Effect of Planar Defects on the Stability of the Bragg Glass Phase of Type-II Superconductors

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(Received 13 April 2006; published 25 October 2006)

It is shown that the Bragg glass phase can become unstable with respect to planar crystal defects as twin or grain boundaries. A single defect plane that is oriented parallel to the magnetic field as well as to one of the main axis of the Abrikosov flux line lattice is always relevant, whereas we argue that a plane with higher Miller index is irrelevant, even at large defect potentials. A finite density of parallel defects with random separations can be relevant even for larger Miller indices. Defects that are aligned with the applied field restore locally the flux density oscillations which decay algebraically with distance from the defect. The current-voltage relation is changed to $\ln V(J) \sim -J^{-1}$. The theory exhibits striking similarities to the physics of Luttinger liquids with impurities.

DOI: 10.1103/PhysRevLett.97.177002 PACS numbers: 74.25.Qt, 61.72.Mm, 72.15.Rn

Type-II superconductors have to contain a certain amount of disorder to sustain superconductivity: the disorder pins magnetic flux lines, hence preventing dissipation due to their motion [1,2]. For some time it was believed that disorder due to randomly distributed impurities destroys the long range translational order (LRO) of the Abrikosov flux line lattice [3]. More recently it was shown that the effect of impurities is much weaker resulting in a phase with quasi-LRO, the so-called Bragg glass [4–7]. In this phase the averaged flux line density is constant but a remnant of its periodic order is seen in the correlations of the oscillating part of the density which decay as a power law. Experimental signatures of this phase have been observed [8]. An important feature of the Bragg glass is the highly nonlinear current-voltage relation related to the flux creep which is of the form $\ln V(J) \sim -J^{-1/2}$, so that the linear resistance vanishes.

Although much of the original transport data on flux creep in high-T_c superconductors was discussed in terms of point disorder (see, e.g., [9]) it was realized later that many samples included planar defects like twin planes or grain boundaries which masked the Bragg glass behavior [10]. Indeed, in clean samples planar defects lead to much more pronounced pinning phenomena than point disorder because of stronger spatial correlations [2,11]. However, the generic experimental situation is a mixture of point disorder and planar defects, a case which has been studied only numerically [12].

Main results.—It is the aim of the present Letter to consider exactly this case, i.e., the question of the influence of planar defects in the Bragg glass phase. Our key results are as follows: a necessary condition for a planar defect to become a relevant perturbation is that it is oriented parallel to the magnetic flux. In this case its influence on the Bragg glass phase can be characterized by the value of a *single* parameter $g \equiv \frac{3}{8}\eta(a/l)^2$ which depends both on the exponent η describing the decay of the density correlations in the Bragg glass phase *and* on the orientation of the defect with respect to the flux line lattice. a and l are the mean spacing of the flux lines in the bulk and the distance between lattice planes of the Abrikosov lattice *parallel* to the defect, respectively. The defect is relevant for an arbitrarily weak defect potential if $g < 1$, which is realized if and only if the defect plane is parallel to one of the main crystallographic planes of the flux line lattice. In the vicinity of the (relevant) defect the density profile shows periodic order with an amplitude decaying algebraically (with exponent g) with the normal distance from the defect. The current-voltage relation for voltage drops perpendicular to a defect plane is of the form $\ln V_D(J) \sim -J^{-1}$. Correlations are destroyed across (relevant) defects. For $g > 1$ a weak (and presumably even a strong) defect is irrelevant and the density profile decays with a larger exponent $2g - 1 > 1$. All defects which are tilted against the magnetic flux are irrelevant and the flux density oscillations decay exponentially. For a finite density of defect planes (with random distances and/or orientations) the Bragg glass is destroyed for $g < \tilde{g}_c$ with $\tilde{g}_c \geq 3/2$ depending on the defect strength.

