M Theory Solution to the Hierarchy Problem

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An old idea for explaining the hierarchy is strong gauge dynamics. We show that such dynamics also stabilizes the moduli in M theory compactifications on manifolds of G_2 holonomy without fluxes. This gives stable vacua with softly broken supersymmetry, grand unification, and a distinctive spectrum of TeV and sub-TeV sparticle masses.

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Stabilizing hierarchies and moduli.—M theory (and its weakly coupled string limits) is a consistent quantum theory including gravity, particle physics, and much more. Although apparently unique, the theory has a large number of solutions, manifested by the presence of moduli: massless scalar fields with classically undetermined vacuum expectation values (VEVs), whose values determine the masses and coupling constants of the low energy physics.

In recent years, there has been substantial progress in understanding mechanisms which stabilize moduli in various corners of the M theory moduli space. In particular, the stabilization of all moduli by magnetic fields (fluxes) in the extra dimensions, perhaps also combined with other quantum effects, has been reasonably well understood in the context of type IIB string theory [1,2], M theory [3], and type IIA string theory [4]. After stabilizing the moduli, one still has to explain why $M_W/m_{\rm pl} \sim 10^{-16}$.

The effective potential of these compactifications fits into the framework of a low energy supergravity theory in four dimensions. A well known property of the latter is that there is a universal contribution to scalar masses of the order of the gravitino mass $m_{3/2}$. Therefore, without miraculous cancellations, in theories in which $m_{3/2}$ is large, the Higgs boson mass is also large. In M theory and type IIA flux vacua, the vacuum superpotential is $\mathcal{O}(1)$ or larger in Planck units. This gives a large $m_{3/2}$ (unless the volume of the extra dimensions is large, ruining standard unification). In heterotic flux vacua [5], $m_{3/2}$ can be smaller but only by a few orders of magnitude. Thus, in these vacua, stabilizing the moduli using fluxes fails to solve the hierarchy problem, viz. to generate and stabilize the hierarchy between the electroweak and Planck scales.

In type IIB theory, this is not so: $m_{3/2}$ can be tuned small by choosing fluxes. One can also address the possibility of generating the hierarchy through warping [6] in this framework [1]. The hierarchy problem is less well understood in other corners of the M theory moduli space.

Our focus will be M theory, and we will henceforth switch off all the fluxes else the hierarchy will be destroyed. Supersymmetry then implies that the seven extra dimensions form a space X with G_2 holonomy. In these vacua, non-Abelian gauge fields are localized along three dimensional submanifolds $Q \subset X$ at which there is an orbifold singularity [7], and chiral fermions are localized at points at which there are conical singularities [8–10].

These vacua can have interesting phenomenological features, independently of how moduli are stabilized: The Yukawa couplings are hierarchical; proton decay proceeds at dimension six with distinctive decays; grand unification is very natural; the μ term is zero in the high scale Lagrangian [8,11–13]. Also, since the Q's generically do not intersect each other, supersymmetry breaking will be gravity mediated in these vacua. Therefore, it is of considerable interest to understand whether or not there exist mechanisms which can (a) stabilize the moduli of such compactifications, (b) generate a hierarchy of scales, and, if so, (c) what is the resulting structure of the soft terms and their implications for LHC?

All the moduli fields s_i have axionic superpartners t_i, which, in the absence of fluxes, enjoy a Peccei-Quinn shift symmetry. This is an important difference with respect to other M theory limits such as heterotic or type IIB. Therefore, in the zero flux sector, the only contributions to the superpotential are nonperturbative. These can arise either from strong gauge dynamics or from membrane instantons. Since the theory of membrane instantons in G_2 manifolds is technically challenging [14], we will restrict our attention to the strong gauge dynamics case henceforth.

