QCD Collisional Energy Loss Reexamined

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It is shown that at a large temperature and $E \to \infty$ the QCD collisional energy loss reads $dE/dx \sim \alpha(m_D^2)T^2$. Compared to previous approaches, which led to $dE^B/dx \sim \alpha^2 T^2 \ln(ET/m_D^2)$ similar to the Bethe-Bloch formula in QED, we take into account the running of the strong coupling. As one significant consequence, due to asymptotic freedom, dE/dx becomes E independent for large parton energies. Some implications with regard to heavy ion collisions are pointed out.

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One of the key arguments for the creation of a "new state of matter" in heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) is the observed jet quenching [1], which *inter alia* probes the parton energy loss in the traversed matter. In a quark gluon plasma (QGP) there are two loss mechanisms: elastic collisions with deconfined partons [2,3], or induced gluon radiation [4–6]. Presuming a dominance of the second effect, experimental findings have often been interpreted in terms of a purely radiative loss. However, the data-adjusted parameters (either $\hat{q}$ or dN_g/dy, depending on the approach) are found to be considerably larger than theoretically expected or even in conflict with a strong constraint from dS/dy (see, e.g., [7])—which indicates a sizable collisional component of jet quenching. The effect of collisions (as estimated within the framework [2,3]) might actually be larger than conceded for a long time [8,9]. The fact that such estimates depend crucially on the *assumed* value of the coupling should motivate us to scrutinize the principal question: "what is the value of α?" Aside from its phenomenological relevance it will also lead to interesting theoretical insight.

Following Bjorken [2], let us consider the propagation of an energetic parton ("jet") through a static QGP at a temperature $T \gg \Lambda$, where the coupling is small. Its mean energy loss per length can be calculated from the rate of binary collisions with partons of the medium, as determined by the flux and the cross section,

$$\frac{dE_j}{dx} = \sum_s \int_{k^3} \rho_s(k)\Phi \int dt \frac{d\sigma_{js}}{dt}\omega. \tag{1}$$

Here $\rho_s = d_s n_s$ is the density of scatterers, with $d_g = 16$ and $d_q = 12n_f$ being the gluon and quark degeneracies for n_f light flavors, and $n_\pm(k) = [\exp(k/T) \pm 1]^{-1}$ in the ideal gas approximation. Furthermore, Φ denotes a dimensionless flux factor, t the 4-momentum transfer squared, and $\omega = E - E'$ the energy difference of the incoming and outgoing jet. Focusing on the dominating t-channel contributions, the cross sections can be approximated by

$$\frac{d\sigma_{js}}{dt} = 2\pi C_{js}\frac{\alpha^2}{t^2}, \tag{2}$$

with $C_{qq} = \frac{4}{9}$, $C_{qg} = 1$, and $C_{gg} = \frac{9}{4}$. For E and E' much larger than the typical momentum of the thermal scatterers, $k \sim T$, the relation of t to the angle θ between the jet and the scatterer simplifies to

$$t = -2(1 - \cos\theta)k\omega, \tag{3}$$

and the flux factor can be approximated by $\Phi = 1 - \cos\theta$.

At this point, Bjorken integrated in Eq. (1)

$$\Phi \int_{t_1}^{t_2} dt \frac{d\sigma_{js}}{dt}\omega = \frac{\pi C_{js}\alpha^2}{-k}\int_{t_1}^{t_2}\frac{dt}{t} = \frac{\pi C_{js}\alpha^2}{k}\ln\frac{t_1}{t_2}, \tag{4}$$

imposing both an IR and UV regularization. The soft cutoff is related to the Debye mass, $|t_2| = \mu^2 \sim m_D^2 \sim \alpha T^2$, describing the screening of the exchanged gluon in the medium. The upper bound of $|t|$ was reasoned to be given by the maximum energy transfer: very hard transfers, say $\omega \approx E$, effectively do not contribute to the energy loss; in this case the energy is collinearly relocated to the scatterer. Assuming $\omega_{\max} = E/2$ implies $t_1 = -(1 - \cos\theta)kE$, hence $dE_j^B/dx = \pi\alpha^2\sum_s C_{js}\int_{k^3} k^{-1}\rho_s \ln[(1 - \cos\theta)kE/\mu^2]$. Replacing now, somewhat pragmatically, the logarithm by $\ln(2\langle k\rangle E/\mu^2)$ and setting $\langle k\rangle \to 2T$, Bjorken obtained