It is worthwhile to mention that some aspects of our results are similar to other theories at their critical dimension like two-dimensional classical or $(1 + 1)$-dimensional quantum models. Adding a planar defect in the Bragg glass resembles the presence of a line defect in the classical or a frozen impurity in the quantum case [13–16] when the coupling constant g is identified with temperature or the Luttinger liquid parameter, respectively. The periodic order seen in the vicinity of the defect plane has its counterpart in the Friedel oscillations in Luttinger liquids close to an isolated impurity. Whereas in those cases the relevance of a defect (i.e., an impurity) can be changed by tuning the interaction strength or the temperature, respectively; in the present case a change of g can be accomplished by changing the *orientation* of the defect with respect to the flux line lattice. The current-voltage relations are, however, different from the two-dimensional cases. Experimentally microtwinned crystals with one direction of twin planes have been studied, e.g., in [17].

Model.—First, we consider the effect of a single planar defect in a system of interacting flux lines which are pinned by randomly distributed impurities. Since we are interested in the behavior on large length scales, it is appropriate to describe the interacting flux lines in terms of a continuum elastic approximation with a displacement field $\mathbf{u}(\mathbf{r})$. The Hamiltonian $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_D$ includes then the elastic energy and the impurity interaction,

$$\mathcal{H}_0 = \frac{1}{2}\int dz d^2\mathbf{x}\{c_{11}(\nabla_\mathbf{x}\cdot\mathbf{u})^2 + c_{66}(\nabla_\mathbf{x}\times\mathbf{u})^2 + c_{44}(\partial_z\mathbf{u})^2 + E_{\text{pin}}(\mathbf{u},\mathbf{r})\}. \quad (1)$$

In general the elastic moduli of compression (c_{11}), shear (c_{66}), and tilt (c_{44}) are nonlocal on length scales smaller then the penetration depth λ. Since disorder effects considered below become relevant only on length scales $L \gg \lambda$ [7] this nonlocality can be neglected here.

The pinning energy can be written as $E_{\text{pin}} = U(\mathbf{r})\rho(\mathbf{r},\mathbf{u})$, where $U(\mathbf{r})$ is a potential arising from randomly distributed impurities and $\mathbf{r} \equiv (\mathbf{x}, z)$. The local flux line density can be expressed as $\rho(\mathbf{r},\mathbf{u}) = \rho_0\{-\nabla_\mathbf{x}\mathbf{u}(\mathbf{r}) + \sum_\mathbf{G} e^{i\mathbf{G}[\mathbf{x}-\mathbf{u}(\mathbf{r})]}\}$ [7]. Here $\rho_0 = 2/\sqrt{3}a^2$, and $\mathbf{G} \equiv \mathbf{G}_{mn} = m\mathbf{b}_1 + n\mathbf{b}_2$ is a vector of the reciprocal lattice with integer m, n [18]. The set of the six smallest reciprocal lattice vectors will be denoted by $\{\mathbf{G}_0\}$. The energy of a planar defect has the same form as E_{pin} but with $U(\mathbf{r})$ replaced by the potential well of the defect $V(\mathbf{r}\cdot\mathbf{n}_D - \delta)$. Here $\mathbf{n}_D$ and δ denote the normal vector of the defect plane and its distance (along $\mathbf{n}_D$) from the origin of the coordinate system, respectively. Bragg glass physics is dominated by disorder fluctuations on large length scales where microscopic details become irrelevant and hence $V(x) \approx -v\delta(x)$. Since the superconducting order is reduced in the defect plane it is plausible to assume $v > 0$ (for more details see Section IX of [2]).

