# Difficulties for Five-Dimensional Gauge-Higgs Unification

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(Received 26 April 2006; published 22 November 2006)

We consider five-dimensional gauge theories where all fields propagate in the bulk and the fifth direction is compactified on the orbifold $S^1/Z_2$, and where the fifth components of the gauge bosons play the role of the standard model Higgs boson (gauge-Higgs unification). The gauge symmetry breaking is realized through the appropriate orbifold boundary conditions and through the Hosotani mechanism. We show that for any such theory (with neither brane gauge kinetic terms nor anomalous gauge-group factors) the assumption that the low-energy vector-boson spectrum consists of the $W^\pm$, $Z$, and $\gamma$ only, is inconsistent with the experimental requirements $\sin^2\theta_W \simeq 1/4$ and $\rho \equiv m_W^2/(m_Z^2\cos^2\theta_w) = 1$.

DOI: 10.1103/PhysRevLett.97.211602 PACS numbers: 11.10.Kk, 11.15.−q, 12.10.−g

*Introduction.*—In the standard model (SM), the Higgs mechanism is responsible for generating fermion and vector-boson masses. Although the model is renormalizable and unitary, it has severe naturality problems associated with the so-called "hierarchy problem." At loop-level this problem reduces to the fact that the quadratic corrections tend to increase the Higgs boson mass up to the UV cutoff of the theory. Extra-dimensional extensions of the SM offer a novel approach to gauge symmetry breaking in which the hierarchy problem could be either solved or at least reformulated in terms of the geometry of the higher-dimensional space. A particularly attractive scenario is offered by the Hosotani mechanism [1] where gauge symmetry breaking is generated by the vacuum expectation value of the extra component of the gauge field, $A_4$, whose Kaluza-Klein zero mode plays a role of the four-dimensional Higgs boson, a setup known as gauge-Higgs unification (GHU). Though five-dimensional gauge symmetry and locality prevent a tree-level potential for $A_4$, radiative effects generate a nontrivial effective potential, leading to a prediction for the Higgs boson mass and the scale of gauge symmetry breaking. In such models the boundary conditions determine the gauge group of the light sector (presumably $SU(3) \times SU(2) \times U(1)$), and the vacuum expectation value $\langle A_4 \rangle$ provides a second stage of breaking, presumably to $U(1)_{EM}$. The fact that the symmetry breaking pattern of the low-energy theory is predicted by the gauge and fermion (fermions enter the effective potential for the zero mode of $A_4$ at a loop level) structure of the fundamental theory is indeed very appealing. Other inherent problems of the SM could also be addressed in extra-dimensional scenarios. For instance, within the SM the amount of *CP* violation is not sufficient to explain the observed barion asymmetry [2]; the GHU scenario offers a possible solution since in such models the geometry can be a new source of explicit and spontaneous *CP* violation [3].

The most economic realization of the GHU paradigm uses $SU(3)_c \times SU(3)_w$ as the gauge group of the full theory [4]; however, the model predicts the phenomenologically unacceptable value of the weak-mixing angle (we define $\theta_W$ as the angle that diagonalizes the $Z$-$\gamma$ mass matrix) $\theta_W = \pi/3$ [5]. Though there exist various remedies to this problem (localized gauge kinetic terms [6,7] or allowing a low-energy gauge group with an extra—anomalous—$U(1)$ [5,8]), in this Letter we will restrict ourselves to the simplest (hence more attractive) scenario and we will not pursue such options. Other models also have serious problems, for example, when the gauge group is $SU(5)$ it is natural to expect spontaneous breaking of $SU(3)_c$ [9]. The next minimal choice, $SU(6)$, again suffers from the presence of an extra light $U(1)$ that must be broken by an extra elementary Higgs field [10]. So, in the simplest 5D examples of the GHU either $\sin^2\theta_W$ is not phenomenologically acceptable, or the low-energy gauge group is larger than $SU(3) \times SU(2) \times U(1)$.

Because of these observations it is natural to ask whether there exist 5D GHU models with all fields propagating in the bulk without localized excitations and where the light sector is an $SU(3) \times SU(2) \times U(1)$ gauge theory broken to $U(1)$ by the Hosotani mechanism, and such that the predictions for the weak-mixing angle and the oblique parameters are close to the experimental values. We will argue below that these constraints cannot be satisfied; *no* such model is phenomenologically viable.

