Phase Transition and Critical Dynamics in Site-Diluted Josephson-Junction Arrays

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Measurements of the IV characteristics of site-diluted Josephson-junction arrays have revealed intriguing effects of percolative disorder on the phase transition and the vortex dynamics in a two-dimensional XY system. Different from other types of phase transitions, the Kosterlitz-Thouless transition was eliminated with the introduction of percolative disorder far below the percolation threshold. Even after the Kosterlitz-Thouless order had been removed, the system remained superconducting at low temperatures by establishing a different type of order. Near the percolation threshold, evidence was found that, as a consequence of the underlying fractal structure, the critical dynamics of the phase degrees of freedom persisted down to zero temperature.

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Percolation has been used as an idealized simple model for understanding a variety of irregular systems. The effects of such a geometry on phase transitions have been of continuous interest in condensed matter physics. The Kosterlitz-Thouless (KT) phase transition [1] is an essential factor for understanding various systems in two dimensions, such as superconductors, easy-plane ferromagnets, crystals, and liquid crystals, which can be represented by the XY model. In addition, the KT transition is singular with regard to excitations of topological defects, which drive the transition from a quasi-long-range ordered phase to a disordered phase. Nevertheless, the relationship between percolative disorder and the two-dimensional (2D) phase transition has not been sufficiently explored and remains unsettled [2,3]. Also of interest are the effects of the underlying fractal geometry on the dynamics of the phase degrees of freedom of XY systems near the percolation threshold. In this Letter, we report an experimental study of the effects of percolative disorder on both the 2D phase transition and the critical dynamics in proximity-coupled Josephson-junction arrays (JJA's), which provide a near-ideal realization of the XY model. Measurements of the current-voltage (IV) characteristics demonstrate that the nature of the phase transition in the 2D XY system is completely altered with progressive addition of percolative disorder. Near the percolation threshold, experimental evidence suggests that the underlying fractal structure results in dynamical criticality being maintained down to zero temperature.

Experiments were performed on a series of site-diluted square arrays of Nb/Cu/Nb Josephson junctions. Nb crosses were periodically disposed on a Cu film with a lattice constant of 13.7 μm, a junction width of 4 μm, and a junction separation of 1.4 μm to form a 400×600 square lattice. Nb islands were randomly removed from the lattice to introduce percolative disorder. Arrays with the concentration of filled sites $p = 1$, 0.94, 0.86, 0.8, 0.75, 0.7, 0.65, and 0.6 were fabricated. For the samples, the same random-number seed was used to select the sites to be removed. The percolation-threshold concentration p_c for the series of samples, below which no percolating cluster of connected junctions was formed, was 0.5906 [4].

The IV characteristics were measured by employing the phase-sensitive voltage-signal-detection method with a lock-in voltmeter and a square-wave current at 23 Hz. The single-junction critical current i_c and the single-junction coupling energy J ($=\hbar i_c/2e$) were determined by using the de Gennes formula [5] for proximity-coupled junctions in the dirty limit to extrapolate the i_c vs T data at low temperatures and by using the numerically obtained I vs dV/dI data for site-diluted arrays in Ref. [6] to compensate the systematic error arising from the presence of diluted sites. The i_c vs T data at low temperatures were obtained from the I vs dV/dI measurements. The magnetic field or the frustration f, defined as the number of flux quanta per unit cell, was adjusted by using the R vs f curve of the sample exhibiting distinct resistance minima at integral f's. The temperature during the measurements was controlled to have fluctuations $\lesssim 1$ mK. Additional details of the measurements are given in Ref. [7].

Figure 1 presents some of the results of the IV characteristics measurements on the samples of $p = 1$, 0.86, 0.7, and 0.65, with the ambient magnetic fields expelled from the sample space by the μ-metal shield and solenoid. The measurements were carried out at 17–19 different temperatures (at intervals shorter than shown in the figure). The IV curves were obtained by averaging 15–240 measurements for each current. It is visible that the IV characteristics change completely with decreasing p. Unlike the $p = 1$ (or no site disorder) sample experiencing a KT transition [8], the samples with $p = 0.7$ and 0.65 exhibit low-temperature IV characteristics with an activated character. The variation of the voltage signal with temperature both above and below the transition is significantly slower for $p = 0.7$ and 0.65 than for $p = 1$ [9]. For $p = 0.86$, the IV characteristics are more like those of a KT system at $T > T_c$, but they exhibit an activated character at $T < T_c$. The IV characteristics for $p = 0.7$ and 0.65 are never similar to those of a KT system [10]. The low-temperature curves for $p = 0.7$ and 0.65 can be fitted to an exponential form

