## Astrophysical Reaction Rate for the Neutron-Generator Reaction  $^{13}{\rm C}(\alpha,n)^{16}{\rm O}$ **in Asymptotic Giant Branch Stars**

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The reaction  ${}^{13}C(\alpha, n)$  is considered to be the main source of neutrons for the *s* process in asymptotic giant branch stars. At low energies, the cross section is dominated by the  $1/2^+$  6.356 MeV subthreshold resonance in <sup>17</sup>O whose contribution at stellar temperatures is uncertain by a factor of 10. In this work, we performed the most precise determination of the low-energy astrophysical *S* factor using the indirect asymptotic normalization (ANC) technique. The  $\alpha$ -particle ANC for the subthreshold state has been measured using the sub-Coulomb  $\alpha$ -transfer reaction (<sup>6</sup>Li, *d*). Using the determined ANC, we calculated *S*(0), which turns out to be an order of magnitude smaller than in the nuclear astrophysics compilation of reaction rates.

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About half of all elements heavier than iron are produced in a stellar environment through the *s* process, which involves a series of subsequent neutron captures and  $\beta$ decays. The reaction  ${}^{13}C(\alpha, n){}^{16}O$  is considered to be the main source of neutrons for the *s* process at low temperatures in low mass stars in the asymptotic giant branch (AGB) [\[1\]](#page-3-1). Two factors determine the efficiency of this reaction: the abundance of <sup>13</sup>C and the rate of the <sup>13</sup>C( $\alpha$ , *n*) reaction. Accurate knowledge of the <sup>13</sup>C $(\alpha, n)$ <sup>16</sup>O reaction rates at relevant temperatures  $(0.8-1.0 \times 10^8 \text{ K})$  would eliminate an essential uncertainty regarding the overall neutron balance and will allow for better tests of modern stellar models with respect to  ${}^{13}$ C production in AGB stars (see [\[2\]](#page-3-2), and references therein).

The rate of the <sup>13</sup>C( $\alpha$ , *n*) reaction at temperatures of  $\sim$ 10<sup>8</sup> K is uncertain by  $\sim$ [3](#page-3-3)00% [3] due to the prohibitively small reaction cross section at energies below 300 keV. A directly measured  ${}^{13}C(\alpha, n)$  cross section is available only at energies above 279 keV (see [[3\]](#page-3-3), and references therein). Below this energy, the cross section has to be extrapolated. It was shown  $[3,4]$  $[3,4]$  $[3,4]$  that this extrapolation can be strongly affected by the  $1/2^+$  subthreshold resonance in <sup>17</sup>O at 6.356 MeV excitation energy, which is just 3 keV below the  $\alpha$  threshold. It was assumed in the recent nuclear astrophysics compilation of reaction rates (NACRE) [\[3\]](#page-3-3) that this resonance has a well developed  $\alpha$  cluster structure. This assumption leads to a strong enhancement of the cross section at low energies [\[3\]](#page-3-3). Recently, Kubono *et al.* [\[5\]](#page-3-5) determined the contribution of the subthreshold state at 6.356 MeV in  $^{17}$ O to the astrophysical factor for the  ${}^{13}C(\alpha, n)$  reaction at low energies by measuring the  $\alpha$ -particle spectroscopic factor of this state by the  $\alpha$ -transfer reaction <sup>13</sup>C(<sup>6</sup>Li, *d*) at 60 MeV. The extracted spectroscopic factor was found to be very small,  $S_\alpha \approx$  0*:*011 [\[5\]](#page-3-5), making the influence of this subthreshold state on the astrophysical factor negligible. However, it was shown in Ref. [\[6](#page-3-6)] that the same experimental data was compatible with a large  $S_\alpha$  factor for the subthreshold state in question. It is the main goal of this work to resolve this difference and to develop a technique which determines the contribution of the subthreshold resonances to the  $(\alpha, n)$ reaction cross sections using a model-independent approach. Until now, the asymptotic normalization (ANC) method has been applied to determine the astrophysical factors for radiative capture processes  $[7-9]$  $[7-9]$  $[7-9]$  $[7-9]$ . Here we present the first case of the application of the ANC method to determine the astrophysical factor for the  ${}^{13}C(\alpha, n){}^{16}O$ reaction.