Furthermore, unlike its weakly coupled string limits, in M theory the nonperturbative superpotential, in general, depends upon all of the moduli. Hence, one would expect that the effective supergravity potential has isolated minima. Our main conclusion is that strong gauge dynamics produces an effective potential which indeed stabilizes all moduli and generates an exponential hierarchy of scales. After describing this result, we also briefly describe the pattern of soft breaking terms which these vacua predict and begin to discuss the consequences for the LHC.

The moduli potential.—The moduli Kahler potential is difficult to calculate explicitly. However, a family of Kahler potentials consistent with G_2 holonomy and known to describe accurately some explicit examples of G_2 moduli dynamics was given in Ref. [15]. These are defined by

$$K = -3\ln(4\pi^{1/3}V_X), \qquad V_X = \prod_{i=1}^{N} s_i^{a_i}, \qquad \text{with} \quad \sum_{i=1}^{N} a_i = 7/3, \tag{1}$$

where V_X is the volume of the G_2-holonomy manifold as a function of the N scalar moduli s_i (in 11D units). The superpotential for the simple case of a hidden sector without charged matter is, therefore,

$$W = \sum_{k=1}^{M} A_k e^{ib_k f_k}, \qquad f_k = \sum_{i=1}^{N} N_i^k z_i = \frac{\theta_k}{2\pi} + i\frac{4\pi}{g_k^2}. \tag{2}$$

M is the number of hidden sectors whose gauginos condense, $b_k = 2\pi/c_k$, with c_k the dual coxeter number of the kth gauge group whose $4d$ gauge coupling function f_k is an integer linear combination of the moduli fields $z_i = t_i + is_i$. The A_k are (renormalization-group-scheme-dependent) numerical constants. More general cases will be described in Ref. [16].

Note that all of the "parameters" which enter the potential, i.e., (b_k, A_k, N_i^k) are constants. b_k and N_i^k are straightforward to determine from the topology of X. The one-loop factor A_k is more difficult to obtain, but, e.g., the threshold corrections calculated in Ref. [12] show that they can be computed and can take a reasonably wide range of values in M theory.

At this point, the simplest possibility would be to consider a single hidden sector gauge group. While this does in fact stabilize all the moduli, it is (a) nongeneric and (b) fixes the moduli in a place which is strictly beyond the supergravity approximation. Therefore, we will consider two such hidden sectors, which is more representative of a typical G_2 compactification as well as being tractable enough to analyze. The superpotential, therefore, has the following form:

$$W^{\mathrm{np}} = A_1 e^{ib_1 f_1} + A_2 e^{ib_2 f_2}. \tag{3}$$

The scalar potential can be computed from K and W, and after integrating out the axions (without loss of generality, we chose $A_k > 0$), it is given by, in $4d$ Planck units,

$$V = \frac{1}{48\pi V_X^3}\Bigg[\sum_{k=1}^{2}\sum_{i=1}^{N} a_i \nu_i^k(\nu_i^k b_k + 3) b_k A_k^2 e^{-2b_k\vec{\nu}^k\cdot\vec{a}} + 3\sum_{k=1}^{2} A_k^2 e^{-2b_k\vec{\nu}^k\cdot\vec{a}} - 2\sum_{i=1}^{N} a_i \prod_{k=1} 2\nu_i^k b_k A_k e^{-b_k\vec{\nu}^k\cdot\vec{a}} - 3\Big(2 + \sum_{k=1}^{2} b_k\vec{\nu}^k\cdot\vec{a}\Big)\prod_{j=1}^{2} A_j e^{-b_j\vec{\nu}^j\cdot\vec{a}}\Bigg], \tag{4}$$

where we introduced a variable

$$\nu_i^k \equiv \frac{N_i^k s_i}{a_i} \text{ (no sum)}; \qquad \mathrm{Im} f_k = \vec{\nu}^k \cdot \vec{a}. \tag{5}$$

Vacua.—Vacua of the theory correspond to stable critical points of the potential. Although, as we will see, the potential has stable vacua with spontaneously broken supersymmetry, it is instructive to analyze the supersymmetric vacua. For simplicity, we will describe here only the special case when the two groups have the same gauge coupling (explicit examples are given later). See [16] for the more elaborate general case.