*The models.*—The Lagrangian is assumed to have the form $\mathcal{L} = -(F^a_{MN})^2/4+$ fermion, ghost and gauge-fixing terms, where $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f_{abc} A^b_M A^c_N$ [with $f_{abc}$ gauge-group structure constants and $g_5 \sim (\text{mass})^{-1/2}$ the gauge coupling] and $M, N, \ldots = (0, 1, 2, 3, 4)$ the five-dimensional space-time indices with the first four corresponding to Minkowski space (labeled by Greek letters $\mu, \nu, \ldots$). The last index corresponds to the compact direction; we use $x^4 = y$.

 

We will consider a space of the form $\mathbb{M} \otimes (\mathbb{R}/\mathcal{Q})$ where $\mathbb{M}$ denotes the four-dimensional Minkowski space-time and $\mathcal{Q}$ is a discrete group with two elements: (i) translation, $y \to y + L$, where $L$ is the size of the compact subspace; and (ii) reflection, $y \to -y$.

We assume that under $\mathcal{Q}$ the gauge fields transform according to [11]

$$A_N^a(y+L) = \mathbb{V}_{ab} A_N^b(y), \qquad A_N^a(-y) = (-1)^{\delta_{N,4}} \tilde{\mathbb{V}}_{ab} A_N^b(y), \tag{1}$$

where $\mathbb{V}$, $\tilde{\mathbb{V}}$ are real and orthogonal matrices (in a basis where the structure constants are real) representing involutions of the gauge algebra. Note that the orbifolding (1) allows also for the generalized twisting discussed in [11]. We will first assume that the gauge group is simple and then generalize.

For a simple group the transformations (1) leave the Lagrangian invariant provided

$$\mathbb{V}_{da}\mathbb{V}_{eb}\mathbb{V}_{fc} f_{def} = f_{abc}; \qquad \tilde{\mathbb{V}}_{da}\tilde{\mathbb{V}}_{eb}\tilde{\mathbb{V}}_{fc} f_{def} = f_{abc}. \tag{2}$$

In addition, Eq. (1) must provide a representation of $\mathcal{Q}$. Using the fact that $-y = [-(y+L)] + L$ and that $-(-y) = y$ we find

$$\mathbb{V}\tilde{\mathbb{V}}\mathbb{V} = \tilde{\mathbb{V}}; \qquad \tilde{\mathbb{V}}^2 = 1. \tag{3}$$

The models we consider are then defined by the Lagrangian $\mathcal{L}$, which specifies the dynamics, as well as by the matrices $\mathbb{V}$, $\tilde{\mathbb{V}}$ that determine the behavior under $\mathcal{Q}$. Similar matrices are associated with the transformation rules for the fermions [11]; however, those will not be relevant for the arguments presented hereafter.

*Light spectrum.*—Higher-dimensional theories must satisfy the minimum constraint of generating the experimentally observed light spectrum; because of this it is of interest to derive the general properties of these excitations. To this end it proves convenient to expand the various fields in Fourier modes in the compact coordinate $y$; the coefficients are then four-dimensional fields for which the action of $\partial_y$ generates a mass term; all $y$-dependent modes will then be heavy (mass $\sim 1/L$) while light excitations are associated with $y$-independent modes.

The light gauge bosons will be denoted by $A_\mu^{\hat{a}}$; the light modes associated with $A_{N=4}$ behave as four-dimensional scalars and will be denoted by $\phi_{\hat{r}} = A_{N=4}^{\hat{r}}$. Using the $y$ independence of these modes and the behavior of the field under $\mathcal{Q}$ we find [11]

$$A_\mu^{\hat{a}} = \mathbb{V}_{\hat{a}\hat{b}} A_\mu^{\hat{b}} = \tilde{\mathbb{V}}_{\hat{a}\hat{b}} A_\mu^{\hat{b}}, \qquad \phi^{\hat{r}} = \mathbb{V}_{\hat{r}\hat{s}}\phi^{\hat{s}} = -\tilde{\mathbb{V}}_{\hat{r}\hat{s}}\phi^{\hat{s}}. \tag{4}$$

If we denote by $P^+$ the subspace of generators characterized by $+1$ eigenvalues of $\tilde{\mathbb{V}}$ and $\mathbb{V}$ and $N^+$ the subspace of generators characterized by $-1$ eigenvalues of $\tilde{\mathbb{V}}$, and $+1$ eigenvalues of $\mathbb{V}$, then the light gauge bosons and scalars are associated with $P^+$ and $N^+$, respectively. Denoting by $R$ the set of remaining generators we find that (2) and (3) imply that

$$[N^+, P^+] \subset N^+, \qquad [N^+, N^+] \subset P^+, \qquad [N^+, R] \subset R. \tag{5}$$

Extracting from $\mathcal{L}$ the terms that contain only light fields, we find the usual gauge terms for the $A^{\hat{a}}$ and the gauge-invariant (under the subgroup associated with the $A^{\hat{a}}$) kinetic terms for the $\phi$. Note, however, that the form of $\mathcal{L}$ disallows any tree-level potential for $\phi$; it follows that *at tree level* all four-dimensional bosons are either massless or have a mass $\sim 1/L$.

