Examining the Necessity to Include Event-By-Event Fluctuations in Experimental Evaluations of Elliptical Flow

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Elliptic flow at BNL RHIC is computed event by event with NEXSPHERIO. We show that when symmetry of the particle distribution in relation to the reaction plane is assumed, as usually done in the experimental extraction of elliptic flow, there is a disagreement between the true and reconstructed elliptic flows (15%–30% for $\eta = 0$, 30% for $p_\perp = 0.5$ GeV). We suggest a possible way to take into account the asymmetry and get good agreement between these elliptic flows.

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Hydrodynamics is one of the main tools used to study the collective flow in high-energy nuclear collisions. Here we discuss results on elliptic flow obtained with the hydrodynamical code NEXSPHERIO. It is a junction of two codes: NEXUS and SPHERIO. The SPHERIO code is used to compute the hydrodynamical evolution. It is based on smoothed particle hydrodynamics, a method originally developed in astrophysics and adapted to relativistic heavy ion collisions [1]. Its main advantage is that any geometry in the initial conditions can be incorporated. The NEXUS code is used to compute the initial conditions $T_{\mu\nu}$, j^μ, and u^μ on a proper time hypersurface [2]. NEXSPHERIO is run many times, corresponding to many different events or initial conditions. At the end, an average over final results is performed. This mimics experimental conditions. This is different from the canonical approach in hydrodynamics where initial conditions are adjusted to reproduce some selected data and are very smooth. This code has been used to study a range of problems concerning relativistic nuclear collisions: the effect of fluctuating initial conditions on particle distributions [3], energy dependence of the kaon effective temperature [4], interferometry at RHIC [5], transverse mass distributions at SPS for strange and nonstrange particles [6], the effect of the different theoretical and experimental binnings [7], the effect of the nature of the quark-hadron transition and of the particle emission mechanism [8]. Here we study the evaluation of elliptic flow using the so-called standard method. The version of NEXSPHERIO used here has a first order quark-hadron transition, sudden freeze out, and no strangeness conservation. The only parameter, the freeze out temperature, was assumed to be 150 MeV, since this gives good agreement for the charged particle pseudorapidity and transverse momentum distributions for all PHOBOS centrality windows.

Theoretically, the impact parameter $\vec{b}$ is known and varies in the range of the centrality window chosen. The theoretical, or true, elliptic flow parameter at a given pseudorapidity η is defined as

$$\langle v_2^b(\eta)\rangle = \left\langle \frac{\int d^2N/d\phi d\eta \cos[2(\phi - \phi_b)]d\phi}{\int d^2N/d\phi d\eta d\phi} \right\rangle. \quad (1)$$

ϕ_b is the angle between $\vec{b}$ and some fixed reference axis. The average is performed over all events in the centrality bin.

Experimentally, the impact parameter angle ϕ_b is not known. An approximation, ψ_2, is estimated. Elliptic flow parameter with respect to this angle, $v_2^{\rm obs}(\eta)$, is calculated. Then a correction is applied to $v_2^{\rm obs}(\eta)$ to account for the reaction plane resolution, leading to the experimentally reconstructed elliptic flow parameter $v_2^{\rm rec}(\eta)$. For example, in a Phobos-like way [9]

$$\langle v_2^{\rm rec}(\eta)\rangle = \left\langle \frac{v_2^{\rm obs}(\eta)}{\sqrt{\langle \cos[2(\psi_2^{<0} - \psi_2^{>0})]\rangle}} \right\rangle, \quad (2)$$

where

$$v_2^{\rm obs}(\eta) = \frac{\sum_i d^2N/d\phi_i d\eta \cos[2(\phi_i - \psi_2)]}{\sum_i d^2N/d\phi_i d\eta} \quad (3)$$

and

$$\psi_2 = \frac{1}{2}\tan^{-1}\frac{\sum_i \sin 2\phi_i}{\sum_i \cos 2\phi_i}. \quad (4)$$

In the hit-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $\eta < 0$ and $\eta > 0$, respectively, and if v_2 is computed for a positive (negative) η, the sums in ψ_2, Eq. (4), are over particles with $\eta < 0$ ($\eta > 0$). In the track-based method, $\psi_2^{<0}$ and $\psi_2^{>0}$ are determined for subevents $2.05 < |\eta| < 3.2$, the sums in ψ_2, Eq. (4), are over particles in both subevents, v_2 is obtained for particles around $0 < \eta < 1.8$ and reflected [to account for the different multiplicities between a subevent and the sums in Eq. (4), there is also an additional $\sqrt{2\alpha}$ with $\alpha \sim 1$, in the reaction plane correction in Eq. (2)]. Since both methods are in agreement but only the hit-based method covers a large pseudorapidity interval, we use this latter method.