In this special case, we have

$$N_i^1 = N_i^2 = N_i \Rightarrow \nu_i^1 = \nu_i^2 = \nu_i \equiv \frac{N_i s_i}{a_i}. \tag{6}$$

As a result, the F terms [$F_i = \partial_i W + (\partial_i K)W$] simplify significantly. Solving $F_i = 0$ yields:

$$\nu_i \equiv \nu = -\frac{3(\alpha - 1)}{2(\alpha b_1 - b_2)}, \qquad \text{with} \quad \frac{A_2}{A_1} = \frac{1}{\alpha} e^{(7/2)(b_1 - b_2)((\alpha-1)/(\alpha b_1 - b_2))}, \tag{7}$$

where α is determined by the second equation in (7). Since ν_i is independent of i, it is also independent of the number of moduli N, which means that this solution fixes all moduli for a manifold with any number of moduli.

In the limit of large ν, ν is given by [for gauge groups $SU(P)$ and $SU(Q)$]

$$\nu \sim \frac{3}{7(b_2 - b_1)} \log\frac{A_2 b_2}{A_1 b_1} = \frac{3}{14\pi}\frac{PQ}{P - Q}\log\frac{A_2 P}{A_1 Q}. \tag{8}$$

This is a very good approximation for $\nu > O(1)$ and shows that the moduli VEVs can be greater than 1 for gauge group ranks less than 10, yielding solutions within the supergravity approximation. However, there will be an upper bound on the moduli VEVs in these vacua, since we expect that A_1, A_2, P, and Q have upper limits. The dependence of (8) on the input parameters is similar to those obtained for other constructions [17]. Once ν is determined, the moduli s_i are found from (6) and the hierarchy between the moduli VEVs is determined by the ratios a_i/N_i. For cases when PA_2/QA_1 is of order 1, it is not clear if additional corrections change the results significantly. Similar issues were faced in Refs. [17,18].

Minima with spontaneously broken supersymmetry.—Formally, the potential has $2^N - 1$ extrema with spontaneously broken supersymmetry and one supersymmetric one [16]. For simplicity, we will exhibit these for the two moduli case. For example, consider the parameter set

$$\{A_1, A_2, b_1, b_2, N_1, N_2, a_1, a_2\} = \Big\{0.12, 2, \frac{2\pi}{8}, \frac{2\pi}{7}, 1, 1, \frac{7}{6}, \frac{7}{6}\Big\}.$$

The solutions are

$$\begin{aligned}
&s_1^{(1)} \approx 13.05, \quad s_2^{(1)} \approx 13.05 \text{ (supersymmetric extremum)},\\
&s_2^{(2)} \approx 13.59, \quad s_2^{(2)} \approx 13.59 \text{ (de Sitter extremum)},\\
&s_1^{(3)} \approx 2.61, \quad s_2^{(3)} \approx 23.55 \text{ (nonsupersymmetric anti-de Sitter minimum)},\\
&s_1^{(4)} \approx 23.55, \quad s_2^{(4)} \approx 2.61 \text{ (nonsupersymmetric anti-de Sitter minimum)}.
\end{aligned} \tag{9}$$

The supersymmetric extremum in (9) is a saddle point. The two stable minima spontaneously break supersymmetry. Visual plots of the potential can be seen in Ref. [19]. The stable minima appear symmetrically, though generically, for $a_1 \neq a_2$ and/or $N_1 \neq N_2$; one of the minima will be deeper than the other. For the case under investigation, the volume is stabilized at the value $V_X = 122.3$, which is presumably large enough for the supergravity analysis to hold.

