Detection Scheme for Acoustic Quantum Radiation in Bose-Einstein Condensates

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Based on doubly detuned Raman transitions between (meta)stable atomic or molecular states and recently developed atom counting techniques, a detection scheme for sound waves in dilute Bose-Einstein condensates is proposed whose accuracy might reach down to the level of a few or even single phonons. This scheme could open up a new range of applications including the experimental observation of quantum radiation phenomena such as the Hawking effect in sonic black-hole analogues or the acoustic analogue of cosmological particle creation.

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Gaseous atomic or molecular Bose-Einstein condensates [1] are in several ways superior to other superfluids: apart from a very good theoretical understanding via the mean-field formalism (in the dilute-gas limit), they offer unprecedented options for experimental manipulation and control. It is possible to influence the shape, density, flow profile, and coupling strength of Bose-Einstein condensates via external electromagnetic fields. Finally, these condensed gases are rather robust against the impact of the environment such that one may reach extremely low temperatures.

In view of all these advantages, the question naturally arises whether it could be possible to measure so far unobserved quantum radiation phenomena in a suitable setup. These exotic quantum effects include cosmological particle creation (due to the amplification of quantum fluctuations in an expanding or contracting universe [2,3]) as well as the acoustic analogue of Hawking radiation [4,5] in "dumb holes" [3,6,7].

For wavelengths which are much longer than the healing length ξ, the propagation of phonons in Bose-Einstein condensates is analogous to a scalar field in a curved space-time described by the effective metric [6]

$$g_{\rm eff}^{\mu\nu} = \frac{1}{\varrho_0 c_s}\begin{pmatrix} 1 & \boldsymbol{v}_0 \\ \boldsymbol{v}_0 & \boldsymbol{v}_0 \otimes \boldsymbol{v}_0 - c_s^2 \mathbf{1} \end{pmatrix}, \tag{1}$$

which is determined by the density ϱ_0 and velocity $\boldsymbol{v}_0$ of the background fluid. For example, assuming an effectively one-dimensional stationary flow, the point where the fluid velocity v_0 exceeds the local speed of sound c_s corresponds to the sonic analogue of the horizon of a black hole. The corresponding Hawking temperature is determined by the velocity gradient [6]

$$T_{\rm Hawking} = \frac{\hbar}{2\pi k_B}\left|\frac{\partial}{\partial r}(v_0 - c_s)\right|, \tag{2}$$

i.e., the characteristic length scale λ over which the flow changes. Since this length scale should be large compared to the healing length ξ (typically of order micrometer) for the curved space-time analogy to apply, a speed of sound of order mm/s leads to an upper bound for the typical energy of the Hawking phonons of order 10^{-13} eV corresponding to a temperature on the nano-Kelvin level.

Moreover, since the fluid velocity equals the sound speed at the acoustic horizon, only a limited number of these low-energy phonons will be created by the Hawking effect—unless one has a very large reservoir for the condensate flow: Since the Hawking radiation is thermal, the typical distance between two emitted Hawking phonons is given by their characteristic wavelength λ and hence it is much larger than the healing length ξ. In addition, Bose-Einstein condensates are formed by atoms (or molecules) whose interparticle distance a_d is far bigger than the s-wave scattering length a_s (dilute-gas limit). As a result, a healing length $\xi \propto a_d\sqrt{a_d/a_s}$ typically contains many atoms $\xi \gg a_d$; i.e., we have a hierarchy of length scales $\lambda \gg \xi \gg a_d \gg a_s$. Consequently, the number of Hawking phonons is extremely small compared to the number of atoms in the condensate $a_d/\lambda \lll 1$.

Similar arguments apply to the analogue of cosmological particle creation, which require a nonstationary setup. Considering the effective metric in Eq. (1), there are basically two possibilities for simulating the cosmic expansion in Bose-Einstein condensates: an expansion of the condensate or a temporal variation of the speed of sound (which can be achieved via varying a_s by means of a Feshbach resonance, for example). For simplicity, we shall focus on the second possibility in the following, but the general ideas apply to both scenarios. The typical wavelength λ of the created phonons is determined by the rate of change $\lambda = \mathcal{O}(c_s^2/\dot{c}_s)$ of c_s and should again be large compared to the healing length ξ for the curved space-time analogy to apply. In the absence of amplification mechanisms such as resonances, the typical number of created phonons per wavelength is again of order one [5].