We want to check whether the theoretical and experimental estimates are in agreement, i.e., $\langle v_2^b(\eta)\rangle = \langle v_2^{\rm rec}(\eta)\rangle$. A necessary condition for this, from Eq. (2), is, $\langle v_2^b(\eta)\rangle \geq \langle v_2^{\rm obs}(\eta)\rangle$. In Fig. 1, we show the results for $\langle v_2^b(\eta)\rangle$ (solid line) and $\langle v_2^{\rm obs}(\eta)\rangle$ (dashed line). We see that $\langle v_2^b(\eta)\rangle \leq \langle v_2^{\rm obs}(\eta)\rangle$ for most η's. So, as shown also in the figure, dividing by a cosine to get $\langle v_2^{\rm rec}(\eta)\rangle$ (dotted curve) makes the disagreement worse: $\langle v_2^b(\eta)\rangle$ and $\langle v_2^{\rm rec}(\eta)\rangle$ *are* different. This is true for all three Phobos centrality windows and more pronounced in the most central window.

Since the standard way to include the correction for the reaction plane resolution [Eq. (2)] seems inapplicable, we need to understand why. When we look at the distribution $d^2N/d\phi d\eta$ obtained in a NEXSPHERIO event (presumably also in a true event), it is not symmetric with respect to the reaction plane. (We recall that the reaction plane is the plane defined by the impact parameter vector and the beam axis.) This happens because (i) the incident nuclei have a granular structure, (ii) the number of produced particles is finite. The symmetry might be better with respect to the plane with inclination ψ_2 in relation to the reference axis and containing the beam axis. Therefore, we must write for each event

$$\frac{d^2N}{d\phi d\eta} = v_0(\eta)\Big\{1 + \sum_n 2v_n^b(\eta)\cos[n(\phi-\phi_b)] + \sum_n 2v_n'^b(\eta)\sin[n(\phi-\phi_b)]\Big\} \quad (5)$$

$$= v_0(\eta)\Big\{1 + \sum_n 2v_n^{\rm obs}(\eta)\cos[n(\phi-\psi_2)] + \sum_n 2v_n'^{\rm obs}(\eta)\sin[n(\phi-\psi_2)]\Big\}. \quad (6)$$

It follows that

$$v_2^{\rm obs}(\eta) = v_2^b(\eta)\cos[2(\psi_2-\phi_b)] + v_2'^b(\eta)\sin[2(\psi_2-\phi_b)]. \quad (7)$$

We see that due to the sine term, we can indeed have $\langle v_2^{\rm obs}(\eta)\rangle > \langle v_2^b(\eta)\rangle$, and therefore $\langle v_2^{\rm rec}(\eta)\rangle > \langle v_2^b(\eta)\rangle$ as in Fig. 1. The sine term does not vanish upon averaging on events because if a choice such as Eq. (4) is done for ψ_2, $v_2'^b(\eta)$ and $\sin[2(\psi_2-\phi_b)]$ have the same sign. This can be visualized with Fig. 2(a). If the momentum distribution, instead of being symmetric with respect to the reaction plane, (for example $v_2^b > 0$, $v_2'^b = 0$) has a positive sine term added ($v_2'^b > 0$), it now points at an angle between 0 and $\pi/4$ above the reaction plane. This angle is in fact ψ_2 and is determined experimentally with Eq. (4). Therefore $v_2'^b\sin[2(\psi_2-\phi_b)] > 0$. Similarly, if $v_2'^b < 0$, ψ_2 is between $-\pi/4$ and 0 and $v_2'^b\sin[2(\psi_2-\phi_b)] > 0$. [Rigorously, this sign condition is true if ψ_2 is computed for the same η as $v_2'^b(\eta)$. Because of the actual way of experimentally extracting ψ_2, we expect this condition is approximately satisfied only for particles with small or moderate pseudorapidity, which are close enough to where ψ_2 was computed.]