Explicit examples.—To prove the existence of a G_2-holonomy metric on a compact 7-manifold, X is a difficult problem. There is no analogue of Yau's theorem for Calabi-Yau manifolds which allows an "algebraic" construction. Nevertheless, Joyce and Kovalev have successfully constructed many smooth examples [20]. Furthermore, dualities with heterotic and type IIA string vacua also imply the existence of many singular examples. The vacua discussed here have two gauge groups so X will have two submanifolds Q_1 and Q_2 of orbifold singularities.

Kovalev constructs G_2 manifolds which can be described as the total space of a fibration, where the fibers are $4d$ $K3$ surfaces with orbifold singularities which vary over a 3-sphere. One then obtains G_2 manifolds with orbifold singularities along the sphere. For example, if the generic fiber has both an $SU(4)$ and an $SU(5)$ singularity, then the G_2 manifold will have two such singularities, both parametrized by disjoint copies of the sphere. In this case, N_i^1 and N_i^2 are equal because Q_1 and Q_2 are in the same homology class, which is precisely the special case that we consider above.

A similar picture arises from the dual perspective of the heterotic string on a $T3$-fibered Calabi-Yau. Then, if the hidden sector E_8 is broken by the background gauge field to, say, $SU(5) \times SU(2)$, the $K3$ fibers of the dual G_2 manifold generically have $SU(5)$ and $SU(2)$ singularities, again with $N_i^1 = N_i^2$ (or $N_i^1 = kN_i^2$, in general).

Finally, we note that Joyce's examples typically can have several sets of orbifold singularities which often fall into the special class we have considered.

Phenomenology.—As mentioned earlier, there are many local minima with spontaneously broken supersymmetry. One can study the particle physics features of these minima. For illustration, we will compute some phenomenologically relevant quantities for the minima (9):

$$\begin{aligned}
&V_0^{(3),(4)} \approx -(5.1 \times 10^{10}\ \text{GeV})^4 \text{ (cosmological constant)},\\
&m_{3/2}^{(3),(4)} = m_p e^{K/2}|W| \approx 2081 \text{ GeV (gravitino mass)},\\
&M_{11} = \frac{\sqrt{\pi} m_p}{V_X^{1/2}} \approx 3.9 \times 10^{17} \text{ GeV (11dim Planck scale)},\\
&\Lambda_g^{(1)} = m_p e^{-(b_1/3)\Sigma_i N_i s^i} \approx 2.6 \times 10^{15} \text{ GeV},\\
&\Lambda_g^{(2)} \approx 9.7 \times 10^{14} \text{ GeV (gaugino cond. scales)},
\end{aligned} \tag{10}$$

where $m_p = 2.43 \times 10^{18}$ GeV and the hidden sector strong coupling scales are defined as in Ref. [21]. Thus, standard gauge unification is naturally compatible with low scale supersymmetry in our theory. An investigation of the entire "parameter" set shows that a significant fraction of models have similar features [16]. Note that to obtain much lower mass scales requires unnaturally large rank gauge groups *and* large A_2/A_1 ratios. Presumably, the latter cannot reach, say, $\mathcal{O}(100)$, implying a lower bound on the supersymmetry breaking scale in these vacua.

A large negative V_0 is not realistic, and one might worry that the features obtained above may not survive when one obtains (or tunes) V_0 to the correct value. We argue that this is not the case. First, there may exist mechanisms in these vacua similar to the Bousso-Polchinski mechanism [22] in IIB. The M theory dual of IIB fluxes, in principle, "scan V_0," leaving a minimum very close to the one discussed here.

Furthermore, in Ref. [16] we have studied the vacuum structure with additional nonperturbative contributions and hidden sector matter, as in Ref. [23]. These can give rise to vacua with a completely different V_0, e.g., de Sitter vacua, but with essentially identical phenomenology. Moreover, if one assumes that the space of G_2 manifolds is such that one can finely scan the constants A_i, then we have checked that it is possible to scan V_0 to small values without changing the phenomenology.

