Compression and Extraction of Stopped Muons

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Efficient conversion of a standard positive muon beam into a high-quality slow muon beam is shown to be achievable by compression of a muon swarm stopped in an extended gas volume. The stopped swarm can be squeezed into a mm-size swarm flow that can be extracted into vacuum through a small opening in the stop target walls. Novel techniques of swarm compression are considered. In particular, a density gradient in crossed electric and magnetic fields is used.

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Standard muon beams have a relatively high energy and poor phase space quality. The resulting limited stopping characteristics are best improved by using phase space compression (PSC) techniques. There are active techniques like particle detection and steering [1], semiactive techniques like frictional cooling [2], and fully passive techniques relying only on slowing down in matter. In the last class, where some reemission into vacuum is required, two methods have been discovered and implemented: thermal muonium emission from foils with subsequent ionization, and muon emission from rare gas solid layers [3]. PSC is there achieved via the tremendous compression in momentum space resulting from the conversion of MeV muons into eV or sub-eV muons. Although the effective yields are small, the potential of stopping in the first atomic layers of solid materials opens a new field in muon physics research.

Given the cooling power of matter, one can think of using compressing forces induced by electric fields to steer stopped muons and squeeze them into a small volume. This can be done in a gas, and with sufficient volume reduction, the muons can be extracted into vacuum through a tiny hole in the target wall. This leads to a new semiactive PSC concept where spatial PSC is added to the above momentum PSC, both using frictional energy loss in matter.

Efficient stopping requires high gas density and, consequently, high electric fields to induce sufficiently fast drifts. The main limitation, the appearance of a Townsend discharge, can be efficiently inhibited by using crossed electric and magnetic fields [4]. For a sufficiently high ratio of magnetic field to gas density B/N, the electron precession around $\vec{B}$ hinders the acceleration along $\vec{E}$ and the electron multiplication coefficient α drops sharply even at high N. However, the muon with its much greater mass and precession period is less sensitive to B/N. It can reach a sufficiently high drift velocity to allow its steering and compression.

A high magnetic field is also optimal for stopping standard muon beams in a gas target of limited stopping power, since the target can be made long in the direction of the B field: the field gradient in front of the target can be used to strongly focus the entering muon beam to a small diameter, and the precession of the muons around $\vec{B}$ inhibits the effect of multiple scattering. The target can remain quite thin in the transverse direction, and only a limited amount of transverse drift is required to bring the muon swarm out of the stop volume. Such a configuration also inhibits the loss of longitudinal muon polarization during slowing down into the charge exchange region as long as the magnetic field strength is much greater than the μ^+-electron contact field in muonium of 0.1585 T (Paschen-Back decoupling [5]).

How can an efficient swarm compression be made in the direction perpendicular to a strong magnetic field? We propose a most natural but quite unconventional way to realize this operation.

In the absence of collisions, the overall drift in crossed electric and magnetic fields is in the $\vec{E}\times\vec{B}$ direction with velocity $v_{\vec{E}\times\vec{B}} = E/B$. In the presence of collisions, the charged particle diffuses in the gas with an overall drift in the direction of $\vec{E}$ with velocity $v_{\vec{E}}$ superposed on the $\vec{E}\times\vec{B}$ drift. The ratio of $v_{\vec{E}}$ to $v_{\vec{E}\times\vec{B}}$ is directly related to the ratio of the collision rate ν to the angular velocity $\omega = eB/m$ of the precession around the magnetic field. The special case where ν is independent of the particle velocity yields the simple relation [4]

$$\tan\theta = \frac{v_{\vec{E}}}{v_{\vec{E}\times\vec{B}}} = \frac{\nu}{\omega} \tag{1}$$

where θ is the angle between the drift velocity direction and the $\vec{E}\times\vec{B}$ direction. Here, ν is the product of the density N and a collision rate constant k (product of velocity and cross section) which, in the general case, has to be appropriately averaged $[\overline{k/(k^2+\omega^2)}/\overline{1/(k^2+\omega^2)}]$ giving a k_{av} that depends on E/N and B/N:

$$\tan\theta = \frac{v_{\vec{E}}}{v_{\vec{E}\times\vec{B}}} = \frac{k_{\mathrm{av}}(E/N, B/N)}{\omega}N. \tag{2}$$