If these models are to be phenomenologically viable, they must be able to generate masses for the appropriate vector bosons at a characteristic scale $v \sim 100$ GeV. This symmetry breaking step can result from radiative corrections since these will generate a nonvanishing (effective) potential $V_{\rm eff}$ for the $\phi$ at $\geq 1$ loops. This opens the possibility that these models will undergo two stages of symmetry breaking: the first generated by the behavior under $\mathcal{Q}$ and the second, at a presumably lower scale, generated radiatively by the scalars $\phi$. Since the scale of $V_{\rm eff}$ is $1/L$ most models predict both scalar and vector-boson masses of $O(1/L)$, in particular, the $m_\phi$ is too light. This problem can find a natural solution by choosing the gauge group, boundary conditions, and fermion content [5,10]; obtaining such a realistic symmetry breaking pattern is a fundamental issue in the GHU scenario.

*Phenomenological constraints.*—Here we consider those five-dimensional models which contain only gauge boson fields and whose light excitations are described by an $SU(3) \times SU(2) \times U(1)$ gauge group. We assume that the $\phi$ effective potential will lead to the expected pattern of spontaneous symmetry breaking; in addition we require

$$\sin^2\theta_W \sim 0.25; \qquad \rho \simeq 1 \tag{6}$$

at tree level. Since we will exhibit a serious problem associated with the minimal requirements (6) we will not investigate whether there exist models where the scalar effective potential produces the correct pattern of spontaneous symmetry breaking. It is possible that no such model exists, in which case our arguments can only be strengthened. We then assume that the zero modes of the $A_4$ acquire vacuum expectation values $v \ll 1/L$ from an effective potential generated at one loop.

We denote by $E_{\boldsymbol{\alpha}}$ and $H_i$ the roots and Cartan generators of Lie algebra of the full theory normalized such that ${\rm Tr} H_i H_j = \delta_{ij}$, ${\rm Tr} E_{-\boldsymbol{\beta}} E_{\boldsymbol{\alpha}} = \delta_{\boldsymbol{\alpha},\boldsymbol{\beta}}$. Then it is straightforward to show that the generators of any $SU(2)$ subgroup [a possible choice for the SM $SU(2)$] will be of the form [12]

$$J_0 = \frac{1}{|\boldsymbol{\alpha}|^2}\boldsymbol{\alpha}\cdot\mathbf{H}, \qquad J_+ = \frac{\sqrt{2}}{|\boldsymbol{\alpha}|}E_{\boldsymbol{\alpha}}, \qquad J_- = (J_+)^\dagger. \tag{7}$$

The SM hypercharge generator $Y$ generates a $U(1)$ subgroup and commutes with $J_{0,\pm}$; we then have

$$Y = \hat{\mathbf{y}}\cdot\mathbf{H}; \qquad \hat{\mathbf{y}}\cdot\boldsymbol{\alpha} = 0. \tag{8}$$

The light scalars that can contribute to the vector-boson mass matrix (in case they acquire a vacuum expectation value) can be arranged according to their $SU(2)$ representations. Assume first that one scalar containing light modes is associated with a linear combination of Cartan generators $\mathbf{x}\cdot\mathbf{H}$. Then, for any root vector $\boldsymbol{\gamma}$ such that $\mathbf{x}\cdot\boldsymbol{\gamma}\neq 0$ we have $[\mathbf{x}\cdot\mathbf{H}, E_{\boldsymbol{\gamma}}] = (\mathbf{x}\cdot\boldsymbol{\gamma})E_{\boldsymbol{\gamma}}$ which is consistent with (5) only if $E_{\boldsymbol{\gamma}}\in R$; in particular, this implies that $\mathbf{x}\cdot\mathbf{H}$ commutes with all generators associated with the SM $SU(2)\times U(1)$, so the associated scalar will be a singlet and cannot contribute to the mass structure of the light vector bosons.