The amplitude of the reaction  $x + A \rightarrow b + B$  proceeding through the subthreshold resonance  $F$  is given in the *R*-matrix approach by [\[10\]](#page-3-9)

<span id="page-0-0"></span>
$$
M = \frac{1}{2\sqrt{k_{xA}k_{bB}}}\sqrt{\frac{P_l(k_{xA}, r_0)}{\mu_{xA}r_0}}\tilde{W}_{-\eta, l+1/2}(2\kappa r_0)
$$

$$
\times \frac{\tilde{C}_{xA}^F \Gamma_{bB}^{1/2}(E_{bB}, r_0)}{E_{xA} + \varepsilon + i\Gamma_F(E_{bB}, r_0)/2} \exp(i\delta), \qquad (1)
$$

where  $P_l(k_{xA}, r_0)$  is the Coulomb-centrifugal barrier penetration factor in the entrance channel,  $\tilde{W}_{-\eta, l+1/2}(2\kappa r_0)$  =  $W_{-\eta, l+1/2}(2\kappa r_0)\Gamma(l+1+\eta)$  is the Coulomb-modified Whittaker function,  $r_0$  is the channel radius,  $\tilde{C}_{xA}^F =$  $C_{xA}^F/\Gamma(l + 1 + \eta)$  stands for the Coulomb-modified ANC for the virtual decay (synthesis)  $F \leftrightarrow x + A$ ,  $\eta$  and *l* are the Coulomb parameter and relative orbital angular momentum of the subthreshold bound state (*xA*), respectively, and  $\Gamma_{bB}(E_{bB}, r_0)$  is the resonance width for the decay to the final channel  $b + B$ . The total width of the resonance *F* is

equal to  $\Gamma_F$ ,  $E_{ij} = k_{ij}^2/(2\mu_{ij})$  is the relative kinetic energy of particles *i* and *j*,  $\kappa = \sqrt{2\mu_{xA}\epsilon}$ , and  $\epsilon$  is the binding energy for the virtual decay  $F \rightarrow x + A$ . In this case,  $\Gamma_{bB}$   $\equiv \Gamma_F \equiv \Gamma_n = 124 \pm 12$  keV [[11](#page-3-10)] is a known neutron partial width. The total cross section is then calculated as  $\sigma = 4\pi g_J |M|^2 k_{b} / k_{xA}$ , where  $g_J = (2J + 1)/[(2S_x +$  $1(2S_A + 1)$ . Thus, the ANC is the only missing quantity needed to calculate the cross section for the <sup>13</sup>C( $\alpha$ , n)<sup>16</sup>O reaction proceeding through the subthreshold state. Because of the peripherality of the sub-Coulomb transfer reactions, the overall normalization of the  $\alpha$ -transfer reaction cross section is determined by the product of the squares of the initial and final ANCs rather than the spectroscopic factors. The initial ANC for the  $\alpha + d \rightarrow {}^{6}Li$  is known,  $(C_{\alpha d}^{\text{6Li}})^2 = 5.3 \pm 0.5 \text{ fm}^{-1}$  [\[12\]](#page-3-11). Hence, by normalizing the distorted-wave Born approximation (DWBA) cross section to the experimental one, we can determine the ANC for  $\alpha + {}^{13}C \rightarrow {}^{17}O$  (6.356 MeV, 1/2<sup>+</sup>).

In this Letter, we report the application of the ANC technique to determine the astrophysical *S* factor of the  ${}^{13}C(\alpha, n)$ <sup>16</sup>O reaction at astrophysically relevant energies by measuring the ANC for the virtual synthesis  $\alpha + {}^{13}C \rightarrow$ <sup>17</sup>O (6.356 MeV,  $1/2^+$ ) using the  $\alpha$ -transfer reaction  ${}^{6}$ Li( ${}^{13}$ C, *d*), performed at two sub-Coulomb energies, 8.0 and 8.5 MeV, of  $^{13}$ C at the Florida State University Tandem-LINAC facility. The choice of inverse kinematics,  $13^{\circ}$ C beam and <sup>6</sup>Li target, allowed measurements to be made at very low energies in the c.m. and to avoid background associated with the admixture of  ${}^{12}C$  in the  ${}^{13}C$  target.