In the standard approach, for example, as in Phobos analysis, it is *assumed* that $d^2N/d\phi d\eta$ is symmetric with

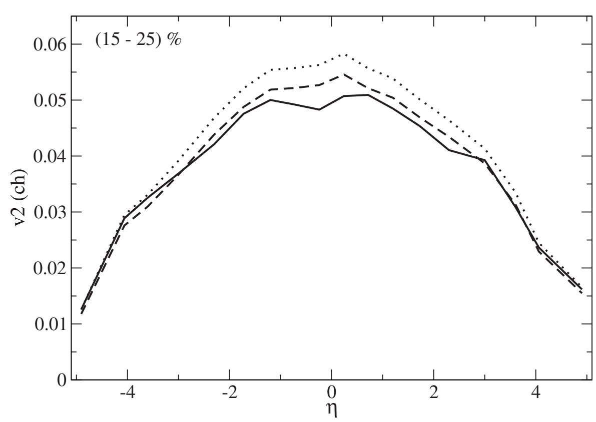

FIG. 1. Comparison between various ways of computing v_2 using NEXSPHERIO for Phobos 15%–25% centrality window [9]: solid line is v_2^b, obtained using the known impact parameter angle ϕ_b, dashed (dotted) line is $v_2^{\rm obs}$ ($v_2^{\rm rec}$), obtained using the reconstructed impact parameter angle ψ_2 without (with) reaction plane correction.

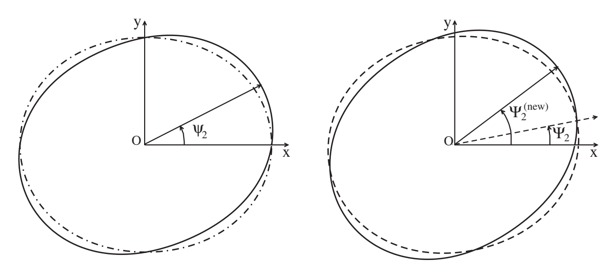

FIG. 2. Assuming (top) $d^2N/d\phi d\eta = 1 + 2v_2^b\cos[2(\phi-\phi_b)] + 2v_2'^b\sin[2(\phi-\phi_b)]$ with $\phi_b = 0$: dash-dotted momentum distribution is symmetric with respect to the reaction plane ($v_2^b > 0$, $v_2'^b = 0$) and solid is asymmetric ($v_2^b > 0$, $v_2'^b > 0$); assuming (bottom) $d^2N/d\phi d\eta = 1 + 2v_2^{\rm obs}\cos[2(\phi-\psi_2)] + 2v_2'^{\rm obs}\sin[2(\phi-\psi_2)]$ with $\phi_b = 0$: dashed momentum distribution is symmetric with to the plane inclination ψ_2 above the impact parameter and containing the beam axis ($v_2^{\rm obs} > 0$, $v_2'^{\rm obs} = 0$) and solid is asymmetric ($v_2^{\rm obs} > 0$, $v_2'^{\rm obs} > 0$).

respect to the reaction plane and there are no sine terms in the Fourier decomposition in Eq. (5); Eq. (7) leads to (for the hit-based or track-based method)

$$\langle v_2^b(\eta)\rangle = \langle v_2^{\rm obs}(\eta)\rangle/\langle\cos[2(\psi_2-\phi_b)]\rangle. \qquad (8)$$

Then, using $\langle\cos[2(\psi_2-\phi_b)]\rangle = \langle\cos[2(\psi_2^>-\phi_b)]\rangle = \langle\cos[2(\psi_2^<-\phi_b)]\rangle$ and $\langle\cos[2(\psi_2^>-\psi_2^<)]\rangle = \langle\cos[2(\psi_2^>-\phi_b)]\rangle\langle\cos[2(\psi_2^<-\phi_b)]\rangle = \langle\cos[2(\psi_2^>-\phi_b)]\rangle^2$ (where it is assumed that the distributions of $\psi_2^>-\phi_b$ and $\psi_2^<-\phi_b$ are symmetrical with respect to the reference axis and $\psi_2^>-\phi_b$ and $\psi_2^<-\phi_b$ are independent), Eq. (2) follows. However, as explained above, the use of NEXUS initial conditions leads to $d^2N/d\phi d\eta$ not symmetric with respect to the reaction plane (and presumably this is also the case in each real event), so Eq. (2) and (8) are not valid.

As already mentioned, the symmetry might be better with respect to the plane with inclination ψ_2 in relation to the reference axis and containing the beam axis. From (5) and (6), we have

$$v_2^b(\eta) = v_2^{\rm obs}(\eta)\cos[2(\psi_2-\phi_b)] + v_2'^{\rm obs}(\eta)\sin[2(\psi_2-\phi_b)]. \qquad (9)$$