Therefore, the light scalar state which is an eigenvector of $J_0$ with the eigenvalue $I$ that belongs to a multiplet of isospin $I_{\max}(I_{\max}+1)$ should be of the form (in the adjoint representation we will identify a state $|X_a\rangle$ with a generator $X_a$; the action of a generator on such a state is given by $X_b|X_a\rangle = |[X_b, X_a]\rangle$)

$$|I\rangle = \sum_{\boldsymbol{\beta}} v^{\boldsymbol{\beta}}|E_{\boldsymbol{\beta}}\rangle, \tag{9}$$

then $J_0|I\rangle = I|I\rangle$ implies

$$\boldsymbol{\alpha}\cdot\boldsymbol{\beta} = |\boldsymbol{\alpha}|^2 I. \tag{10}$$

Note that $\boldsymbol{\beta}$ cannot be parallel to $\boldsymbol{\alpha}$.

Next we consider the repeated application of the lowering and rising operators to $|I\rangle$,

$$J_\pm^n|I\rangle = \sum_{\boldsymbol{\beta}} v_{\pm n}^{\boldsymbol{\beta}}|E_{\boldsymbol{\beta}\pm n\boldsymbol{\alpha}}\rangle. \tag{11}$$

Note that not all such states will vanish (otherwise $|I\rangle$ would be an $SU(2)$ singlet), hence $E_{\boldsymbol{\beta}\pm n\boldsymbol{\alpha}}\in N^+$ for some integers $n$. Using (5) we then find

$$[E_{\boldsymbol{\beta}\pm n\boldsymbol{\alpha}}, E_{-\boldsymbol{\beta}\mp n\boldsymbol{\alpha}}] = (\boldsymbol{\beta}\pm n\boldsymbol{\alpha})\cdot\mathbf{H}\in P^+, \tag{12}$$

which implies that the set of generators $\{(\boldsymbol{\beta}\pm n\boldsymbol{\alpha})\cdot\mathbf{H}\}$, such that $\boldsymbol{\beta}\pm n\boldsymbol{\alpha}$ is a root, are in $P^+$.

Suppose first that there are two root vectors $\boldsymbol{\beta}, \boldsymbol{\beta}'$ that contribute to the sum (9), then $P^+$ will contain generators proportional to $(\boldsymbol{\beta}+n\boldsymbol{\alpha})\cdot\mathbf{H}$, $(\boldsymbol{\beta}'+n'\boldsymbol{\alpha})\cdot\mathbf{H}$ (for some integers $n, n'$); in addition, $P^+$ will also contain $J_0$. But this is impossible since the electroweak group has rank 2. It then follows that a single root vector $\boldsymbol{\beta}$ can contribute to the sum in (9): the constraint on the rank allows a single light scalar multiplet.

This also implies that the hypercharge generator (8) must be of the form $(r\boldsymbol{\alpha}+s\boldsymbol{\beta})\cdot\mathbf{H}$ for some constants $r$ and $s$; using (8) then implies

$$\hat{\mathbf{y}} = \frac{\boldsymbol{\beta}-(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta})\hat{\boldsymbol{\alpha}}}{|\boldsymbol{\beta}-(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta})\hat{\boldsymbol{\alpha}}|}. \tag{13}$$

Then the (canonically normalized) electroweak bosons correspond to the zero modes of the gauge fields associated with the generators $\hat{\boldsymbol{\alpha}}\cdot\mathbf{H}$, $E_{\pm\boldsymbol{\alpha}}$, $\hat{\mathbf{y}}\cdot\mathbf{H}$; we denote these zero modes by $W^0$, $W^\pm$, and $B$, respectively. Then we can write

$$A_\mu = W_\mu^+ E_{\boldsymbol{\alpha}} + W_\mu^- E_{-\boldsymbol{\alpha}} + W_\mu^0\hat{\boldsymbol{\alpha}}\cdot\mathbf{H} + B_\mu\hat{\mathbf{y}}\cdot\mathbf{H} + \cdots$$
$$A_4 = \phi E_{\boldsymbol{\beta}} + \phi^\star E_{-\boldsymbol{\beta}}. \tag{14}$$

