Interaction-Induced Localization of Anomalously Diffracting Nonlinear Waves

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We study experimentally the interactions between normal solitons and tilted beams in glass waveguide arrays. We find that as a tilted beam, traversing away from a normally propagating soliton, coincides with the self-defocusing regime of the array, it can be refocused and routed back into any of the intermediate sites due to the interaction, as a function of the initial phase difference. Numerically, distinct parameter regimes exhibiting this behavior of the interaction are identified.

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Solitary waves, in general, and optical solitons, in particular, are the most important manifestations of the interplay between nonlinear self-focusing and diffraction or dispersion, which are inescapable characteristics of linear wave-packet propagation. Periodic structures, such as weakly coupled waveguide arrays (WGAs), introduce two new important features to this problem. First, they support discrete solitons (DSs), which are robust against collapse with increasing input power and propagation length [1,2]. Second, they allow control over the diffraction properties of tilted beams, giving rise to regions of normal, zero, or *anomalous* diffraction for linear excitations [3] and, accordingly, regions of self-focusing (SF) or self-defocusing (SDF) for nonlinear excitations [4]. These controllable characteristics not only make such structures favorable candidates for applications in optical switching, they also make optical WGAs a model for a diverse array of classical and quantum systems that have nontrivial dispersion properties and support localized nonlinear modes, from Bose-Einstein condensates in periodic optical lattices [5,6] to localized lattice vibrations [7] and nonlinear excitations in arrays of mesoscopic-mechanical cantilevers [8].

Recently, there has been a growing interest in interactions between copropagating beams in these WGAs [9,10], including interactions between spatially separate solitons [11–13] as well as between copropagating solitons and linear waves [14–18]. In homogeneous Kerr media, mutually coherent in-phase and out-of-phase solitons attract or repel each other, respectively [11]. It was found theoretically [9] and experimentally [14,15] that the collision between a DS and a weak nondiffracting signal beam can lead to dragging of the blocker soliton towards the weak signal beam. In Ref. [12], Meier *et al.* have experimentally studied the interaction between two narrow collinear DSs in Kerr-nonlinear WGAs fabricated of AlGaAs. It was found that a pair of DSs, separated by 3 sites, is stable (i.e., noninteracting) when the relative phase difference between them is around π and unstable (i.e., attractive) when it is close to zero, in accordance with theoretical predictions [9,10,16]. However, interactions involving *tilted* beams, at different diffraction regimes, have never been studied experimentally.

In this Letter, we report results for the interaction between a normally injected soliton (NS) and a nonlinear tilted beam (TB) that propagates *away* from the NS. We find that the characteristics of the interaction are considerably different in the SF and SDF regimes of TB propagation. Specifically, in the SDF regime, we demonstrate for the first time that it is possible to *restore* self-focusing to the TB, and to redirect it into any of the intermediate sites, by varying the relative phase difference.

We use a WGA fabricated of silica glass, as in Ref. [19], with the periodicity $d = 12$ μm and coupling constant $C = 0.23$ mm^{-1}. The sample is 13 mm long, corresponding to 1.8 coupling lengths. As the Kerr coefficient in glass is ~500 times smaller than in AlGaAs [11], shorter pulses with higher peak powers are required to generate nonlinear effects. Importantly, the lowest-order nonlinear loss mechanism in glass is six-photon absorption, which results in an extremely low loss rate. DSs in glass WGAs, including TBs, are therefore sustained at high powers without breakup [19], allowing a broader range of powers and tilts to be explored. Glass is also characterized by strong stimulated Raman scattering, which expresses itself as a self-frequency shift of the soliton's spectrum to longer wavelengths [18]. The formation of a soliton is therefore accompanied by a change of the focusing plane, due to the chromatic aberration of the output microscope objective lens. This sudden change of focus enables us to distinguish between strong solitary features and the linear background without explicit spectral measurements, as demonstrated in Fig. 1.