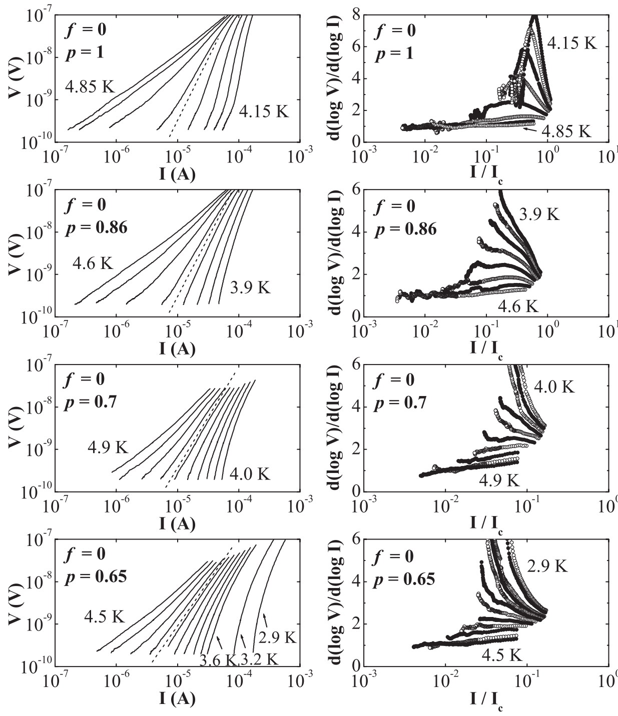

FIG. 1. Development of the IV characteristics with the filled-site concentration p for $f = 0$. The I vs V isotherms differ by a temperature interval of 0.1 K. The panels on the right show the slopes of the isotherms as functions of I/I_c. The dashed lines are drawn to show where the phase transition occurs. For $p = 1$, the dashed line represents the IV curve at the temperature where $V \sim I^3$. The dashed lines for $p = 0.7$ and 0.65 are drawn at the temperatures where the IV curves are estimated to become straight. For $p = 0.86$, $V \sim I^3$ at the transition temperature where the IV curve becomes straight. See the text for discussions.

$V \sim I\exp[-(I_T/I)^{\mu}]$, with $\mu = 0.9$–1.1. The low-temperature curves of an exponential form indicate that, at strong site disorder, the arrays have genuine superconductivity with long-range phase coherence. The $d(\log V)/d(\log I)$ vs I/I_c plots for $p = 0.7$ and 0.65 in Fig. 1, showing the proposed criterion for a non-KT-type superconducting transition [11] to be satisfied, confirm a finite-temperature non-KT-type superconducting transition for the strongly disordered samples. The evolution of the IV characteristics with p demonstrates that percolative disorder far below the percolation threshold alters the nature of the 2D phase transition in an unfrustrated XY system from a KT-type transition to a non-KT-type transition.

Figure 2 shows the IV characteristics of the same samples with frustration $f = 2/5$. The IV plots for $f = 2/5$ demonstrate that, at strong disorder or at lower concentrations of filled sites, the IV characteristics are affected little by the imposition of frustration, excluding the f-dependent phase-transition temperature. It is surprising that, at strong disorder, the IV characteristics of the unfrustrated system become similar to those of frustrated

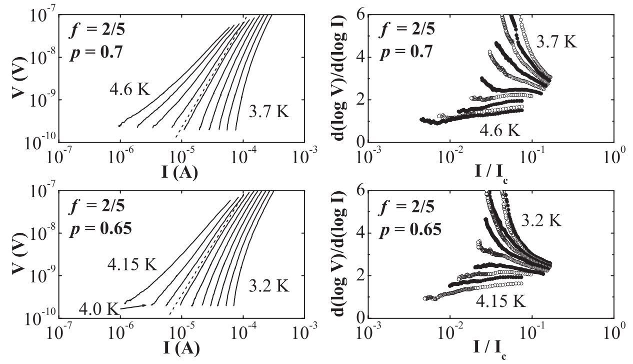

FIG. 2. I vs V and I/I_c vs $d(\log V)/d(\log I)$ plots for the samples with $p = 0.7$ and 0.65 at $f = 2/5$.