Without $\mathcal{H}_D$ this model has been studied in detail using different approaches [4–7]. It was shown that thermal fluctuations are irrelevant (zero temperature fixed point) and that the pinned flux line lattice exhibits a power law decay of the translational order parameter $\Psi_\mathbf{G}(\mathbf{x},z) = e^{i\mathbf{G}\mathbf{u}(\mathbf{x},z)}$, similar to pure 2D crystals at finite temperatures. In particular, $\langle\Psi_\mathbf{G}\rangle = 0$ but $\langle\Psi_\mathbf{G}(\mathbf{x},0)\Psi_{-\mathbf{G}}(\mathbf{0},0)\rangle \sim |\mathbf{x}|^{-\eta_\mathbf{G}}$, where $\langle ...\rangle$ denotes both the thermal and disorder average and $\eta_\mathbf{G} = \eta(G/G_0)^2$. From a Gaussian, variational treatment in $d = 3$ dimensions follows $\eta = 1$ [6] whereas a functional renormalization group analysis in $d = 4 - \epsilon$ dimensions yields a nonuniversal exponent η that varies with the elastic constants of the vortex lattice [7]. Extrapolating to $d = 3$, one finds only a very weak variation with $1.143 < \eta < 1.159$ [7]. Since (despite of the glassy nature of the phase) the structure factor shows still Bragg peaks the notation Bragg glass was coined [6]. Next we discuss the influence of $\mathcal{H}_D$. In order to integrate over the delta function of the defect potential, it is convenient to introduce an explicit parameterization for the position vector $\mathbf{r}_D$ of the defect plane which obeys $\mathbf{r}_D\cdot\mathbf{n}_D = \delta$. With

$$\mathbf{r}_D = (\mathbf{x}_D, z_D) + \delta\mathbf{n}_D, \qquad z_D = t\cos\beta,$$
$$\mathbf{x}_D = (s\sin\alpha - t\cos\alpha\sin\beta,\, s\cos\alpha + t\sin\alpha\sin\beta) \quad (2)$$

we introduce in-plane coordinates s, t, and two angles α and β which determine the rotation of the plane with respect to the y and z axis, respectively, (see Fig. 1). The defect energy reads then

$$\mathcal{H}_D = v\rho_0\int dtds\left\{\nabla_\mathbf{x}\mathbf{u}(\mathbf{r}_D) - \sum_{\mathbf{G}\neq\mathbf{0}} e^{i\mathbf{G}[\delta\mathbf{n}_D + \mathbf{x}_D - \mathbf{u}(\mathbf{r}_D)]}\right\}. \quad (3)$$

Since the displacement field $\mathbf{u}(\mathbf{r}_D)$ varies slowly on the scale of the flux line lattice constant a, the integrals over s and t vanish for all $\mathbf{G}$ except those for which the oscillatory factor $e^{i\mathbf{G}\mathbf{x}_D}$ becomes one (for all s, t). This condition can be satisfied only if $\sin\beta = 0$, i.e., if the defect plane is *parallel* to the applied magnetic field. There remains a second condition for the angle α which results from the constraint that $\mathbf{G}$ has to be perpendicular to $\mathbf{x}_D$. Expressing the defect plane (for $\sin\beta = 0$) as $\mathbf{x}_D = (c_1\mathbf{a}_1 - c_2\mathbf{a}_2)s/a$ where $\mathbf{a}_i\mathbf{b}_j = 2\pi\delta_{ij}$, this results in the condition $m/n = c_2/c_1$. Hence if c_1/c_2 is irrational the effect of the defect plane is always averaged to zero. On the other hand, for rational c_2/c_1 we may choose m_D, n_D to be the smallest coprime pair with $c_2/c_1 = m_D/n_D$. Then m_D, n_D are the Miller indices of the defect plane and only those $\mathbf{G}$ which are integer multiples of $\mathbf{G}_D \equiv \mathbf{G}_{m_D n_D}$ contribute in Eq. (3). In the following, we will concentrate on the contribution from these $\mathbf{G}$ vectors only. The flux line lattice planes (of the ideal lattice) parallel to a defect plane with Miller indices m_D, n_D have a separation of $l = \frac{\sqrt{3}}{2}a/\sqrt{m_D^2 + m_D n_D + n_D^2}$ and hence $G_D = 2\pi/l$. The number of flux lines in the defect plane increases with increasing l, rendering the defect more relevant.