The experimental setup for the 2-beam experiment is sketched in Fig. 2. A train of 70-fs pulses at $\lambda_0 = 1.52$ μm, with ≤ 20 MW peak power, was split into two identical beams. The normally injected beam forming the NS was given an adjustable phase delay $\Delta\theta$ using a variable delay stage and shaped using a cylindrical lens. The second beam passed through a rotating cylindrical lens that

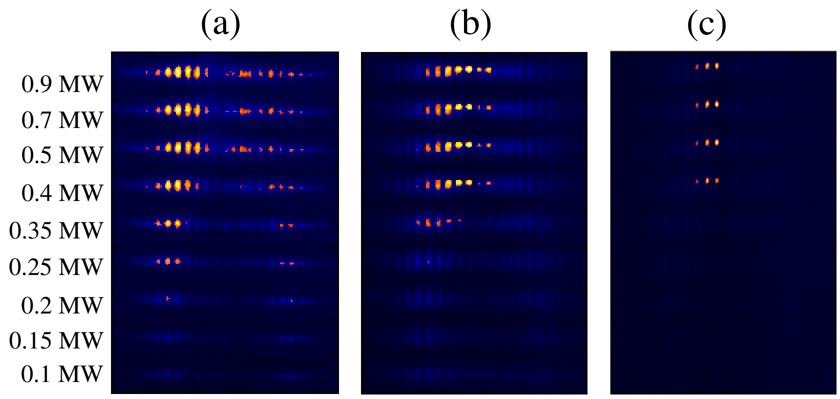

FIG. 1 (color online). Formation of a single normally injected soliton in the glass waveguide array, for a 40 μm wide input beam. The three panels correspond to different imaging conditions, and in each one the input power increases from bottom to top. (a) shows images of the output facet of the sample with the imaging lens adjusted for the laser wavelength. At low powers, the discrete-diffraction pattern is observed; at high powers a soliton forms but is out of focus. (b) The soliton becomes visible at a different focal plane, corresponding to the Raman-shifted wavelength, but its image is saturated. (c) The same image as in (b), with a neutral density filter of $\times 100$ attenuation, showing that the soliton is much stronger than the background. Setting (c) is used to study soliton interactions.

moved the beam on the aspherical coupling lens and, thus, changed its angle of incidence α at the input facet of the sample. This angle determines the transverse wave number K_x and, thus, the phase difference between adjacent sites in the TB excitation. Both beams were coupled into the sample through the same aspherical lens, which has a large numerical aperture (0.68 NA) and is optimized for the operating wavelength. A unique feature of this setup is the large clear aperture of the coupling lens, which, in combination with its small focal length, results in a wide range of accessible incidence angles for the tilted beam while maintaining paraxiality conditions and good coupling efficiency. The edge of the first Brillouin zone for this sample is attained at an incidence angle of $\alpha = \arcsin(\lambda_0/2d) = 3.6°$. In order to have sufficient resolution in K_x that allows probing of the different diffraction regimes for the TB, the lateral width of the input TB had to be larger than ~ 30 μm. The combined input intensity profile for the two beams is shown in the inset in Fig. 2. The TB input profile did not vary under relevant tilts.

Experimental results are shown in Fig. 3. Figures 3(a) and 3(b) show the results obtained when the two beams are at normal incidence. At low input powers [Fig. 3(a)], linear interference fringes are observed in the superposition of two discrete-diffraction patterns. It was found that the slope of the fringes exactly coincided with the phase slope imposed by the translation motor. At high input powers [Fig. 3(b)], the anticipated nonlinear instability around $\Delta\theta = 2\pi$, which partially diverts the beams into intermediate sites, is observed [12].