systems, which are known to experience a melting transition of a vortex solid driven by domain wall excitations [12]. The development of the unfrustrated system into a system similar to frustrated systems with the addition of percolative disorder is also revealed in the p dependence of the reduced phase-transition temperature $\tilde{T}_c$ [$=T_c/(J/k_B)$], shown in Fig. 3. The T_c's in the figure are the temperatures at which $I \sim V^3$ for systems exhibiting KT-like IV characteristics or at which the IV curve becomes straight in a $\log I$-$\log V$ plot for systems exhibiting non-KT-like IV characteristics [13]. The T_c's determined from the IV curves agree, within the error bars, with those independently determined from the resistance measurements. The $\tilde{T}_c$ vs p plot shows that the p dependences of $\tilde{T}_c$ for $f = 0$ and $2/5$, despite the total difference at weak disorder, become qualitatively similar at strong disorder. The rapid drop of T_c to zero in the vicinity of p_c is a common feature of systems with a gap in the excitation spectrum [14]. The close resemblance of the IV characteristics and the p dependence of $\tilde{T}_c$ at strong disorder for two systems, one with and one without frustration, appear to suggest that the non-KT-type low-temperature order in an

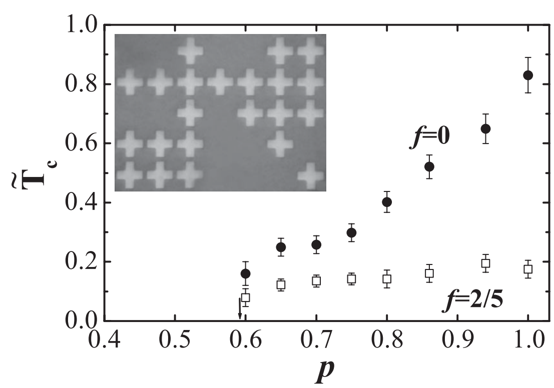

FIG. 3. p dependence of the reduced phase-transition temperature $\tilde{T}_c$ [$= T_c/(J/k_B)$] for $f = 0$ and $2/5$. The arrow points to the percolation threshold $p = 0.5906$. The inset shows a portion of a site-diluted array used in the experiment.

unfrustrated XY system at strong disorder is possibly that of a superconducting vortex solid with long-range phase coherence, as in frustrated systems.

Although a vortex solid in an unfrustrated array sounds peculiar, the possibility of its existence is notable because an unfrustrated array with the KT transition suppressed always contains sufficient unbound vortices to form a vortex solid. A recent numerical study [15] has suggested that, in the presence of a random potential above a critical disorder strength, a breaking of ergodicity due to a large energy barrier against vortex motion may allow unbound vortices to form a vortex solid at finite temperatures. In the equivalent interacting-vortex (or Coulomb-gas) representation of the XY model, random site dilutions produce a random pinning potential acting on vortices. Our speculation is that the random pinning potential eliminates the KT transition far below the percolation threshold and constrains the unbound vortices to form a superconducting solid at low temperatures [16].

Another important feature of the data presented in this Letter is that the IV characteristics of the unfrustrated system completely change again near p_c ($=0.5906$). As Fig. 4 reveals, the low-temperature IV curves shift from exponential-type curves to power-law-type curves when p changes from 0.65 to 0.6. For $p = 0.6$, an approximate power-law character is maintained even at such low temperatures as $\tilde{T} = 0.006$, far below $\tilde{T}_c$ ($=0.16$) [17]. The power-law character never weakens, even when frustration is turned on. In addition, the slope of the isotherm at T_c in the $\log I$- $\log V$ plot is only 1.9 for $p = 0.6$, less than it is for $p \gtrsim 0.65$. Since the slope minus one is the dynamic critical exponent [13], the lower slope for $p = 0.6$ implies that the critical relaxation in a percolating system is faster than it is in less-disordered systems. This is contrary to the general expectation that more diluted sites in a JJA usually result in slower relaxation of the system. Even if the power-law IV relation is characteristic of a KT transition, the f independence of the power-law behavior and the dynamic exponent being much less than 2 indicate that the possibility of a reemerging KT transition near p_c is low. The motion of the vortices created at diluted sites by an injected current does not explain the observed power-law behavior either. If the vortices created at the defects were responsible for the power-law behavior of the $p = 0.6$ system, as expected from numerical studies [18] of diluted arrays at zero temperature, a similar behavior should be visible in systems with comparable defect concentrations. The exponential low-temperature IV curves for $p = 0.65$, however, confirm that the contribution of vortices created at defects must be insignificantly small.