For a small number of phonons with an energy of order 10^{-13} eV, the usual detection mechanisms for sound (such as Bragg spectroscopy [8] or time-of-flight imaging [9]) are extremely difficult to apply: the recoil of a single (optical) photon is sufficient to "kick out" one atom/phonon and the kinetic energy of a single atom with a velocity of a few mm/s already exceeds 10^{-13} eV.

Usually, these measurements involve many atoms/phonons (limit of classical waves). For example, it is possible to excite phonon modes via light scattering [8] and to map out the Bogoliubov coefficients and the dispersion relation, etc. An indirect observation of the phonon number was achieved in ultrasensitive temperature measurements [9].

Fortunately, an alternative detection mechanism may circumvent these obstacles. For example, the ion-trap quantum computer in [10] is based on optically induced transitions which require the simultaneous absorption of a phonon in a given mode (due to the detuning of the laser). Using the occurrence of the transition as an indicator for the existence of the phonon yields an energy amplification over many orders of magnitude. If the phonon-assisted transition mediates between (meta)stable atomic states which can be separated or addressed individually, the problem of detecting a few phonons transforms into the task of counting atoms, which can be achieved using many photons [11–13].

In the following, a scheme for the transformation of low-energy phonons in a given mode into an equal number of atoms in a different atomic state with controlled energy and momentum based on doubly detuned Raman transitions is presented. Let us consider atoms which can be described by a three-level (Λ) system consisting of two (meta)stable states Ψ_1 and Ψ_2 together with a third excited level Ψ_3 with the energies $\omega_1 < \omega_2 < \omega_3$. This three-level system is illuminated by two optical laser beams which consist of many photons and can therefore be treated as rapidly oscillating classical fields described by the effective Rabi frequencies $\Omega_1(t)$ and $\Omega_2(t)$. Within the rotating wave and dipole approximation, the Hamiltonian reads ($\hbar = 1$ throughout)

$$H = \omega_1|\Psi_1^2| + \omega_2|\Psi_2^2| + \omega_3|\Psi_3^2| - [\Omega_1(t)\Psi_1^*\Psi_3 + \Omega_2(t)\Psi_2^*\Psi_3 + \text{H.c.}]. \quad (3)$$

The frequencies of the two doubly detuned laser beams are chosen according to (see Fig. 1)

$$\Omega_1(t) = \Omega_1 \exp\{i(\omega_3 - \omega_1 + \Delta)t\},$$
$$\Omega_2(t) = \Omega_2 \exp\{i(\omega_3 - \omega_2 + \Delta + \delta)t\}, \quad (4)$$

with a large detuning Δ and a small detuning δ (which will later determine the phonon energy). Standard manipulations [14] yield the effective Hamiltonian

$$H_{\text{eff}} = \frac{|\Omega_1^2||\psi_1^2| + |\Omega_2^2||\psi_2^2| + [\Omega_1\Omega_2^* e^{-i\delta t}\psi_1^*\psi_2 + \text{H.c.}]}{\Delta}. \quad (5)$$

Assuming $|\Omega_1| = |\Omega_2| = \Omega$, both levels acquire the same additional shift Ω^2/Δ; otherwise we would obtain an effective detuning $\delta \to \delta'$ shifted by $(|\Omega_1^2| - |\Omega_2^2|)/\Delta$.