As tilts are imposed on the right DS [Fig. 3(c)], it is routed aside. As expected, the sideways shift of the beam increases in the SF regime and decreases again in the SDF regime, where the beam becomes delocalized [the data in Fig. 3(c) were obtained with a very large time delay between the pulses, which inhibits the interaction]. Figures 3(d) and 3(e) show examples of interacting NS and TB in the SF and SDF regimes of the TB, respectively. For small tilts that correspond to a TB in the SF regime [Fig. 3(d)], the observed attractive instability near 2π phase multiples is less pronounced compared to the collinear excitation [Fig. 3(b)]. This may be expected due to the decreasing overlap of the two fields as the TB is routed aside. However, when the tilt angle is in the SDF regime of the TB [Fig. 3(e)], the instability (strong attraction between the beams) is extended for phase differences far beyond a vicinity of 2π. Remarkably, the TB can be routed as a localized DS into any of the intermediate sites, with the

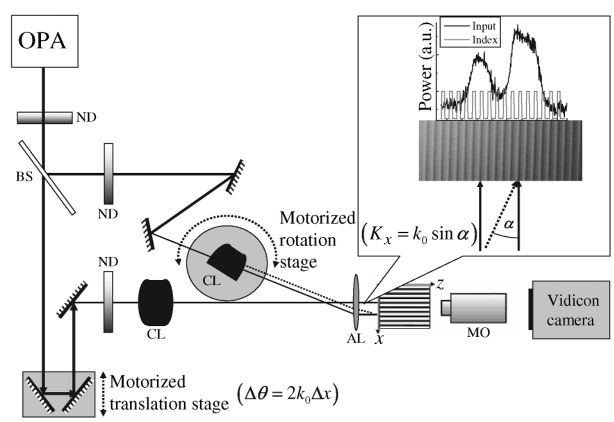

FIG. 2. Experimental setup. OPA, optical parametric amplifier; BS, beam splitter; ND, neutral density filter; CL, cylindrical lens; AL, aspherical lens; MO, microscope objective. Inset: Intensity profile of the input beams. The normal beam excites $\simeq$ 3–4 sites, while the tilted beam excites $\simeq$ 4–5 sites. The excitation parameters $\Delta\theta$ and K_x, described in the text, are determined from applied translations and rotations, respectively, using the free-space wave number $k_0 = 2\pi/\lambda_0$.