If the symmetry is perfect $v_2'^{\rm obs}=0$. Otherwise, looking at Fig. 2(b), if the angular distribution, instead of being symmetric with respect to the axis with inclination ψ_2 in relation to the impact parameter, (e.g., $v_2^{\rm obs}>0$, $v_2'^{\rm obs}=0$) has a positive sine term added ($v_2'^{\rm obs}>0$), it now points at an angle $\psi_2^{\rm new}$ greater than ψ_2. If a negative sine term is added ($v_2'^{\rm obs}<0$), it now points at an angle $\psi_2^{\rm new}$ smaller than ψ_2. Both possibilities are equally likely for a given ψ_2 but lead to opposite signs for $v_2'^{\rm obs}(\eta)\sin[2(\psi_2^{\rm new}-\phi_b)$ (in general). Therefore $\langle v_2'^{\rm obs}(\eta)\sin[2(\psi_2-\phi_b)]\rangle = 0$. So whether the symmetry is perfect or approximate, $\langle v_2^b(\eta)\rangle \sim \langle v_2^{\rm obs}(\eta)\cos[2(\psi_2-\phi_b)]\rangle$ and instead of Eq. (2) we would have

$$\langle v_2^{\rm Rec}(\eta)\rangle = \langle v_2^{\rm obs}(\eta)\sqrt{\langle\cos[2(\psi_2^{<0}-\psi_2^{>0})]\rangle}\rangle. \qquad (10)$$

In Fig. 3, we show $\langle v_2^{\rm Rec}(\eta)\rangle$ (dash-dotted line) and $\langle v_2^b(\eta)\rangle$ (solid line). We see that the agreement between both methods is improved compared to Fig. 1. We have also computed the elliptic flow parameter as function of transverse momentum for charged hadrons with $0<\eta<1.5$ for the 50% most central collisions. We found that $\langle v_2^b(p_\perp)\rangle$ computed as in Eq. (1) is well approximated by $\langle v_2^{\rm Rec}(p_\perp)\rangle$ computed as in Eq. (10).

In summary, from Fig. 1, elliptic flow estimated from the standard method with reaction plane correction is an overestimate of true elliptic flow ($v_2^{\rm rec}>v_2^b$). From Fig. 3, using a method that takes into account the more symmetrical nature of particle distribution in relation to the plane with inclination ψ_2 with respect to the reference axis and containing the beam axis (rather than with respect to the reaction plane), we get a better agreement between reconstructed and true elliptic flows ($v_2^{\rm Rec}\sim v_2^b$).

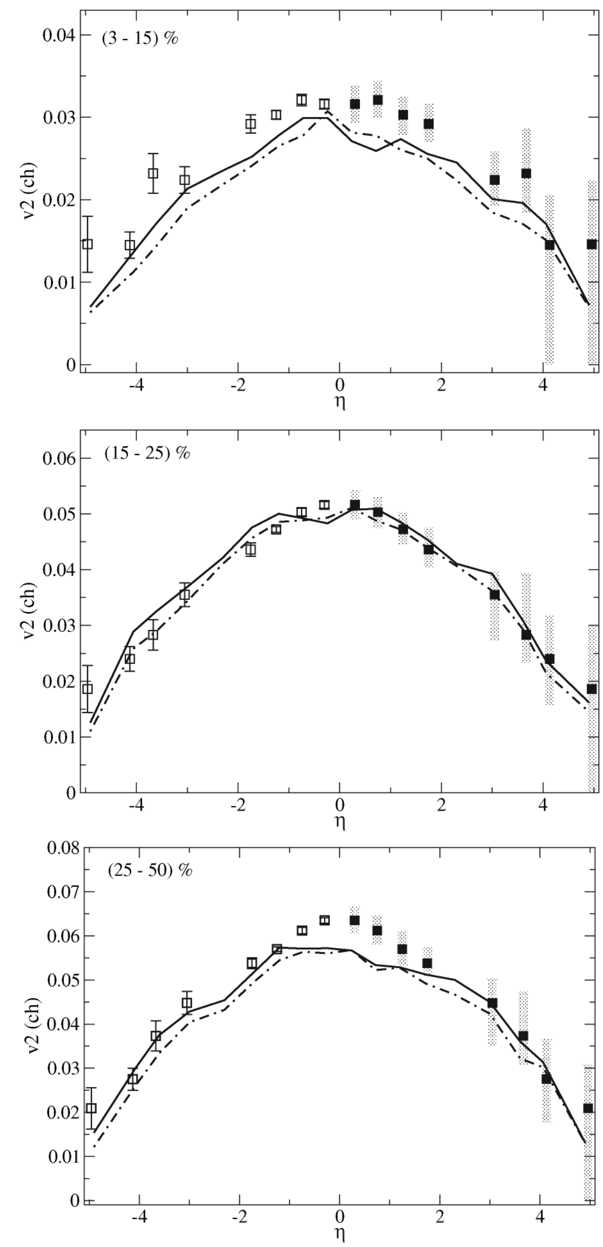

FIG. 3. Comparison between true elliptic flow v_2^b (solid line) and suggested method to compute reconstructed elliptic flow from data $v_2^{\rm Rec}$ (dash-dotted) for the three Phobos centrality windows [9]. Squares represent Phobos data (black error bars are 1σ statistical errors and gray bands, systematic uncertainties at $\sim$90% confidence level).