In order to achieve spatial compression, we require the direction of the total velocity

$$\vec{v} = \vec{v}_{\vec{E}} + \vec{v}_{\vec{E}\times\vec{B}} \tag{3}$$

to be position dependent. What can be varied? Both B (and

ω) are constant, and the effect of varying E depends on the gas properties. The other possible variations are either the direction of $\vec{E}$ or the value of N. We here focus on the variation of N; implementing it leads to what can be called "density gradient compression." It can be illustrated on the basis of Eq. (2). Suppose that at a given position $N = N_0$ and $k/\omega = 1/N_0$. Then $v_{\vec{E}}$ equals $v_{\vec{E}\times\vec{B}}$ and $\theta = 45°$. If there is a density gradient ∇N in the direction perpendicular to $\vec{v}$ (and $\vec{B}$), then, for k independent on N and neglecting diffusion, the swarm size compresses towards zero over a distance $N/|\nabla N|$.

A general expression of the compression rate λ (neglecting diffusion) can be deduced from the variation of θ from Eq. (2) in the direction perpendicular to the flow:

$$\lambda = |\nabla\theta \times \vec{v}_0| = \sin\theta_0 \cos\theta_0 \left(1 + \frac{\delta k_{\mathrm{av}}/\delta N}{k/N}\right) \frac{|\nabla N \times \vec{v}_0|}{N_0} \tag{4}$$

where θ_0, $\vec{v}_0$, and N_0 are, respectively, the central drift angle, drift velocity, and density. Clearly, optimal compression takes place at $\theta_0 = 45°$ with a density gradient perpendicular to $\vec{v}_0$. The second term in the parentheses accounts for the density variation of the collision rate constant which can, depending on its sign, either improve or hinder the compression.

Of the various options available to implement a density gradient, we consider here only the simplest one: a static gas target with one of two opposite target walls either cooled or heated. For parallel walls, the temperature gradient and consequently the density gradient are perpendicular to the walls. If $\vec{E}$ is applied at an angle of 45° with the walls, the flow is focused towards the particles having $v_{\vec{E}} = v_{\vec{E}\times\vec{B}}$ which drift parallel to the walls, and the final swarm width is set by the competition between compression and diffusion. But as the swarm width decreases, the distance between the walls can be reduced, increasing the density gradient and therefore the focusing strength. The swarm is squeezed further by quite an amount until the focusing strength, which at constant $\vec{E}$ will increase inversely with the distance between walls, cannot anymore compete with the diffusion strength, which increases like the inverse square of the swarm width.

Of the two possible target gases, we select helium since the μ^+-He and muonium-He cross sections are sufficiently well known to allow reliable modeling of the method. Momentum transfer cross sections in our energy range can be deduced by applying the classical trajectory approximation to the collision process with helium atoms. At any energy ϵ, the cross sections $Q(\epsilon)$ are mass-independent. In the case of the muon, we use the calculated values for protons [6], and for muonium, we deduce them from the known H-He interaction potential [7]. Use of momentum transfer cross sections means averaging over the angular dependence. This is justified since many collisions are normally needed to change the muon energy appreciably. The charge exchange cross sections are deduced from those of protons [8] by using standard velocity scaling [5].

We apply the method to existing surface muon beams (momentum 28 Mev/c, intensity up to 10^6 μ^+/s for pulsed beams or up to 10^9 μ^+/s for dc beams). The long gas target is placed on the axis of a 5 T solenoid after a degrader on which the muon beam is focused. Most muons remain contained within the 2 cm wide target and slow down into a 14 mm wide volume. The density gradient compression between slanted walls is illustrated in Fig. 1. The swarm shrinks into a concentrated flow of 0.5 mm FWHM in an average time on the order of the muon lifetime. The drift times are shorter in the lower density region because of the increased muon kinetic energy at lower collision rates. Note, however, that the focusing strength decreases at lower densities. It comes out that the compression time λ^{-1} [Eq. (4)] is everywhere smaller than the muon lifetime.