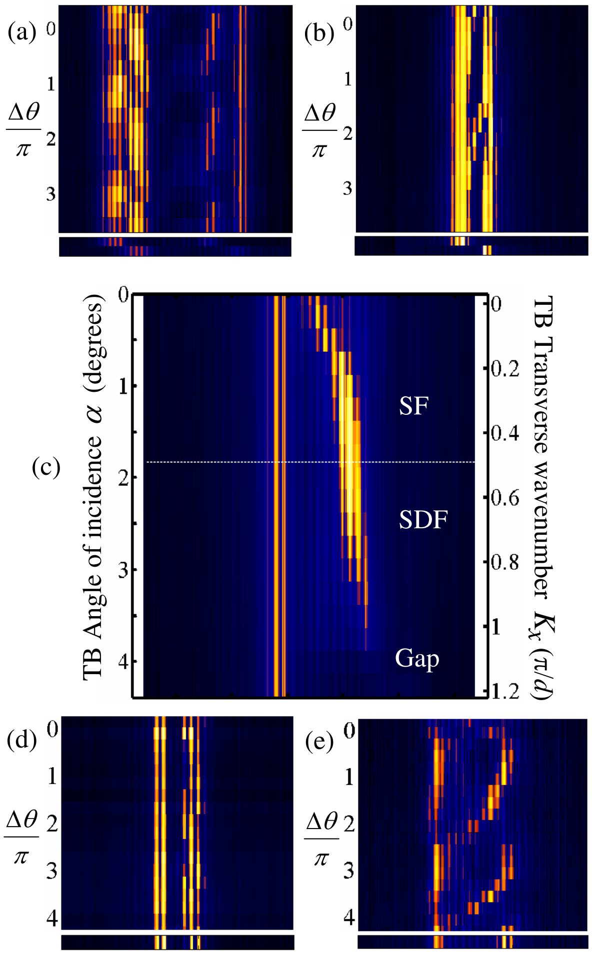

FIG. 3 (color online). (a),(b) Output profiles as a function of the phase difference, with both beams at normal incidence. The input powers are (a) 0.15 and (b) 0.7 MW (linear and soliton regimes, respectively). (c) The output profiles of *noninteracting* 0.7 MW beams as a function of the TB tilt (data obtained with a very large time delay between the pulses). (d),(e) Interactions of 0.7 MW beams as a function of the phase difference for the tilt angles of (d) 0.6° ($K_x = 0.17\pi/d$) and (e) 3° ($K_x = 0.84\pi/d$). Sections beneath the white line at the bottom of (a), (b), (d), and (e) show the output in the absence of the interaction.

appropriate phase difference, while the NS is partially annihilated. Evidently, the WGA favors interaction when one of the beams propagates in the SF regime and the other one in the SDF regime, as demonstrated by the extended range of phases exhibiting interaction dynamics.

In order to gain better understanding of the interaction dynamics, we solved the one-dimensional discrete nonlinear Schrödinger equation in dimensionless form [1,20],

$$i\frac{d\Phi_n}{dz} + h^{-2}(\Phi_{n+1} + \Phi_{n-1} - 2\Phi_n) + |\Phi_n|^2\Phi_n = 0, \quad (1)$$

on a grid of 101 sites; $h^{-2} = Z_{\rm NL}C$, the ratio between the characteristic nonlinear length $Z_{\rm NL}$ and the diffraction length $Z_D = 1/C$, is a single parameter characterizing the sample. The input is taken as two Gaussians centered at waveguide numbers $n_{c1} = 51$ and $n_{c2} = 55$:

$$\Phi_n(0) = \frac{A_{\rm NS}}{\sqrt{P^*}} \exp\left(-\frac{(n-n_{c1})^2}{(w_1)^2}\right) + \frac{A_{\rm TB}}{\sqrt{P^*}} \times \exp\left(-\frac{(n-n_{c2})^2}{(w_2)^2} + iK_x n + i\Delta\theta\right), \quad (2)$$

where P^* is the characteristic soliton power that defines $Z_{\rm NL}$ [20]. We took $P^* = 0.5$ MW. This value and the other experimental parameters (e.g., the diffraction length, mode area, and Kerr coefficient) yield $h^{-2} = 4.9 \times 10^{-3}$. The input widths $w_1 = 2.5$ and $w_2 = 3.5$ correspond to the experimental conditions. Finally, in these units, the first band of linear waves corresponds to $0 < K_x < \pi$.

We have found that, to reproduce the propagation dynamics observed in the experiment, two conditions must hold regarding the excitation amplitudes. First, for the above input widths, the amplitude of the beams must be $45 < A_{\rm TB} < 60$ and $40 < A_{\rm NS} < 90$. In this range of parameters, the diffraction and the nonlinearity are balanced. For higher TB amplitudes, the steering to nearby sites in the SF region (with $A_{\rm NS} = 0$) is suppressed, and the soliton instead becomes trapped in one waveguide [10,21]. Simultaneously, it becomes unstable under small variations of the tilt. The second condition pertains to the relative amplitudes of the NS and the TB. We have found that $A_{\rm NS}$ must be comparable to $A_{\rm TB}$ when the two excitations are sent together with the initial condition (2). Otherwise, the TB vanishes during the propagation, with only a small effect on the NS. In addition to the above constraints on the excitation amplitudes, we have also found that the balance between the strength of diffraction and nonlinearity must be in favor of the nonlinearity, in the form of a small h^{-2} parameter, to avoid breakup of the TB into filaments due to modulational instability [22]. Indeed, all of the above conditions were fulfilled in our experiment.