Soft supersymmetry breaking parameters.—Soft supersymmetry breaking parameters (at M_{unif}) can be calculated in this framework—the gaugino masses $M_{1/2}^a$ are easier to calculate than the scalars and trilinears. The gaugino masses are given by:

$$M_{1/2} = m_p \frac{e^{K/2} K^{i\bar{j}} F_{\bar{j}} \partial_i f_{\text{sm}}}{2i\,\text{Im} f_{\text{sm}}}, \qquad f_{\text{sm}} = \sum_{i=1}^{N} N_i^{\text{sm}} z_i, \tag{11}$$

where the f_{sm} is determined by the homology class of the 3-cycle, Q_{sm}. From (11), the normalized gaugino mass in these vacua can be expressed as

$$|M_{1/2}| = \left[\frac{2(\alpha b_1 - b_2)}{3(\alpha - 1)} \frac{\sum_{i=1}^{N} N_i^{\text{sm}} s_i \nu_i}{\sum_{i=1}^{N} N_i^{\text{sm}} s_i} + 1\right] \times m_{3/2}. \tag{12}$$

At the supersymmetry extremum, using Eq. (7) for ν_i in (12), $M_{1/2}$ vanishes as expected implying a perfect cancellation between the two terms. The moduli for the anti-de Sitter minima with spontaneously broken supersymmetry are such that there is a subtle cancellation (albeit not perfect) between the two terms, leading to a suppression of the gauginos relative to $m_{3/2}$. This will be explained further in Ref. [16].

For the illustrative two moduli case with a pure super Yang Mills hidden sector, take $N_1^{\rm sm} = 2$, $N_2^{\rm sm} = 1$, so that the gauge kinetic function is $f_{\rm sm} = 2z_1 + z_2$. The gaugino masses for the two vacua in (9) are then

$$|M_{1/2}^{(3),(4)}| = m_p \left| \frac{e^{K/2} K^{i\bar{j}} F_{\bar{j}} \partial_i f_{\rm sm}}{2{\rm Im} f_{\rm sm}} \right| \approx \{165, 97\}\ {\rm GeV}. \tag{13}$$

Similar values arise for a significant fraction of the parameters. The tree level gaugino masses are universal but the nonuniversal one-loop anomaly mediated contributions are also non-negligible.

With V_0 tuned, the scalar masses are equal to $m_{3/2}$ times a factor which is generically unsuppressed in these vacua, so the scalar masses are expected to be of $O(m_{3/2})$—heavier than the gauginos. The trilinears (with the Yukawas factored out) turn out to be $\geq m_{3/2}$. Since the scalars are heavier than the gauginos, the lightest supersymmetric particle is a neutralino.

(13) gives a renormalized gluino mass of about $\{500, 300\}$ GeV at the TeV scale and will give a clear signal at the LHC beyond the standard model background. For example, there will be an excess of events with two charged leptons, at least two jets with a transverse momentum greater than 100 GeV, and a large missing energy from the lightest supersymmetric particle. This signal will be seen even with low luminosity.

The fact that the gaugino masses are suppressed but the scalars are not implies that LHC data could distinguish these vacua from the type IIB vacua considered in Ref. [24]. Some large volume type IIB vacua may give a spectrum similar to M theory [25], but we expect that a more thorough study [16], e.g., of the trilinears, will show that the LHC is capable of distinguishing these also, using techniques in Ref. [26].

Remarks and conclusions.—The stabilization of moduli and the hierarchy by strong dynamics in M theory seems to be quite generic and robust. The electroweak scale emerges from the fundamental theory even though the fundamental scale and compactification scale are much larger. Focusing on mechanisms which stabilize the hierarchy was useful and complementary to the approach of "searching for the Calabi-Yau which gives the spectrum at the GUT scale." The μ problem, electroweak symmetry breaking, flavor and CP physics, dark matter, inflation, and LHC physics can all be addressed within this framework, and some of these studies are underway [16].

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