The electron multiplication coefficient α was computed with the CERN MAGBOLTZ program [9] which takes in account all electron-He interactions. In the configuration of Fig. 1, α is near 0.4 cm^{-1} with a slight density dependence, giving an electron gain along the longest electron path $e^{\alpha l_{\max}}$ ($l_{\max} = 2.7$ cm) of 5, which is much less than the value needed to induce a Townsend discharge at low temperature. In fact, gains reaching 200 without breakdown have been achieved at 14° K and below since the feedback via helium ions, metastable helium atoms, or ultraviolet photons is strongly reduced at low temperatures [10]. The adopted value of the electric field is therefore quite conservative, and an experimental test of the present configuration may show that higher fields and smaller compression times may be achieved. Note that while the electrons reach the walls within 1 μs, the slow He^+ ions need 50 μs. The resulting space charge modifies the elec-

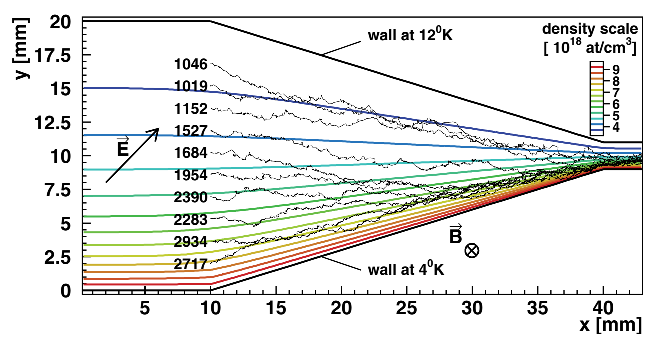

FIG. 1 (color online). Density gradient compression in helium gas for muon trajectories starting 10 mm away from the left target wall. The drift time up to the throat is given in ns near the starting point of each muon trajectory. Constant density lines are shown together with the density scale. The magnetic field is $B_z = 5$ T, the electric field $|\vec{E}| = 1800$ V/cm, and the pressure 4.6 mbar. For a 50 cm long target, 1 liter/hour liquid helium is consumed for cooling the lower wall.

tric field and has to be taken in account at intensities greater than 5×10^4 μ^+/pulse for pulsed muon beams or greater than 10^9 μ^+/s for dc beams.

At the end of the above spatial compression, the average muon kinetic energy ϵ is a few eV. It can be lowered by reducing the electric field along the swarm path. However, the Maxwell equation $\nabla \cdot \vec{E} = 0$ forces thereby the field lines to diverge, causing an expansion of the swarm flow. Nevertheless, continued density gradient compression will efficiently counter the induced expansion since the position dependent directions of flow, according to Eq. (2), will remain converging if the slowing down is done at a sufficiently slow pace. Moreover, the swarm width can even be reduced because the diffusion strength decreases at low energy. In less than a μs, ϵ falls to below 1 eV and the swarm FWHM becomes 0.25 mm. The whole operation achieves a spatial compression of 10^3.

At this level, one could attempt extraction through a thin slit. However, extraction through a hole after compression in the magnetic field direction will provide beams with optimum energy and time definition. Until now, a pure transverse electric field was considered. It is generated by thin successive longitudinal metallic stripes, deposited on the isolated target walls in the same direction as the magnetic field (z), and on which the appropriate potentials, decreasing in direction of the flow, are applied. Starting from a given x, we bend the stripes toward the flow direction x. If the bending takes place from $z = z_0$ to $z = z_{\max}$, a longitudinal potential gradient is created and an electric force directing the muons towards $z = |z_0|$ is created. Specifically, the bending can be made in such a way that except for a small region near z_0 and $z = z_{\max}$, $|E_z|$ is constant for fixed x. The compression is optimally done at very low gas densities. This is obtained by a gradual increase with z of the wall temperatures from 4 K or 12 K, respectively, to room temperature. The gas densities fall by factors between 25 and 75, and E_z/N becomes high at a quite low E_z.

