction of the birth energy, since most α particles are born closer to the magnetic axis than to the periphery.

For $W_{\|\mathrm{res}}$ small, all the α particle energy is perpendicular, so, from Eq. (5), the maximum radial excursion at $z = 0$ for a 3.5 MeV α particle losing its energy is $\Delta r_{\max} = R_{\mathrm{rf}}^{1/2}(k_\phi \rho_{\alpha 0})\rho_{\alpha 0}(\Omega_\alpha/2\omega)$, where $\rho_{\alpha 0}$ is the initial alpha particle gyroradius at z_{rf}. In a 5 T field, that would be 5.3 cm. For $k_\phi = 4\ \mathrm{cm}^{-1}$, we then get an excursion distance of 54 $R_{\mathrm{rf}}^{1/2}$ cm near the α particle resonance (more at the tritium resonance), which is optimal for meter-sized plasma radii. Low azimuthal wave number waves have been coupled into mirrors (e.g., [15,16]), but to achieve this effect with one wave would require higher azimuthal wave numbers; in a tokamak DT reactor the higher wave numbers might be achieved by a mode-converted ion Bernstein wave [17,18].

Since particles diffusing to high energies are not lost, whereas the lost particles are at small energy, eventually all particles will be lost cold, thereby achieving the channeling effect. However, the particles affected are only those in resonance, namely, only those diffusing along the trajectory indicated by Eq. (6). At first glance, it appears that to affect a range of resonant parallel energies a large range of $W_{\|\mathrm{res}}$ will be required, which would mean that the amount of energy lost at the trapped-passing boundary becomes large. However, interestingly, it is possible to access the full range of trapped particles by arranging for several regions of rf as shown in Fig. 2, but still employing a limited range of small $W_{\|\mathrm{res}}$.

To see this, consider multiple regions of rf power, with the rf power in each region having its own prescribed wave frequency and wave number. For ions resonant in region i, say at $z = z_{\mathrm{rf}i}$, diffusion takes place along the trajectory indicated by Eq. (6) for wave parameters and mirror ratios corresponding to the ith region, so that

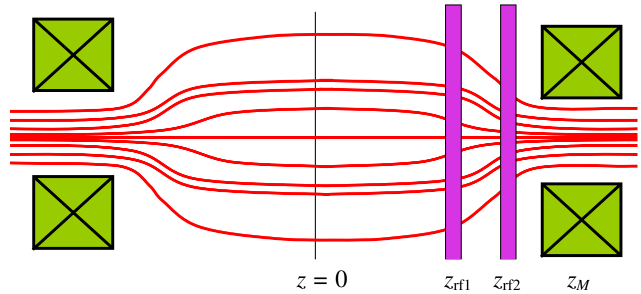

FIG. 2 (color online). Mirror field with rf regions at axial positions $z = z_{\mathrm{rf}1}$ and $z = z_{\mathrm{rf}2}$. The magnetic field maximum is at $z = z_M$.

$$W_{\perp 0} = (W_{\|0} - W_{\|\mathrm{res}_i})/(R_{\mathrm{rf}i} - 1). \tag{9}$$

Particles resonant in one region need not be resonant in a second region. The criteria for resonance in more than one region is that the diffusion paths cross within the trapping region. Figure 3 shows two diffusion paths (dashed lines) due to two regions of rf, one at $z = z_{\mathrm{rf}1}$ corresponding to the mirror ratio $R_{\mathrm{rf}1}$, and one at $z = z_{\mathrm{rf}2}$ corresponding to the mirror ratio $R_{\mathrm{rf}2}$. For the case here, I choose the same rf resonant velocity for both regions. Since each diffusion path is parallel to its own corresponding trapped-passing boundary, these paths meet only at $W_{\perp 0} = 0$, which is outside the trapped-passing boundary. More generally, for multiple rf regions, note from Fig. 3 that diffusion paths do not cross within the trapped particle region so long as the resonant regions at larger z_{rf} are arranged with parallel resonant energies $W_{\|\mathrm{res}}$ not too much smaller than the resonant energies at smaller z_{rf}. If the diffusion paths do not cross, then there is a strong constraint on the energy at exit; resonant particles exiting at the trapped-passing