An ideal quantum gas containing many of these atoms with mass m can be described by the many-particle field operator $\hat{\psi}_r$ with the dynamics (Heisenberg picture)

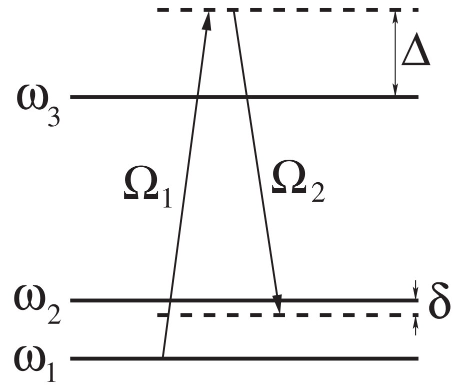

FIG. 1. Sketch (not to scale) of the three-level (Λ) system and the doubly detuned Raman transitions denoted by $\Omega_{1,2}$.

$$i\frac{\partial}{\partial t}\hat{\psi}_r = \left(-\frac{\boldsymbol{\nabla}^2}{2m} + V_r\right)\hat{\psi}_r + \Xi_{rs}\hat{\psi}_s, \quad (6)$$

where r, $s = 1, 2$ are labels for the remaining two levels and V_r the corresponding potentials. The anti-Hermitian space-time dependent transition amplitude $\Xi_{12}(t, x) = \exp\{-i\delta t + i\boldsymbol{\kappa}\cdot\boldsymbol{r}\}\Omega^2/\Delta$ represents the mode coupling in Eq. (5), where $\boldsymbol{\kappa}$ arises from a small angle between the Raman beams and the resulting wave number mismatch $\boldsymbol{\kappa} = \boldsymbol{k}_1^{\text{Laser}} - \boldsymbol{k}_2^{\text{Laser}}$.

An expansion into single-particle energy eigenstates

$$\hat{\psi}_s(t, \boldsymbol{r}) = \sum_\alpha \hat{a}_{s\alpha}(t) f_{s\alpha}(\boldsymbol{r}) \exp\{-iE_{s\alpha}t\}, \quad (7)$$

diagonalizes Eq. (6) apart from the transitions, which are (in the rotating wave approximation) only relevant for $E_{1,\alpha} - E_{2,\beta} = \delta$ (energy conservation) and if the spatial matrix element $\langle f_{1\alpha}|\hat{\Xi}_{12}|f_{2\beta}\rangle$ is large enough. For nearly homogeneous potentials $V_r \approx$ const, the eigenfunctions are plane waves $\alpha \to \boldsymbol{k}$ with $E_{r,\boldsymbol{k}} = \boldsymbol{k}_r^2/(2m) + V_r$ and the latter condition represents momentum conservation $\boldsymbol{\kappa} = \boldsymbol{k}_1 - \boldsymbol{k}_2$. Hence, for a given frequency and wave number mismatch of the lasers (δ, $\boldsymbol{\kappa}$), these energy and momentum conservation conditions determine $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ up to a contribution perpendicular to $\boldsymbol{\kappa}$. For effectively one-dimensional condensates, therefore, we can address single modes $\boldsymbol{k}_1 = k_1\boldsymbol{e}_x$ and $\boldsymbol{k}_2 = k_2\boldsymbol{e}_x$ by adjusting the lasers.

Now let us consider the following gedanken experiment: initially all atoms are in the state $r = 1$ and form a nearly homogeneous and (quasi-)one-dimensional condensate, which is not in its ground state but contains a single phonon with a given wave number $\boldsymbol{k}_p = k_p\boldsymbol{e}_x$. In contrast to Eq. (6), this requires a nonvanishing coupling g. However, if we switch off this interaction g adiabatically (e.g., via a Feshbach resonance), the system stays in this first excited state and finally contains a single atom with the momentum k_p of the original phonon. After applying a Raman π pulse (with the duration $T = \pi\Delta/\Omega^2$) adapted to this wave number, e.g., $\kappa = k_p$ and $\delta = k_p^2/(2m) + V_1 - V_2$, exactly this single atom will be transferred to the other state

$r = 2$, while all the condensate atoms are not affected (assuming that rotating wave approximation applies).