How do the various ions behave in such a configuration? The electrons, which, because of their tiny cyclotron radius are almost insensitive to the transverse electric field, drift rapidly and multiply in the direction of increasing $|z - z_0|$ according to the value of E_z/N. The produced He^+ ions have large cyclotron radii and large collision energy loss. They remain at nearly thermal energies with $\nu \gg \omega$, and, according to Eq. (1), follow the total electric field whose dominating transverse component leads them to the nearby sidewalls. The μ^+ show a unique behavior. As explained by Lin *et al.* [6] (see their Eq. (10) and its consequences) and confirmed by experiments with protons [11], the μ^+-He momentum transfer cross section decreases so rapidly with energy above 1 eV that, if the accelerating field E_a (which reduces here practically to E_z) is such that $E_a/N$is greater than 36.5 Td, the slowing down cannot compete with the acceleration and *runaway* to high energy can take place. In our configuration, the muons starting at any z would then reach z_0 rapidly and, after some time, slow down near $z = z_0$. An extended distribution in z can be compressed in this way.

Large extension in z is allowed even in presence of appreciable electron multiplication, since all three feedback processes contributing to the initiation of a Townsend discharge in helium [10] are here heavily inhibited by the presence of the adjacent sidewalls toward which all He^+ ions drift, most ultraviolet photons are directed, and most metastables diffuse. The path length for multiplication is thereby strongly reduced for most secondary electrons.

After the compression in z, a parabolically increasing potential in $|z - z_0|$ remains applied on the walls in order to counter the diffusion in the z direction. At this point, the gas density is returned to a high value by decreasing the wall temperature, and the swarm is cooled under the action of the compression forces in both y (density gradient compression) and z until it reaches its minimum size at the desired final average kinetic energy as fixed by E/N. As the swarm reaches a minimum extension in both y and z, while flowing in the x direction, it enters the extraction region where the wall temperatures return to a high value (with the associated density decrease), and E_x is reduced to zero giving a nearly pure $\vec{E} \times \vec{B}$ flow in the x direction. Just before the outlet, some helium is pumped in through holes in the sidewalls in order to sustain the outflow of helium through the extraction hole. Inner gas flow that could cause forced convection effects and affect the required density distributions is thereby eliminated.

In vacuum, the gas flow from the extraction hole is pumped out in two successive regions separated by a wall with a hole (skimmer) through which the muon swarm flows. The electric potential distribution (constant in x, constant gradient in y, and parabolic in z) is extended in free space via thin wires stretched in the x direction along two planes between which the muons drift until they reach the second region, where the gas density is low enough to allow a nearly collision free final extraction in the magnetic field (z) direction. This is done periodically, e.g., every μs for a dc beam, by pulsing the wires for a short time from the parabolic potential distribution into a linearly increasing potential distribution in the z direction so that a fixed momentum is imparted to the muons in that direction. After exiting the wire region, an electrode with a high negative potential accelerates them to a higher energy bringing them rapidly outside the solenoid. The near-eV energy spread is thereby conserved and a sharp time definition is obtained.

For many applications, extraction from the embedding magnetic field is required. An efficient method using an iron grid at an adiabatically lowered magnetic field has been demonstrated for electrons [12]. Its application here would use a fine mesh grid at a field of about 0.05–0.1 T. About 50% decay and transmission losses are expected.