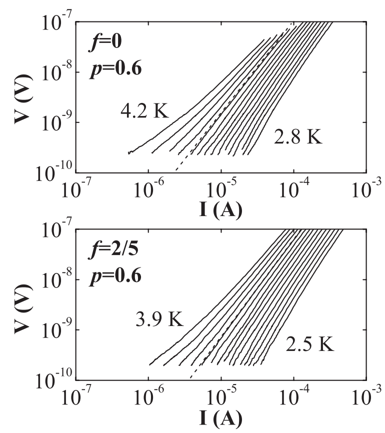

FIG. 4. IV characteristics of the sample with $p = 0.6$ for $f = 0$ and $2/5$. The I vs V isotherms differ by a temperature interval of 0.1 K.

Near the percolation threshold, a self-similar fractal structure with geometrical inhomogeneities and a divergent percolation correlation length characterize percolating systems. Therefore, even at zero temperature where the thermal correlation length, i.e., the spatial extent of the thermal fluctuations vanishes, the *spatial* configuration of the phase angles of the superconducting islands in the percolating array has a self-similar critical structure, as at critical temperature. For non-KT-type superconducting transitions, a power-law IV relation is a salient feature of *dynamical* criticality [13]. The power-law IV curves far below the transition for $p = 0.6$ may then indicate that temperature-independent criticality due to the fractal geometry applies to the dynamics of the phase angles, as well as to their spatial configuration. This means that the relaxation time near the percolation threshold becomes divergent at all temperatures below T_c. It has been suggested [19] from measurements of the frequency-dependent complex conductance of a site-diluted 2D JJA that the fractal structure results in a crossover, at a critical frequency, of the dynamics of the phase angles from a low-frequency Euclidean (homogeneous) region dominated by vortices to a high-frequency fractal (inhomogeneous) region in which the relevant excitations are localized fractons. The critical frequency was found from the ac measurements to be $\gtrsim$1 kHz for arrays comparable to the present sample. However, the results of the IV characteristics measurements on the present sample appear to indicate that the low-frequency vortex dynamics is not Euclidean either but is in another inhomogeneous region in which the fractal geometry leads to critical dynamics.

To conclude, the IV characteristics measurements on site-diluted JJA's have revealed intriguing effects of percolative disorder on the phase transition and the vortex dynamics in a 2D XY system. Unlike other types of phase transitions, the KT transition was eliminated by introduction of percolative disorder far below the percolation threshold. When the KT order was removed, the system unexpectedly remained superconducting by establishing a different type of order with long-range phase coherence until the percolation threshold was attained. Near the percolation threshold, as a consequence of the underlying self-similar geometrical structure, evidence was found for the critical dynamics of the phase degrees of freedom persisting down to zero temperature.

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[9] The horizontal distances between adjacent isotherms are much narrower for $p = 0.7$ and 0.65 than for $p = 1$.

[10] The results differ from those of the previous work by Harris *et al.* [2], in which the IV characteristics were interpreted to be of a power-law type even for strong site disorder. We note that their IV data contain much larger voltage noise than is manifested in the present data. The noise level of 0.2–0.3 nV in the data presented in Ref. [2] is too high to reveal the less-prominent activated character of the IV curves at temperatures near T_c. By reducing the noise level and extending the range of measurements to low temperatures far below T_c, we could reveal the activated character for $p = 0.7$ and 0.65 beyond the apparent power-law behavior found at the same concentration ($p = 0.7$) in Ref. [2].

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