As for overestimating the true elliptic flow, a similar conclusion was reached in [10,11]. In [10] elliptic flow was assumed proportional to eccentricity and eccentricity was computed event by event using a Monte Carlo Glauber calculation. As in our case, $\vec{b}$ is known. It was found that the integrated true v_2^b is smaller than $v_2^{\rm rec}$ computed with a two-particle cumulant method (for all centralities) and larger than $v_2^{\rm rec}$ computed with higher-order cumulants (for centralities 0%–80%). In [11], elliptic flow was computed event by event within the UrQMD model. Again $\vec{b}$ is known. It was found that the integrated true v_2^b is smaller than $v_2^{\rm rec}$ computed with a two-particle cumulant method (for all centralities) and equal to $v_2^{\rm rec}$ computed with higher-order cumulants (for centralities 10%–50%). Differential elliptic flow was also studied leading to similar conclusions.

In these two papers, it is expected [10,11] that there will be differences between v_2^b and $v_2^{\rm rec}$ calculated with the reaction plane method or two-particle cumulant method both because of the so-called nonflow correlations (overall momentum conservation, resonance decays, jet production, etc.) and event-by-event fluctuations (mostly eccentricity fluctuations). In principle, higher-order cumulant methods take care of nonflow effects. If there is still disagreement between the true elliptic flow and higher-order cumulant methods, as in [10], then fluctuations are important. If there is agreement as in [11], then nonflow effects are important and not fluctuations. In addition to the disagreement between their conclusions, [10,11] do not (neither are they expected to) reproduce the RHIC data. So an interesting question is whether a more accepted hydrodynamical description would lead to a sizable effect. Using NEXSPHERIO, we found that true elliptic flow $v_2^b(\eta = 0)$ is overestimated by ~15%–30% (according to centrality) with the reaction plane method, and $v_2^b(p_\perp)$ by ~30% at $p_\perp = 0.5$ GeV. In our case, since $\langle v_2^b \rangle \sim \langle v_2^{\rm Rec} \rangle$, a large part of the difference between the true $\langle v_2^b \rangle$ and reconstructed $\langle v_2^{\rm rec} \rangle$ is due to the (wrong) assumption of symmetry of the particle distribution around the reaction plane, made to obtain $\langle v_2^{\rm rec} \rangle$.

Finally, we would like to emphasize that it is important to have precise experimental determination of elliptic flow, in particular, free from the assumption of symmetry that we discussed. Elliptic flow teaches us about the initial conditions and thermalization, in principle. In this manner, in [12], the author showed that with his hydrodynamical code plus freeze out, he could not reproduce $v_2(\eta)$, in particular, at large η, and therefore concluded that there might be a lack of thermalization for these large η's. In [13], it was shown that agreement with $v_2(\eta)$ data could be obtained for central collisions with a similar hydrodynamical code but with color glass initial conditions and, instead of freeze out, a transport code matched to the hydrodynamical code to describe particle emission. It was therefore concluded that some viscosity was necessary in the hadronic phase. Lastly, in [14] (see Fig. 3 and 4 therein), it was shown that for all centralities, Glauber-type initial conditions plus hadronic dissipation lead to a reasonable agreement with $v_2(\eta)$ data while color glass condensate initial conditions plus hadronic dissipation do not, except in the most central window (unless some additional dissipation occurs in the early quark gluon plasma phase). Both sets of initial conditions without hadronic dissipation tend to underestimate $v_2(\eta)$ data if $T_{f.{\rm out}} = 169$ MeV and overpredict them if $T_{f.{\rm out}} = 100$ MeV. However, these conclusions would be affected if $v_2(\eta)$ data were lower, as we think they should be. [Incidentally, though our objective was not to reproduce data, note that our model with freeze out (no transport code) reproduces reasonably both the $v_2(\eta)$ data as in [14] (Fig. 3) and the $v_2(p_\perp)$ data (not shown).] Therefore, to know, e.g., what the initial conditions are or if there is viscosity and in what phase, we need to settle the question of whether event-by-event fluctuations are important and take them into account in the experimental analysis.

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