The terms in the Lagrangian responsible for the generation of vector bosons masses are $\propto \mathrm{Tr}[A_\mu, A_4]^2$. Using $[E_{\boldsymbol{\gamma}}, E_{\boldsymbol{\delta}}] = N_{\boldsymbol{\gamma},\boldsymbol{\delta}}E_{\boldsymbol{\gamma}+\boldsymbol{\delta}}$ and the standard properties of the $N_{\boldsymbol{\gamma},\boldsymbol{\delta}}$ [12], we find

$$\mathrm{Tr}[A_\mu, A_4]^2 = (N_{\boldsymbol{\alpha},\boldsymbol{\beta}}^2 + N_{-\boldsymbol{\alpha},\boldsymbol{\beta}}^2)W^+\cdot W^- + [(\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta})W^0 + (\hat{\mathbf{y}}\cdot\boldsymbol{\beta})B]^2 + \cdots. \tag{15}$$

Now, $N_{\boldsymbol{\gamma},\boldsymbol{\delta}}^2 = p(\boldsymbol{\gamma}\cdot\boldsymbol{\delta}) + |\boldsymbol{\gamma}|^2 p(p+1)/2$, where $p$ is an integer such that $p\boldsymbol{\gamma}+\boldsymbol{\delta}$ is a root, but $(p+1)\boldsymbol{\gamma}+\boldsymbol{\delta}$ is not. For our case, using (10), we have $p = I_{\max}\mp I$ for $N_{\pm\boldsymbol{\alpha},\boldsymbol{\beta}}$ so

$$N_{\boldsymbol{\alpha},\boldsymbol{\beta}}^2 + N_{-\boldsymbol{\alpha},\boldsymbol{\beta}}^2 = |\boldsymbol{\alpha}|^2[I_{\max}(I_{\max}+1) - I^2]. \tag{16}$$

Assuming that $|I\rangle$ is a member of a multiplet with maximum isospin $I_{\max}$ and it is the component that gets a vacuum expectation value $v/\sqrt{2}$, it is straightforward to show that the mass terms in $\mathcal{L}$ take the form

$$\mathcal{L}_{\text{mass}} = \frac{v^2}{2}\{|\boldsymbol{\alpha}|^2[I_{\max}(I_{\max}+1) - I^2]W^+\cdot W^- + (\hat{\boldsymbol{\alpha}}\cdot\boldsymbol{\beta}W^0 + \hat{\mathbf{y}}\cdot\boldsymbol{\beta}B)^2\}, \tag{17}$$

so the electroweak mixing angle and $\rho$ parameter are given by

$$\sin^2\theta_W = 1 - (\hat{\boldsymbol{\alpha}}\cdot\hat{\boldsymbol{\beta}})^2, \qquad \rho = \frac{I_{\max}(I_{\max}+1)}{2I^2} - \frac{1}{2}. \tag{18}$$

Since either $\boldsymbol{\beta}+\boldsymbol{\alpha}$ or $\boldsymbol{\beta}-\boldsymbol{\alpha}$ is a root (otherwise $I = I_{\max} = 0$), then the commutator $[E_{\boldsymbol{\beta}}, E_{\boldsymbol{\beta}\pm\boldsymbol{\alpha}}]$ either vanishes or it is proportional to $E_{2\boldsymbol{\beta}\pm\boldsymbol{\alpha}}$. But since $E_{\boldsymbol{\beta}}$, $E_{\boldsymbol{\beta}\pm\boldsymbol{\alpha}}$ are roots, then, using (5) shows that a nonzero commutator implies that $E_{2\boldsymbol{\beta}\pm\boldsymbol{\alpha}} = E_{\boldsymbol{\alpha}}$ or $E_{2\boldsymbol{\beta}\pm\boldsymbol{\alpha}} = E_{-\boldsymbol{\alpha}}$, both of which are impossible. Hence

$$[E_{\boldsymbol{\beta}}, E_{\boldsymbol{\beta}\pm\boldsymbol{\alpha}}] = 0. \tag{19}$$

There are then two possibilities: (i) $-\boldsymbol{\beta}\pm\boldsymbol{\alpha}$ is not a root. Then $[E_{-\boldsymbol{\beta}}, E_{\pm\boldsymbol{\alpha}}] = 0$ which, together with (19) and $[E_{-\boldsymbol{\beta}}, E_{\boldsymbol{\beta}\pm\boldsymbol{\alpha}}]\propto E_{\pm\boldsymbol{\alpha}}$, implies $\boldsymbol{\beta}\cdot(\boldsymbol{\beta}\pm\boldsymbol{\alpha}) = |\boldsymbol{\beta}|^2/2$. Combining this with (10) we find