If we can separate the two species $r = 1$ and $r = 2$ or address them individually, the number of atoms in the state $r = 2$ can be counted via fluorescence measurements [11–13] and yields (in the ideal case) the number of phonons in a given mode k_p present initially, i.e., one. For example, a beam with a frequency just between the resonances of the two species $r = 1$ and $r = 2$ is repulsive for one component and attractive for the other one and could be used as optical tweezers.

With fixed momentum k_p (homogeneous condensate), the energy gap between the ground state and the one-particle excited state decreases with diminishing coupling g in view of the Bogoliubov dispersion relation $\omega^2(\boldsymbol{k}) = g\varrho \boldsymbol{k}^2/m + \boldsymbol{k}^4/(2m)^2$. Hence, let us study the application of the Raman transitions in the presence of a nonvanishing coupling g (respecting the altered dispersion relation). With interactions, the field operator $\hat{\psi}_r$ of the two levels $r = 1, 2$ obeys the equation of motion

$$i\frac{\partial}{\partial t}\hat{\psi}_r = \left(-\frac{\boldsymbol{\nabla}^2}{2m} + V_r + g_{rs}\hat{\psi}_s^\dagger\hat{\psi}_s\right)\hat{\psi}_r + \Xi_{rs}\hat{\psi}_s. \tag{8}$$

For simplicity, we assume equal one-particle trapping potentials $V_1 = V_2 = V$ and coupling constants $g_{11} = g_{22} = g_{12} = g_{21} = g$ for the two species (otherwise we would again obtain an effective detuning $\delta \to \delta'$). Initially, all the atoms (and hence also the condensate) are in the state $r = 1$, which facilitates the mean-field expansion

$$\hat{\psi}_r = \begin{pmatrix}\hat{\psi}_1\\ \hat{\psi}_2\end{pmatrix} = \begin{pmatrix}\psi_c + \hat{\chi}\\ \hat{\zeta}\end{pmatrix}\frac{\hat{A}}{\sqrt{N}} + \mathcal{O}(1/\sqrt{N}), \tag{9}$$

with the condensate wave function ψ_c and the one-particle excitations $\hat{\chi}$ and $\hat{\zeta}$. The operator $\hat{N} = \hat{A}^\dagger\hat{A}$ counts the total number of particles.

In complete analogy to the previous example, the condensate ψ_c is not affected by the Raman transitions for $\delta > 0$ (assuming that the rotating wave approximation applies) and hence the dynamics of the excitations read

$$i\frac{\partial\hat{\zeta}}{\partial t} = \left(-\frac{\boldsymbol{\nabla}^2}{2m} + V + g|\psi_c^2|\right)\hat{\zeta} + \Xi^*\hat{\chi}, \tag{10}$$

with $\Xi = \Xi_{12} = \exp\{-i\delta t + i\boldsymbol{\kappa}\cdot\boldsymbol{r}\}\Omega^2/\Delta$ as before. Assuming a nearly homogeneous condensate with $V + g|\psi_c^2| = \mu = \text{const}$, a normal mode expansion yields

$$i\frac{\partial}{\partial t}\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}} = \left(\frac{(\boldsymbol{k}+\boldsymbol{\kappa})^2}{2m} + \mu\right)\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}} + \frac{\Omega^2}{\Delta}e^{i\delta t}\hat{\chi}_{\boldsymbol{k}}. \tag{11}$$

The atomic one-particle excitation operator $\hat{\chi}_{\boldsymbol{k}}$ can be decomposed into phonon creation and annihilation operators $\hat{a}_{\boldsymbol{k}}^\dagger$ and $\hat{a}_{\boldsymbol{k}}$, respectively ($m = 1$ for brevity)

$$\hat{\chi}_{\boldsymbol{k}} = e^{-i\mu t}\sqrt{\frac{\boldsymbol{k}^2}{2\omega_{\boldsymbol{k}}}}\left[\left(\frac{1}{2} - \frac{\omega_{\boldsymbol{k}}}{\boldsymbol{k}^2}\right)\hat{a}_{\boldsymbol{k}}^\dagger + \left(\frac{1}{2} + \frac{\omega_{\boldsymbol{k}}}{\boldsymbol{k}^2}\right)\hat{a}_{\boldsymbol{k}}\right]. \tag{12}$$