Results of the numerical simulations are shown in Fig. 4, for $A_{\rm NS} = 48$ and $A_{\rm TB} = 40$. Figure 4(a) shows the amplitude profile of the input beam and the output intensity distribution for individual (noninteracting) beams, as a function of the TB tilt. Figures 4(b) and 4(c) show the output intensity distribution as a function of the initial phase difference, for a TB that is SF and SDF, respectively. The calculated results are in good qualitative agreement with the experimental data displayed in Fig. 3. In particular, Fig. 4(c) indeed shows routing of a relocalized TB to nearby sites, which changes gradually over the entire range of the phase difference $0 < \Delta\theta < 2\pi$. The propagation maps as a function of z [Figs. 4(d)–4(i)] demonstrate how the NS is rephasing the otherwise defocusing TB to form a localized soliton. The initial rephasing generates a DS around one of the intermediate sites, which then periodically hops outwards. While the initial site in which the DS forms is a function of $\Delta\theta$, the hopping period is always

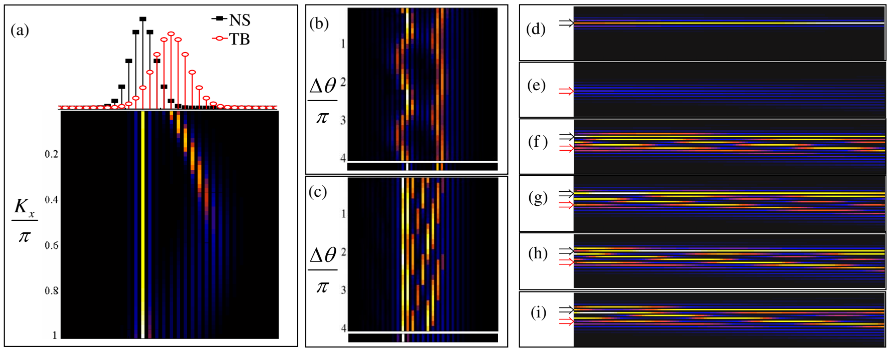

FIG. 4 (color online). Results of numerical simulations. (a) Top: The amplitude distribution in the input $|\Phi_n(0)|$. Bottom: Output intensity distributions as a function of the tilt K_x for noninteracting beams (i.e., superposition of individual solutions). (b),(c) Output intensity distribution as a function of the phase difference for: (b) $K_x = 0.24\pi$ and (c) $K_x = 0.9\pi$. (d)–(i) Propagation maps of the intensity distribution as a function of z for $0 < z < 13$ mm and $K_x = 0.9\pi$: (d) NS without TB; (e) TB without NS; (f)–(i) both beams present, with the relative initial phase differences: (f) 0, (g) $\pi/2$, (h) π, and (i) $3\pi/2$.

the same for a given K_x, corresponding to a constant propagation angle. The good agreement with the experimental data suggests that the spectral and temporal dynamics are not crucial for the understanding of the observed interactions.

In conclusion, we have studied experimentally the interactions among two noncollinear discrete solitons in glass waveguide arrays. We have demonstrated for the first time that a TB launched in a self-defocusing direction, while deviating away from its normally propagating counterpart (NS), can be relocalized and routed into any intermediate site by the nonlinear interaction with the NS. The routing is controlled by the initial phase difference between the solitons. In simulations, this behavior was reproduced under several conditions, in which (i) the nonlinear length is considerably smaller than the diffraction length, (ii) the NS and TB have comparable input amplitudes, and (iii) the TB input amplitude is moderate, to enable soliton steering and to avoid its trapping and instability. When two on-site excitations exist simultaneously, $\Phi_n = A + B$, the nonlinear Kerr term in the coupled equations (1) acts as a nontrivial interference term $|\Phi_n|^2\Phi_n = (|A|^2 + |B|^2 + 2|A||B|\sin\Delta\theta)(A + B)$, and a new propagation mode that is in balance with the weak intersite coupling is obtained. Under the above conditions for the initial collective excitation, this mode favors localization of the initially SDF TB, which can indeed be controlled with the initial phase difference as a single parameter. The result is a unique recapturing of a defocusing beam into a soliton, which should have analogs in many other physical systems that have nontrivial dispersion properties and support localized nonlinear modes. In addition, the interaction that we observe has potential applications in all-optical switching devices.

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