Angular distributions of the deuterons from transfer reactions at sub-Coulomb energies in inverse kinematics peak at forward angles. Four Si  $\Delta E - E$  telescopes were positioned at forward angles to identify deuterons. Thicknesses of the  $\Delta E$  detectors ranged from 15 to 25  $\mu$ m. 50  $\mu$ g/cm<sup>2</sup> Li targets (enriched to 98% of <sup>6</sup>Li) were prepared and transported into a scattering chamber under vacuum to prevent oxidation. The telescope at the smallest angle  $(6^{\circ}$  in the laboratory frame) was shielded from the Rutherford scattering of  $^{13}$ C on Li target with a 5  $\mu$ m Havar foil. A spectrum of deuterons at  $6^{\circ}$  (169 $^{\circ}$  in the c.m. for the  $1/2^{+}$  6.356 MeV state) at a beam energy 8.5 MeV is shown in Fig. [1.](#page-1-0) The typical experimental resolution in the c.m. system was about 250 keV (FWHM). The angular distributions of  ${}^{6}$ Li(<sup>13</sup>C, d)<sup>17</sup>O(1/2<sup>+</sup>, 6.356 MeV) are shown in Fig. [2.](#page-1-1) Their shape is typical for sub-Coulomb transfer reactions. The absolute normalization of the cross section was performed by measuring the elastic scattering of 6.868 MeV protons on the 6Li target. The cross section of this reaction at 95 is known with 3% accuracy [\[13\]](#page-3-12). Each telescope was sequentially placed at 95 , and the product of the target thickness times the telescope solid angle was determined for each telescope.

The code FRESCO (version FRXY.3h) [[14](#page-3-13)] was used to calculate the angular distribution of the  ${}^{13}C({}^{6}Li, d)$  reaction in the DWBA approach. The DWBA calculations were

<span id="page-1-0"></span>

FIG. 1. Spectrum of deuterons from the  ${}^{6}Li(^{13}C, d)$  reaction, at 6 with the 8.5 MeV 13C beam. The inset shows the level scheme of  $17O$  [[11](#page-3-10)]. The solid line is a Gaussian fit.

performed for beam energies at the center of the target. It was found that the normalization factor is the same for both energies. The maximum contribution of the CN mechanism can be estimated using the measured cross section for the population of the first  $1/2^+$  state of <sup>17</sup>O at 0.87 MeV (see Fig. [1](#page-1-0)), which is 24  $\mu$ b/sr at 8.31 MeV at 170<sup>°</sup> in c.m. We will assume that all of it is due to the CN mechanism. According to the Hauser-Feshbach statistical model, it is 6 times less probable to populate the  $1/2^+$  state at 6.356 MeV than the one at 0.87 MeV. Thus, the maximum

<span id="page-1-1"></span>

FIG. 2. Angular distribution of the <sup>13</sup>C(<sup>6</sup>Li,  $d$ )<sup>17</sup>O(1/2<sup>+</sup>) reaction. Data taken at 8.5 MeV of 13C are shown as diamonds and 8.0 MeV data as boxes. The dashed and solid lines are DWBA calculations at 13C energies 8.31 and 7.81 MeV, respectively (see text).

<span id="page-2-0"></span>TABLE I. Parameters of optical potentials used in DWBA calculations.  $R = r_0 A_T^{1/3}$ ;  $r_0 = 1.25$  fm and  $a = 0.68$  fm were used for  $\alpha + d$  and  $\alpha + ^{13}C$  form factor potentials with *V* fitted to reproduce binding energy.

Channel	Potential	$V_{0}$ (MeV)	$a_V$ (fm)	$r_V$ (fm)	W (MeV)	$W_{S}$ (MeV)	$a_W$ (fm)	$r_W$ (fm)	$r_{c}$ (fm)	$V_{so}$ (MeV)	$a_{so}$ (fm)	$r_{so}$ (fm)	Ref.
${}^{6}Li + {}^{13}C$	LC <sub>1</sub>	134.0	0.68	1.50	$\ldots$	11.1	0.68	1.50	1.50	$\ldots$ .	$\cdots$	$\ldots$	$\lceil 15 \rceil$
$d + {}^{17}O$	DO <sub>1</sub>	105.0	0.86	1.02	$\ldots$	15.0	0.65	1.42	1.40	6.0	0.86	1.02	$[16]$
$d + {}^{13}C$	DC <sub>1</sub>	79.5	0.80	1.25	10.0	$\cdots$	0.80	1.25	1.25	6.0	0.80	1.25	$\lceil 17 \rceil$

possible cross section due to the CN mechanism for the 6.356 MeV state is 6% of the measured 70  $\mu$ b/sr.