Inserting the time dependences $\hat{\zeta}_{\boldsymbol{k}}(t) = \hat{\zeta}_{\boldsymbol{k}}e^{-i[\mu + \boldsymbol{k}^2/(2m)]t}$ and $\hat{a}_{\boldsymbol{k}}(t) = \hat{a}_{\boldsymbol{k}}e^{-i\omega_{\boldsymbol{k}}t}$ as well as applying the rotating wave approximation, only the second term survives $i\partial\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}/\partial t = \hat{a}_{\boldsymbol{k}}\sqrt{\boldsymbol{k}^2/2\omega_{\boldsymbol{k}}}(1/2 + \omega_{\boldsymbol{k}}/\boldsymbol{k}^2)\Omega^2/\Delta$ and we obtain the expected resonance condition

$$\delta = \omega_{\boldsymbol{k}} - \frac{(\boldsymbol{k}+\boldsymbol{\kappa})^2}{2m}, \tag{13}$$

which implies energy conservation. Of course, for $\boldsymbol{\kappa}^2$, $\boldsymbol{k}^2 \gg 1/\xi^2$, we reproduce the previous result (5). Far below the healing length $\boldsymbol{\kappa}^2$, $\boldsymbol{k}^2 \ll 1/\xi^2 = mg\varrho$, i.e., in the phonon regime, Eq. (13) simplifies to $\delta = \omega_{\boldsymbol{k}}$ and we get $i\partial\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}/\partial t = \sqrt{\mu/\delta}\hat{a}_{\boldsymbol{k}}\Omega^2/\Delta$. Note that the prefactor $\sqrt{\mu/\delta}$ is a consequence of the interactions and illustrates the difference between phonons ("dressed" atoms) and free-particle excitations as in Eq. (5).

The effective interaction Hamiltonian,

$$\hat{H}_{\text{int}} = \frac{\Omega^2}{\Delta}\sqrt{\frac{\boldsymbol{k}^2}{2\omega_{\boldsymbol{k}}}}\left(\frac{1}{2} + \frac{\omega_{\boldsymbol{k}}}{\boldsymbol{k}^2}\right)(\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}^\dagger\hat{a}_{\boldsymbol{k}} + \hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}\hat{a}_{\boldsymbol{k}}^\dagger), \tag{14}$$

has the following intuitive interpretation: because of the detuning of the Raman beams, the missing energy δ prohibits transitions from the multiparticle ground state of the condensate (which has a sharp and well-defined energy) in component $r = 1$ to the other level $r = 2$ and must be compensated by absorbing a phonon with this (for $\boldsymbol{k} = -\boldsymbol{\kappa}$) or an even higher energy $\omega_{\boldsymbol{k}} \geq \delta$.

If there are n phonons to annihilate ($\hat{a}_{\boldsymbol{k}}$), n atoms can be transferred to the $r = 2$ state ($\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}^\dagger$) such that the final number of these transferred atoms measures the initial number of phonons. Vice versa, if the component 2 is not empty initially, the Raman beams transfer atoms ($\hat{\zeta}_{\boldsymbol{k}+\boldsymbol{\kappa}}$) from the state 2 to the level 1 with simultaneous emission of an equal number of phonons ($\hat{a}_{\boldsymbol{k}}^\dagger$).

The energy-momentum balance (13) is a bit more complicated than in the previous case without interactions, but exhibits a similar direction degeneracy, which can again be eliminated by considering effectively one-dimensional condensates. In the phonon limit ($\lambda \gg \xi$ and $\delta \ll \mu$), we obtain a unique solution for the phonon energy $\omega_{\boldsymbol{k}} \approx \delta$ which allows us to address single modes with suitably tuned lasers. If we choose δ and $\boldsymbol{\kappa}$ to lie on the phonon dispersion curve $\delta = \omega(\boldsymbol{\kappa})$, we annihilate one phonon with energy δ and momentum $\boldsymbol{\kappa}$ and create one particle in the component $r = 2$ in the ground state.