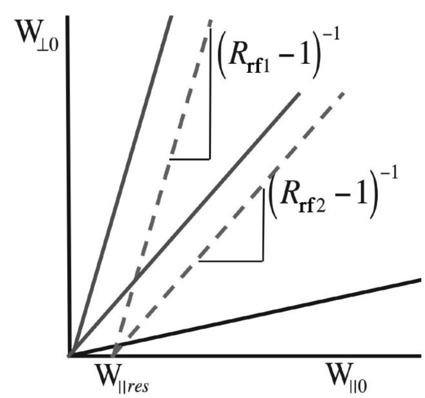

FIG. 3. Diffusion paths in midplane energy coordinates for particles resonant in the rf regions shown in Fig. 2. The solid rays are the particle coordinates for particles mirroring at $z = z_{\mathrm{rf}1}$ (upper ray) or $z = z_{\mathrm{rf}2}$ (lower ray). The dashed rays are the diffusion paths for particles, resonant at instantaneous parallel energy $W_{\|\mathrm{res}}$, at $z = z_{rf1}$ (upper ray) or $z = z_{rf2}$ (lower ray). The dashed rays are parallel to the associated solid rays and offset by the resonant parallel energy.

boundary must exit with the exit energies given by Eqs. (7) and (8).

Although particles resonant at $z = z_{rf1}$ diffuse into the rf region $z = z_{rf2}$ before becoming untrapped, when they are in the rf region at $z = z_{rf2}$, their parallel velocity is not resonant with the rf waves at $z = z_{rf2}$. Thus, each set of resonant particles maintains the diffusion path set by one rf region. The resonance conditions can be arranged at each axial location to correspond to the same relatively low parallel energy $W_{\|res}$. Fitting the wave into the rf region with manageable antennas will require sufficiently small parallel wavelengths, $\lambda_\| = 2\pi v_\| /(\omega - \Omega_D)$, like on the order of a meter. For $\omega - \Omega_D \rightarrow 0$, $\lambda_\|$ as small as necessary can clearly be accommodated by small $W_{\|res}$, yet it remains that $\Delta W_\| / \Delta W_\perp = k_\| v_\| / \Omega = (\omega - \Omega)/\Omega \rightarrow 0$. For $W_{\|res}$ small, essentially all the energy is extractable as can be seen from Fig. 3, or from Eqs. (7) and (8).

If the diffusion paths do cross within the trapped region of velocity space, then the strong condition on the exit energy may not obtain. However, the probability distribution of particles interacting with multiple waves along multiple paths might still highly favor exit at low energy, similar to the channeling effect in tokamaks [19,20].

In addition to capturing their energy, the prompt ejection of the α particles is itself useful. At ignition, the α-particle heating $P_\alpha = n_\alpha \epsilon_\alpha / \tau_\alpha$ must exceed the ion heat loss $P_i = nT_i/\tau_i$, where ϵ_α is the α birth energy, τ_α is the slowing down times of energetic α particles, and τ_i is the ion energy containment time. Rewriting this as $n_\alpha / n_i > (T_i/\epsilon_\alpha)(\tau_\alpha/\tau_i)$, and assuming $\tau_\alpha > \tau_i$, it can be seen that for ions of several hundred keV the α-particle density is very significant [2], although for lower ion temperatures it will be smaller [21]. For α-particle density 30% of the ion density [2], the fuel ions are much diluted. Since the reactivity scales as the ion density squared, at constant confined positive charge, for this case the prompt loss of α particles alone increases the fusion reactivity by a factor of 2.8. Moreover, if the α-particle energy is captured in waves that are absorbed by ions, then not only is the α-particle energy optimally converted, but higher disparities in ion and electron temperatures, or in temperatures of different ions, may be tolerated. These effects are also particularly useful for mirror concepts in which the ions are much hotter than the electrons [22], or at least one species of ions is [23].

Another advantageous effect would be if the cooling of the α particles were accompanied by the heating of the trapped fuel ions along the reverse diffusion path, like in a tokamak. A high phase-space density cold fuel source near the periphery might create a population inversion opposite to that of the α particles, since there are no MeV deuterium ions in the mirror center. For the simple mirror discussed here, the same rf field might then heat the fuel ions, drawing their turning points closer to the midplane and their gyrocenters closer to the mirror axis. This fueling effect might also profit from concepts for which there are many cold fuel ions near the trapped-passing boundary [24].

In summary, α particles born in mirror machines might in principle be made to slow down collisionlessly on azimuthally and axially propagating rf fields rather than on electrons. The quick expulsion of the cold fusion byproducts would reduce the mirror electric potential, leading to better confinement of the fuel ions, while the energy diverted to the waves might be used for better purposes, including the possible heating and fueling of ions. Although only the simplest mirror geometry is considered here, which is not a practical fusion device, the basic channeling mechanisms predicted here should apply to a variety of open field configurations. It remains to identify how the reactor prospects of more promising open field configurations, such as the tandem mirror rather than the simple mirror, may be enhanced through the wave channeling effects predicted here. It also remains to show in detail how waves with the characteristics identified here may be coupled efficiently into these devices.

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