The extracted ANC, unlike the spectroscopic factor, does not depend on the number of nodes of the  $\alpha$  - $^{13}C_{1/2}$  bound state wave function or the geometrical parameters of the Woods-Saxon potential. Parameters of the optical model potentials of the usual Woods-Saxon form used in the DWBA calculations are given in Table [I](#page-2-0). The LC1 potential was used for the  ${}^{6}Li + {}^{13}C$ channel. It reproduces the experimental data of the elastic scattering of  ${}^{6}$ Li by  ${}^{13}$ C at energies ranging from 3 to 23 MeV in c.m. Experimental data of elastic scattering of deuterons on  $^{17}O$  at low energies are not available. Thus, several potentials for the  $d + {}^{17}O$  channel were used [\[16](#page-3-14)[,18\]](#page-3-15). The angular distributions shown in Fig. [2](#page-1-1) were calculated with the DO1 potential; however, it was verified that other potentials [[16](#page-3-14),[18](#page-3-15)] produce essentially identical results, with variations in the normalization factor of less than 7%. This demonstrates that the transfer reaction cross section at sub-Coulomb energies only weakly depends on the parameters of the optical potentials, which was the main point of making this measurement. In fact, a calculation with no nuclear part of the optical potentials changes the absolute value of the cross section at large angles by only  $\sim$ 40%. Further investigation of the sensitivity of the cross section to the parameters of the optical potentials was performed and was found to be less than 20% if the parameters were kept within reasonable limits. We found no sensitivity of the extracted ANC to the parameters of the core-core DC1 interaction potential in the full DWBA transition operator.

Our determined Coulomb-modified ANC squared for  $^{13}C + \alpha \rightarrow ^{17}O(1/2^+, 6.356 \text{ MeV})$  is  $(\tilde{C}_{\alpha^{13}C}^{^{17}O(1/2^+)})^2 =$  $0.89 \pm 0.23$  fm<sup>-1</sup>. The contribution of the  $1/2^+$  state to the astrophysical *S* factor calculated using Eq.  $(1)$  $(1)$  is shown as a dashed curve in Fig. [3.](#page-2-1) It was verified that this result is insensitive to variations of the channel radius.

Six sources of uncertainty associated with the  $S(0)$  factor of the 6.356 MeV  $1/2^+$  state can be identified: statistical 7%, combined systematical 7% (in determination of the target thickness times solid angle), 20% uncertainty associated with the theoretical analysis, 10% uncertainty in the total resonance width, 10% due to the initial  $\alpha - d$  ANC. and 4% due to 8 keV uncertainty in the excitation energy of the  $1/2^+$  resonance. Thus, the  $S(0)$  value of the 6.356 MeV  $1/2^+$  state in <sup>17</sup>O determined in this experiment is  $(2.5 \pm 1)$ 

 $(0.7) \times 10^6$  MeV  $\times b$ , which is 10 times smaller than that adopted in the NACRE compilation [\[3\]](#page-3-3) and a factor of 5 larger than in Ref. [[5](#page-3-5)].

The two channel *R*-matrix approach was used to calculate the total *S* factor of the <sup>13</sup>C( $\alpha$ , *n*) process. All resonances in 17O from an excitation energy of 4.14 (neutron decay threshold) to 8.2 MeV were included. The parameters of the resonances were taken from Ref. [\[11\]](#page-3-10). The  $\alpha$ -reduced widths of the resonances were fitted to reproduce the <sup>13</sup>C( $\alpha$ , *n*) experimental data [\[4\]](#page-3-4). It was found that it is necessary to introduce a constant nonresonance contribution of  $0.4 \pm 0.2 \times 10^6$  MeV  $\times b$  to fit the experimental *S* factor data in the lowest energy region. This contribution is due to the direct <sup>13</sup>C( $\alpha$ , *n*)<sup>16</sup>O reaction, with the dominant partial wave  $1/2^-$  ( $\ell = 0$  in  $\alpha + {}^{13}C$ 