$$|I|/2 = (\hat{\boldsymbol{\alpha}} \cdot \hat{\boldsymbol{\beta}})^2 = m/4, \tag{20}$$

where $m$ is an integer, $0 \leq m \leq 4$ [12]. Of these choices only $m = 1, 4$ allow $\rho = 1$, but in this case $\sin^2\theta_W = 0.75$, 0, both of which are phenomenologically uninteresting [13]. (ii) If $-\boldsymbol{\beta} \pm \boldsymbol{\alpha}$ is a root then $[E_{-\boldsymbol{\beta}}, E_{\pm\boldsymbol{\alpha}}] \propto E_{-\boldsymbol{\beta}\pm\boldsymbol{\alpha}}$, but now $[E_{-\boldsymbol{\beta}}, E_{-\boldsymbol{\beta}\pm\boldsymbol{\alpha}}]$ must vanish [or else it would belong to $P^+$ and so must be $\propto E_{\boldsymbol{\alpha}}$ or $\propto E_{-\boldsymbol{\alpha}}$ which is impossible; i.e., for the same reasons leading to (19)]. In this case $\boldsymbol{\beta} \cdot (\boldsymbol{\beta} \pm \boldsymbol{\alpha}) = |\boldsymbol{\beta}|^2$ whence $\sin^2\theta_W = 1$ which is again phenomenologically uninteresting.

*Nonsimple groups.*—When the gauge group is not simple Eq. (2) is replaced by $\sum_{def} g_d \mathbb{V}_{da} \mathbb{V}_{eb} \mathbb{V}_{fc} f_{def} = g_a f_{abc}$ (and an equivalent expression for $\tilde{\mathbb{V}}$) where the $g_a$ denote the gauge coupling constants taking the same value for all indices $a$ belonging to one group factor. These imply that if $\mathbb{V}$ maps the gauge fields of some factor group $G_i$ into those of another factor $G_j$, then these groups must have the same algebras and gauge couplings. Models where this is not trivial ($i \neq j$) have a gauge group of the form $G^N \times \cdots$ where the $N$ factors of $G$ have the same couplings constants and so have an additional permutation symmetry $\mathcal{P}$ which is respected by $\mathbb{V}$ and $\tilde{\mathbb{V}}$.

Phenomenologically we must require that the low-energy gauge fields be singlets under $\mathcal{P}$ [else the light gauge bosons would be members of a nontrivial $\mathcal{P}$ multiplet so that the electroweak gauge group would be of the form $SU(2)^n \times U(1)^l$ for some integers $n$, $l > 1$]. The $SU(2) \times U(1)$ generators will be a direct sum of generators of the form (7) and (8) with one contribution from each of the $N$ factor groups (each term containing the same $\boldsymbol{\alpha}$ and $\hat{\mathbf{y}}$ as a result of the invariance under $\mathcal{P}$). Hence the crucial expressions (10) and (17) remain unchanged and the same problems associated with $\rho$ and $\theta_W$ occur.

*Conclusions.*—We have shown that within the gauge-Higgs unification scenario (with neither brane gauge kinetic terms nor anomalous gauge-group factors) in 5D the phenomenological conditions (6) necessarily imply a light electroweak gauge group $G_{\text{light}}$ larger than $SU(2) \times U(1)$. This general statement is illustrated by specific cases that have appeared in the literature, e.g., [5] ($\sin^2\theta_W = 3/4$) and [10] [extra $U(1)$ factor in $G_{\text{light}}$]. It is unlikely that GHU model with an extended $G_{\text{light}}$ can be phenomenologically viable since this would require the Hosotani mechanism to generate a two stage breaking, $G_{\text{light}} \to SU(2) \times U(1)$ at a scale $V$ by one 4D scalar mode $\Phi$, and $SU(2) \times U(1) \to U(1)$ at a scale $v \ll V$ by another mode $\phi$. But this hierarchy is determined by the 1-loop effective potential generated by all bosonic and fermionic modes, which mix $\Phi$ and $\phi$. Then, in the absence of fine-tuning, the hierarchy $V \gg v$ cannot be maintained.

These results do not necessarily generalize to more than 5 dimensions[14]. The conditions under which models in $\geq 6$ dimensions are phenomenologically viable will be examined in a future publication.

This work is supported in part by the State Committee for Scientific Research (Poland) under Grant No. 1 P03B 078 26 in the period 2004–07, and by funds provided by the US Department of Energy under Grant No. DE-FG03-94ER40837.

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