With sufficiently long pulses leading to a good energy resolution, it should be possible to "see" the discrete nature of the phonon spectrum, i.e., to address single (or a few) phonon modes. In order to annihilate all phonons in the $r = 1$ condensate with a given energy or momentum and to transfer the same number of atoms to the $r = 2$ component, we apply an effective Raman π pulse with the duration [cf. Eq. (14)]

$$T = \frac{\pi\Delta}{\Omega^2}\sqrt{\frac{2\omega_{\boldsymbol{k}}}{\boldsymbol{k}^2}}\left(\frac{1}{2} + \frac{\omega_{\boldsymbol{k}}}{\boldsymbol{k}^2}\right)^{-1} \approx \frac{\pi\Delta}{\Omega^2}\sqrt{\frac{\delta}{\mu}}, \tag{15}$$

where the $\approx$ sign applies to the phonon limit.

Of course, the approximations used in the presented derivations must be checked for a potentially realistic set of experimental parameters. Let us assume a speed of sound of a few millimeters per second and a healing length around 1 μm. In this case, the wavelength λ of the phonons to be detected would typically be several micrometers $1/\kappa = \mathcal{O}(10\ \mu\text{m})$ and their frequency a few hundred Hertz $\delta = \mathcal{O}(100\ \text{Hz})$. Using lasers in the optical range $\mathcal{O}(10^{15}\ \text{Hz})$, the large detuning Δ depends on the atomic level structure and would be a little bit below this value, say $\Delta = \mathcal{O}(10^{13} - 10^{14}\ \text{Hz})$. With quite moderate Rabi frequencies $\Omega = \mathcal{O}(10^4 - 10^7\ \text{Hz})$, we can achieve an effective Raman transition rate $\sqrt{\mu/\delta}\Omega^2/\Delta$ of a few tens of Hertz. Consequently, the duration of the effective Raman π pulse in Eq. (15) would be of the order of hundred milliseconds $T = \mathcal{O}(100\ \text{ms})$ leading to a energy resolution of circa ten Hertz. In view of the aforementioned parameters, the assumptions and approximations (e.g., the adiabaticity $\Delta \gg \Omega$) used in the derivation are reasonably well justified. The major constraint is given by the energy resolution of the effective Raman π pulse peaked around $\delta = \mathcal{O}(100\ \text{Hz}) \pm \mathcal{O}(10\ \text{Hz})$. Apart from a few excitations (i.e., phonons), the beams illuminate many atoms in the ground state (zero energy) and one has to make sure that the probability of transferring an atom from the ground state of the condensate in component $r = 1$ into the state $r = 2$ is small enough. Thus the negative-frequency tail of the Fourier transform of the pulse (which is peaked around δ in frequency space) must be suppressed accordingly [12,15]. A similar restriction applies to the width of the phonon's energy, i.e., the phonons must live long enough (without experiencing decoherence such as damping due to collisions with the thermal cloud, for example) to be detected. Fortunately, for sufficiently low temperatures (nano-Kelvin level), their typical lifetime (e.g., for sodium) is much longer than 100 ms, cf. [16].

With the ability of measuring a few low-energy phonons, it might become possible to observe some of the exotic quantum effects mentioned in the Introduction. The analogue of cosmological particle production is probably easier to realize experimentally than Hawking radiation since it can be done with a condensate at rest and a practically unlimited measurement time. The same advantage applies to a small wiggling stirrer in the condensate, which would act as a pointlike noninertial scatterer and generate the analogue of moving-mirror radiation [5]. In contrast, the detection of the Hawking radiation requires either a flowing condensate or a motion of the horizon via a space-time dependent sound velocity $c_s(t, x)$ cf. [17]. Apart from measuring this striking effect, these experiments may also shed light onto the trans-Planckian problem, i.e., impact of the short-range physics on the long-wavelength Hawking radiation: even though the Hawking effect seems to be quite robust against modifications of the dispersion relation at short wavelengths (see, e.g., [18]), only very little is known about the impact of interactions.

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