$$\frac{dE_{q,g}^B}{dx} = \left(\frac{2}{3}\right)^{\pm 1} 2\pi\left(1 + \frac{1}{6}n_f\right)\alpha^2 T^2 \ln\frac{4TE}{\mu^2} \tag{5}$$

for quarks (+) and gluons (−), respectively. This expression can be regarded as an adaption of the QED Bethe-Bloch formula [10], which describes the ionization/excitation energy loss of charged particles in matter, with the logarithm reflecting the relativistic kinematics and the long-range Coulomb-type interaction.

There are various refinements of Bjorken's intuitive calculation, aiming at the precise determination of the cutoffs. Worth accentuating is the approach of Braaten and Thoma [3] who studied, within HTL perturbation theory, the propagation of a fermion through a QED plasma (and applied their method also to QCD). For light quarks, their result reproduces the generic form (5), with $\mu \to m_D$ in the logarithm and the factor 4 replaced by some function of n_f.

A remark concerning a pragmatic usage of such "Bjorken-type" formulas seems apposite here. Applied to experimentally relevant temperatures and rather low E, a *formally* resulting negative loss has been interpreted, at times, as an energy transfer of the thermal medium to the "soft jet." This interpretation, however, is untenable in the given framework: the jet always loses energy in a collision, $\omega > 0$; cf. (3). A negative result for dE^B/dx is *de facto* the consequence of interchanged boundaries in the integration (4). Since μ^2 is the minimal $|t|$ by definition, dE^B/dx should instead be set to zero for $4E < \mu^2/T$. This concern, though, will prove irrelevant by the following considerations.

In Bjorken's derivation, α is a *fixed* parameter. Conceptionally, such a tree-level approximation may be appropriate for QED. The strong interaction, however, is known to vary considerably over the range of scales which can be probed in heavy ion collisions. Thus, in QCD one should study quantum corrections to the tree-level amplitudes, whose renormalization will specify unambiguously the value of "the" coupling.

For the sake of transparency of the argument, consider first the analogous case of electron scattering in massless QED. There are three types of (divergent) loop corrections to the t-channel tree-level process (see, e.g., [11]). First, the exchanged photon is dressed by a self-energy. Then, encoded in the quantity Z_1, there are vertex corrections and finally, via the field strength renormalization Z_2, self-energy corrections to the external fermions. Yet, due to the identity $Z_1 \equiv Z_2$, in QED the renormalized coupling is determined only by the boson self-energy.

It is appropriate to renormalize the theory (i.e., to fix its parameters) by a scattering experiment at $T = 0$. The relevant part of the amplitude leading to the (vacuum) cross section corresponding to (2) is $\alpha/[P^2 - \Pi_{\text{vac}}(P^2)]$. Here α denotes the *bare* coupling, and Π_{vac} is the *unrenormalized* boson self-energy at $T = 0$. In dimensional regularization, $\Pi_{\text{vac}}(P^2) = \alpha b_0[\epsilon^{-1} - \ln(-P^2/\mu^2)]P^2$, where $b_0 = 4\pi\beta_0$ and β_0 is the leading coefficient of the β function. For a specific momentum transfer $P_r^2 = t_r$, the matrix element reads explicitly

$$\frac{1}{t_r}\frac{\alpha}{1 - \alpha b_0[\epsilon^{-1} - \ln(-t_r/\mu^2)]} \equiv \frac{\alpha(t_r)}{t_r}.$$

A measurement then specifies the *renormalized* coupling $\alpha(t_r)$ at the scale t_r (as introduced in the right-hand side), which is related to the (infinite) bare coupling by

$$\alpha^{-1}(t_r) = \alpha^{-1} - b_0[\epsilon^{-1} - \ln(-t_r/\mu^2)]. \tag{6}$$

An equivalent relation holds for the coupling $\alpha(t)$ at an arbitrary scale t, consequently $\alpha^{-1}(t) = \alpha^{-1}(t_r) + b_0 \ln(t/t_r)$ or, in a common alternative form,

$$\alpha(t) = [b_0 \ln(|t|/\Lambda^2)]^{-1}. \tag{7}$$

It is underlined that the momentum dependence of the renormalized ("running") coupling is fully specified by its value at t_r or, equivalently, by the parameter Λ.