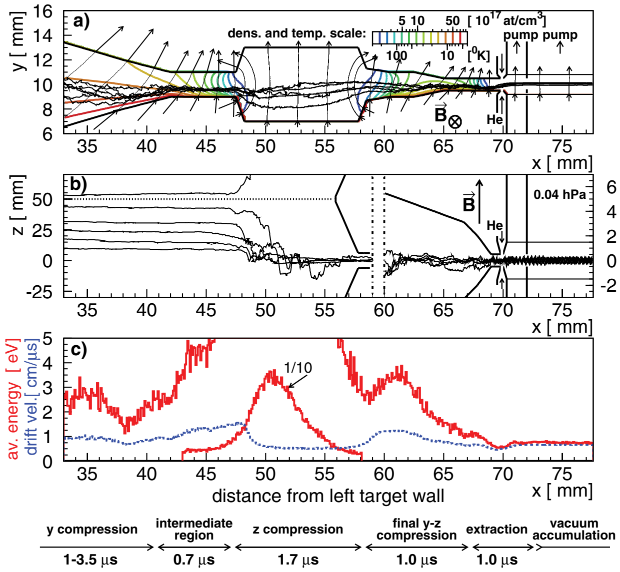

FIG. 2 (color online). Compression and extraction of the muon swarm. (a) x-y plane with constant density lines and electric field lines with field vector at their end. (b) x-z plane with a factor 10 increase in z scale on the left. (c) Average muon kinetic energy and drift velocity in x (dotted curved line). Trajectories shown start at $x = 10$ mm, $y = 17$ mm, and $z = 5$, 15, 25, 35, 45, 55, in mm. Only the end of the initial y compression region (see Fig. 1) is shown. It is followed by an intermediate region where the temperature is slowly increased to 300 K and E_z increased from zero to 66 V/cm. The low density z compression region, operated at a low transverse field of 150 V/cm, where 100 eV muon energies are reached, is followed by a cold region where the gas density is high enough to allow cooling and final compression of the muon swarm before extraction through a 1 mm diameter hole. The average delay time in each region is given at the bottom of the figure.

Simulations and optimizations of the muon drift from stopping to accumulation in high vacuum have been done. For the selected wall temperatures (constant along z) and electric potentials, the temperature, density, and electric field distributions were calculated with the ANSYS software [13]. Gas flow and density at the exit hole and in the vacuum regions were obtained via the DS2G package [14]. An arbitrarily long stopping target in the z direction made of successive z compression regions of $2z_{\max}$ length was considered. The stages following the initial transverse compression are shown in Fig. 2. The compression in z extends up to $z_{\max} = 5$ cm (10 cm distance between holes) at a density of 1.5×10^{17} at/cm^2 and $E_z/N = 54$ Td, giving a maximum electron multiplication of 15 [15] over the full 5 cm gain length of the low density channel. Sufficient wall distances and region lengths have been selected in order to insure minimum wall losses due to muon diffusion. These are 5% near the extraction holes and below 1% elsewhere besides 5% losses for muons starting near $|z| = z_{\max}$. In a more compact design allowing more wall losses, the dominating decay losses can be strongly reduced and the overall efficiency increased. Note that about 20% of the losses result from the neutralization reaction, partly during the initial transverse (y) compression, as some muonium atoms are formed at energies too low to be reionized before slowing down, and partly during z compression at low density where muonium atoms formed at high energy diffuse towards the walls.

The outcome of the operation is the conversion of an initial stopping volume of 7 mm radius and 10 cm length in the z direction with a central density of 50×10^{17} at/cm^3 into a volume 0.5 mm long in y, 2 mm long in z, and in x its length depends on the period between the final pulsed beam extraction: it is 7 mm for a 1 μs period. In the center of this volume, the gas density is 0.05×10^{17} at/cm^3 and falls rapidly in the y-z plane to 0.0001×10^{17} at/cm^3, i.e., 4×10^{-4} mbar for pumping powers of 200 1/s per hole. A 50 cm long target will stop a substantial fraction of standard surface μ^+ beams giving an efficiency greater than 10^{-3} for the whole operation or an intensity greater than 10^6 μ^+/s for the most intense available muon beams. With a final swarm average kinetic energy of 0.75 eV, the increase in phase space density relative to the initial muon beam is greater than 10^{10}. This allows, for example, a beam of 10 ± 0.5 eV energy and less than 1 cm^2 area to stop on the first layer of a target with a timing of 10 ns. At energies above 10 keV, a muon microscope with a position resolution below 10μ can be achieved.

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