<span id="page-2-1"></span>

FIG. 3. *S* factor of the <sup>13</sup>C( $\alpha$ , *n*) reaction. The experimental data, corrected for electron screening, are from Refs. [[4,](#page-3-4)[19](#page-3-18)]. The best fit, obtained with a  $0.4 \times 10^6$  MeV  $\times b$  nonresonance contribution, is shown as the solid curve. The contribution of the  $1/2$ <sup>+</sup> state is shown as the dashed curve. The dashed-dotted curves represent a 28% uncertainty band due to the uncertainty in the  $1/2$ <sup>+</sup> S(0) factor. The dotted curves were calculated using a 0.2 and  $0.6 \times 10^6$  MeV  $\times b$  nonresonance contribution with  $1/2$ <sup>+</sup> S(0) factor of 1.8 and 3.2  $\times$  10<sup>6</sup> MeV  $\times$  *b*, respectively, and represent the overall uncertainty band.

<span id="page-3-19"></span>

FIG. 4 (color online). Rate of the <sup>13</sup>C( $\alpha$ , *n*) reaction. The dashed-dotted curve is from Ref. [[3\]](#page-3-3); the solid curve is the rate obtained in this work. The yellow (light) band in the inset represents uncertainties from Ref. [\[3\]](#page-3-3); new uncertainties are shown in red (dark).

channel); thus, possible interference with the  $1/2^+$  resonance is small and was not taken into account.

Our calculated reaction rates are shown in Fig. [4.](#page-3-19) The best fit rate is shown as a solid curve in Fig. [4](#page-3-19) in comparison with the NACRE adopted rate (dashed-dotted curve). At temperatures above 0.3 GK, where the contribution of  $1/2$ <sup>+</sup> is small, it is identical to the curve adopted in NACRE; however, at temperatures which are most significant for the *s* process in AGB stars, 0.08–0.1 GK, the reaction rate is smaller by a factor of 3 than that adopted in the NACRE compilation (still inside the NACRE uncertainty band). The uncertainty in this astrophysically important reaction rate is now reduced to 25%. The inset in Fig. [4](#page-3-19) shows the NACRE compilation adopted rate (dashed-dotted curve) with a yellow (light) uncertainty band and the reaction rate obtained in this work (solid line) with an uncertainty band shown in red (dark). Numerical values of the reaction rate for the 0.08– 0.1 GK temperature range are given in Table [II](#page-3-20).

In summary, in this Letter we described an indirect technique which allows for the measurement of the astrophysical  $S(0)$  factor of subthreshold, particle unbound resonances and applied this technique to measure the contribution of the  $1/2$ <sup>+</sup> 6.356 MeV resonance in <sup>17</sup>O to the  ${}^{13}C(\alpha, n)$  reaction rate at stellar temperatures. The combination of the sub-Coulomb  $\alpha$ -transfer reaction and the application of the ANC technique in the analysis of experimental data practically eliminates all dependence of the results on model parameters, making this approach a valuable tool for future studies of astrophysically impor-

<span id="page-3-20"></span>TABLE II. The rate of <sup>13</sup>C( $\alpha$ , *n*) reaction at temperatures from 0.08 to 0.1 GK. The rate obtained in this work is compared with the rate published in the NACRE [[3](#page-3-3)] compilation. Units are  $[cm<sup>3</sup> mol<sup>-1</sup> s<sup>-1</sup>]$ ; exp stands for  $10<sup>exp</sup>$ .

		This work					
$T_{\rm o}$	Low	Adopt High		Low	Adopt	High	exp
0.08	1.17	1.49	1.87	1.22	4.80	5.80	$-16$
0.09	1.90	2.41	3.01	2.03	6.99	8.45	$-15$
0.10	2.11	2.64	3.30	2.28	6.99	8.49	$-14$

tant reaction rates with both stable and radioactive beams. Verification of the results presented here using an independent experimental approach (e.g., the Trojan horse technique) is highly desirable. The  ${}^{13}C(\alpha, n)$  reaction rate at stellar temperatures was found to be lower by a factor of 3 than the previously adopted rate  $[3]$  $[3]$  $[3]$ ; also, the uncertainty in this reaction rate was greatly reduced. It would be of great interest to incorporate the new reaction rate into the *s*-process calculations in AGB stars.

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