In a thermal medium, the boson self-energy has the generic structure

$$\Pi^i = \alpha b_0\{[\epsilon^{-1} - \ln(-P^2/\mu^2)]P^2 + f^i(p_0, p)\},$$

where the finite "matter" contributions f^i differentiate transverse and longitudinal modes ($i = t, l$). Then, utilizing (6), the in-medium scattering matrix can easily be rewritten in terms of the renormalized coupling,

$$\begin{aligned}\frac{\alpha}{P^2 - \Pi^i} &= \frac{P^{-2}}{\alpha^{-1} - b_0[\epsilon^{-1} - \ln(-P^2/\mu^2) + f^i/P^2]}\\ &= \frac{P^{-2}}{\alpha^{-1}(P^2) - b_0 f^i/P^2} = \frac{\alpha(P^2)}{P^2 - \Pi^i_{\text{mat}}}.\end{aligned} \tag{8}$$

This distinct form, where the divergent vacuum contribution of the self-energy is "absorbed" in the running coupling, elucidates that the amplitude depends only on the physical parameter Λ and the matter part of the self-energy, $\Pi^i_{\text{mat}} = \alpha(P^2)b_0 f^i$. Thus the effective IR cutoff for the energy loss is, as expected, related to the Debye mass [12]. The main emphasis here, though, is on the renormalized coupling in Eq. (8) and, consequently, also in the resulting differential cross section: the scale of the running coupling is set by the virtuality $P^2 = t$.

It is physically intuitive that this fact is generic. Thus, instead of a detailed analysis of loop corrections in QCD (which is more complex), it is useful to invoke a more comprehensive argument. The vacuum differential cross section, as a quantity with an unambiguous normalization, obeys a Callan-Symanzik equation with $\gamma \equiv 0$ [11],

$$\left[M\frac{\partial}{\partial M} + \beta(g)\frac{\partial}{\partial g}\right]\frac{d\sigma(t, \ldots; M, g)}{dt} = 0, \tag{9}$$

where $g = [4\pi\alpha(M^2)]^{1/2}$ is related to the coupling at a given renormalization point M. The general solution of this linear partial differential equation is a function $h(x)$, whose argument $x = g(M)$ satisfies $M dg/dM + \beta(g) = 0$. To investigate the dependence on the momentum scale Q, with $Q^2 = t$, note that, in the limit of small t, the most general form of the cross section is $d\sigma/dt = \mathcal{S}(Q/M, g)/t^2$. Consequently, the function $\mathcal{S}$ obeys $[-Q\partial_Q + \beta(g)\partial_g]\mathcal{S} = 0$ (mind the minus sign). The corresponding characteristic equation, with $\beta(g) = -\beta_0 g^3$ at leading order, then leads readily to the running coupling as introduced above. In other words, renormalization group invariance implies that loop corrections to the differential cross section can be "absorbed" in the tree-level expression by replacing $\alpha \to \alpha(t)$.

A running coupling, as given by Eq. (7) with $b_0 = (11 - \frac{2}{3}n_f)/(4\pi)$ in QCD, alters the integral (4) completely [13],

$$\Phi \int_{t_1}^{t_2} dt \frac{d\sigma_{js}}{dt} \omega = -\frac{\pi C_{js}}{k b_0^2} \int_{t_1}^{t_2} \frac{dt}{t \ln^2(|t|/\Lambda^2)} = \frac{\pi C_{js}}{k b_0^2} \frac{1}{\ln(|t|/\Lambda^2)} \Big|_{t_1}^{t_2} = \frac{\pi C_{js}}{k b_0} [\alpha(\mu^2) - \alpha(|t_1|)]. \quad (10)$$

Opposed to Bjorken's expression (4), the weighted cross section is actually UV finite—due to the asymptotic weakening of the strong interaction, large-t processes are less effective. For hard jets, the integral (10) becomes independent of the energy E; i.e., it is then controlled solely by the coupling at the screening scale. The necessary condition, $|t_1| \sim TE \gg \mu^2 \sim m_D^2 \sim \alpha T^2$, is, for representative parameter values, in line with the previous assumption $E \gg T$ to simplify the kinematics [14]. Hence from this perspective, the collisional loss can become E independent already for typical jet energies—provided, of course, that the resummation improved perturbative framework gives at least a semiquantitative guidance at larger coupling (which will be advocated below).

In this case, the collisional energy loss approaches

$$\frac{dE_j}{dx} = \pi \frac{\alpha(\mu^2)}{b_0} \sum_s C_{js} \int_{k^3} \frac{\rho_s(k)}{k}. \quad (11)$$

In the ideal gas limit, the remaining integration yields

$$\frac{dE_{q,g}}{dx} = \left(\frac{2}{3}\right)^{\pm 1} 2\pi \left(1 + \frac{1}{6} n_f\right) \frac{\alpha(\mu^2)}{b_0} T^2, \quad (12)$$

which differs *structurally* from former expressions as Bjorken's (5). Aside from the energy independence discussed above, dE/dx is proportional to α (instead of α^2). It is highlighted that the considerations above also show that the relevant scale for the coupling is the (perturbatively soft) screening mass rather than a "characteristic" thermal (hard) scale $\sim T$, as commonly presumed.

In order to quantitatively compare Eq. (12) to previous estimates it is necessary to specify parameters, namely Λ in (7) and the cutoff μ, i.e., the Debye mass. Similar renormalization arguments as above led in [15] to an implicit equation for m_D; to leading order

$$m_D^2 = (1 + \tfrac{1}{6} n_f) 4\pi\alpha(m_D^2) T^2, \quad (13)$$

whose solution can be given in terms of Lambert's function. Obviously, also this improved perturbative formula is justified strictly only at temperatures $T \gg \Lambda$. Notwithstanding this, it is found in *quantitative* agreement with lattice QCD calculations down to $\tilde{T} \approx 1.2 T_c$.

It may come as a surprise that a leading order formula does reproduce nonperturbative results [16]. Thus, it is worth emphasizing that the adjusted parameter $\Lambda = 0.2$ GeV for $n_f = 2$ [15] is right in the expected ballpark, refuting a possibility of an uninterpretable fit. Moreover, the 1-loop running coupling (7), with the *same* Λ, reproduces lattice calculations for another quantity, namely, the heavy quark potential $V(r)$ at $T = 0$, notably up to large distances corresponding to $\alpha(r^{-2}) \approx 1$ [15]. Although at first sight rather different quantities, $V(r)$ and $m_D(T)$ are in fact closely related by the renormalized t-channel scattering discussed above—which determines also the collisional energy loss. In other words (tidying the order of the arguments): one can renormalize the theory at $T = 0$ [i.e., determine once and for all Λ from $V(r)$], verify the applicability of the approach for larger couplings as relevant near T_c by successfully calculating $m_D(T)$, and make then predictions for dE/dx.

With this justification, an extrapolation of Eq. (12) as presented in Fig. 1 might be not too unreasonable. Assumed here is $T_c = 160$ MeV and $\mu^2 = [\frac{1}{2}, 2] m_D^2$ to estimate the uncertainty due to the IR cutoff. Shown for comparison are results from (5); here $\mu = m_D$ [likewise from (13)] albeit α in the prefactor (unjustified, but as often presumed) fixed at the scale $Q_T = 2\pi T$. It turns out that already near T_c, the estimates from Eq. (12) exceed those from (5), even for rather large values of E.

For phenomenological implications it is instructive to take further into account a main effect of the strong interaction near T_c. From the distinct decrease of the QGP entropy seen in lattice QCD calculations [18], one can, on general grounds, infer a similar behavior for the number densities ρ_s. In the framework of the quasiparticle model [19], the distribution functions in Eq. (11) are to be evaluated with $n_\pm[\sqrt{m_s^2(T) + k^2}]$, where the effective coupling in the quasiparticle masses, $m_s^2 \propto \alpha_{\rm eff}(T) T^2$, is adjusted to lattice results for the entropy. As shown in Fig. 2, near T_c—despite the strong interaction—the plasma becomes transparent due to the reduced number of "active" degrees of freedom. Qualitatively, this characteristic behavior is in

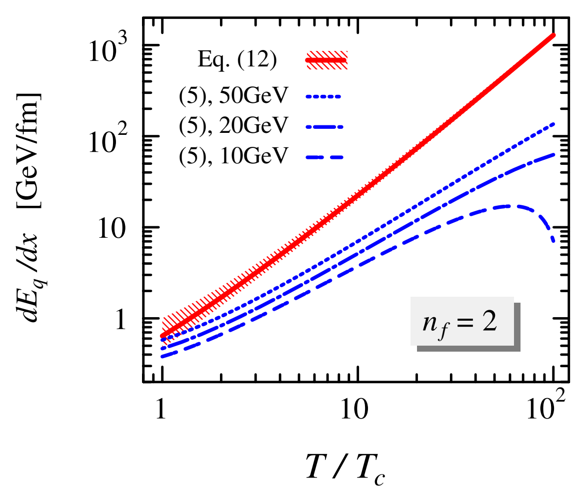

FIG. 1 (color online). Light quark collisional energy loss: Eq. (12) vs the prevalent expression (5) (which yields negative values for very large T) for representative jet energies. For details, see text.

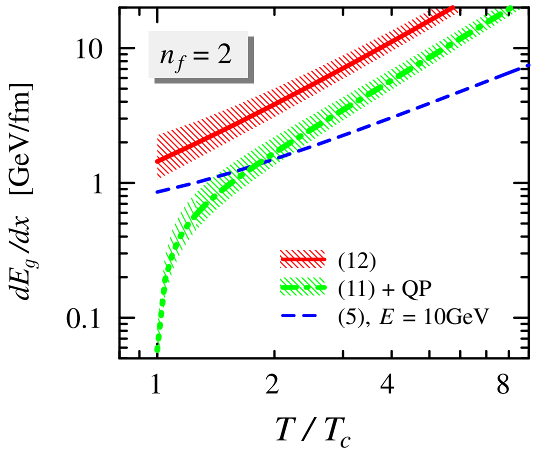

FIG. 2 (color online). Influence of reduced scatterer density near T_c on the gluon energy loss. Below $\tilde{T} \approx 1.2T_c$, Eq. (13) overestimates the Debye mass as obtained in lattice QCD; hence dE/dx could be slightly larger than depicted by the dotted line.

line with the observed quenching measure R_{AA}, when going from SPS (Super Proton Synchrotron) to RHIC energies.

In conclusion, it has been demonstrated in the context of thermal field theory that renormalization does not only dictate the value of the running coupling for a given quantity, but that the momentum dependence can also influence crucially the structure of results. For the QCD collisional energy loss, the relevant scale in $\alpha(t)$ is the (perturbatively soft) screening mass $m_D \sim \sqrt{\alpha}T$ (instead of T, as commonly presumed). The increasing coupling at soft momenta leads to a parametric enhancement compared to previous calculations; see Eqs. (12) vs (5). On the other hand, due to asymptotic freedom, dE/dx becomes independent of the jet energy as $E \to \infty$. Thus, the asymptotic behavior of the underlying interaction makes the energy loss qualitatively different in QCD and QED [where an analog of Eq. (5) indeed holds].

Except very near T_c, Eq. (12) suggests a larger collisional energy loss than previously estimated [2,3]. This finding can be interpreted as a facet of the "strongly coupled" QGP (sQGP), which is characterized by large interaction rates. In fact, $\sigma = \int dt d\sigma/dt$ with *running* coupling can actually be significantly larger than expected from the widely used expression $\sigma_{\alpha \mathrm{fix}} \propto \alpha^2(Q_T^2)/\mu^2 \lesssim$ 1 mb. Thus, the present approach hints also at a natural explanation of the phenomenologically inferred large cross sections $\mathcal{O}(10$ mb) [20,21].

Close to T_c, though, the particle density is known to be substantially reduced. This implies that, irrespective of the strong coupling, the sQGP becomes transparent near the (phase) transition. Such a distinct temperature dependence of the energy loss should be observable. The quantification of this effect (including a discussion of implications of the running coupling for the radiative energy loss) will be the subject of a forthcoming study.

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[13] Aiming here only at the leading order result, details related to the modified propagator structure in the presence of a medium can again be omitted.
[14] Anticipating an extrapolation of the final result for dE/dx to larger coupling (where the hierarchy of scales $\sqrt{\alpha}T \ll T$ breaks down) it is mentioned that $m_D \lesssim 3T$ (see, e.g., [15]).
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