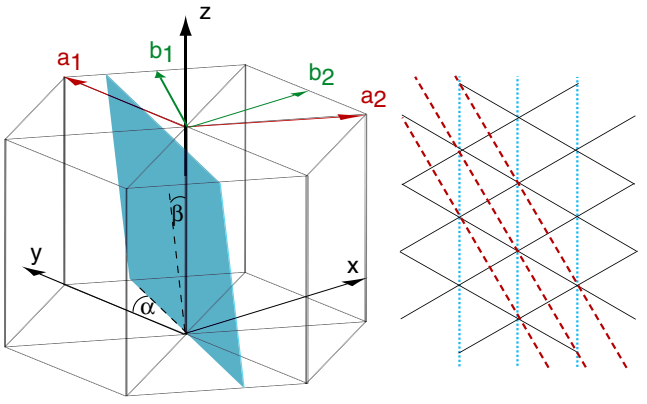

FIG. 1 (color online). Left: Triangular flux line lattice with vectors of the direct ($\mathbf{a}_1$ and $\mathbf{a}_2$) and the reciprocal lattice ($\mathbf{b}_1$ and $\mathbf{b}_2$) and the orientation of the defect plane. Right: Two lowest order orientations corresponding to $g = \eta/2$ (dotted lines), $3\eta/2$ (dashed lines).

Single defect.—Using the result for the average of $\Psi_{\mathbf{G}}(\mathbf{r})$ in the Bragg glass phase [7], one finds for the disorder averaged defect energy $\mathcal{H}_D$ on scale L

$$\langle \mathcal{H}_D \rangle_0 \sim \sum_{k=1}^{\infty} v_k \cos(k\delta G_D) \left(\frac{L}{L_a}\right)^{2-k^2 g}, \qquad g \equiv \frac{3}{8}\frac{\eta a^2}{l^2}, \tag{4}$$

where $\langle \ldots \rangle_0$ denotes the average with $\mathcal{H}_0$. The gradient term in (3) scales $\sim L$ if the defect size is $\sim L^2$. Since the elastic energy of Eq. (1) scales in the same way, the gradient term is a marginal perturbation. Indeed, it can be eliminated by the transformation $\mathbf{u} \to \mathbf{u} + \nabla_{\mathbf{x}} \phi(\mathbf{x})$ with $\phi(\mathbf{x})$ a scalar field obeying $\nabla^2_{\mathbf{x}} \phi = \frac{v\rho_0}{2c_{11}} \delta(\mathbf{x} \cdot \mathbf{n}_D - \delta)$. This transformation does not change the terms $\sim c_{66}$, c_{44} in Eq. (1) but shifts the flux line density on the defect. However, the periodic terms in Eq. (3) can grow faster than the elastic energy, depending on g.

Their coefficients $\cos(k\delta G_D)$ reflect the periodicity of the defect energy under translations by l normal to the plane. The Gaussian approximation used here is believed to be correct to order ϵ [19]. It is important to remark that the L dependence of Eq. (4) holds only on length scales larger than the positional correlation length $L_a \approx L_\xi (a/\xi)^{1/\zeta_{\mathrm{rm}}}$, where ξ is the maximum of the coherence and the disorder correlation length and L_ξ denotes the Larkin length on which the typical flux line displacement is of the order ξ. $\zeta_{\mathrm{rm}} \approx 0.175$ is the roughness exponent of the elastic distortions on scales smaller than L_a (in the so-called random manifold regime). There the correlations of $\Psi_{\mathbf{G}}(\mathbf{r})$ decay as a stretched exponential and the effect of the defect plane is reduced by disorder fluctuations. Because of these fluctuations on intermediate length scales the initial value of the defect strength is reduced to $v_k \approx v(L_a/a)^2 e^{-c(G_D k a)^2}$ where c is a constant [20].

To linear order in v_k, the renormalization-group flow equation of the v_k is obtained by comparison of the defect energy scaling in Eq. (4) with the scaling of $\mathcal{H}_0$ at the Bragg glass fixed point, yielding

$$dv_k/d\ln L = (1 - k^2 g) v_k. \tag{5}$$

Hence, v_1 is a relevant perturbation for $g < 1$, i.e., if

$$\eta(m_D^2 + m_D n_D + n_D^2) < 2 \quad \text{or} \quad l > \sqrt{\frac{3\eta}{8}} a \approx 0.66a, \tag{6}$$

which is compatible only with $l = \sqrt{3}a/2 \approx 0.87a$. Hence the defect plane must be oriented parallel to one of the three main crystallographic planes of the flux line lattice (i.e., $\cos 2\beta = \cos 6\alpha = 1$).

The transition described by Eq. (5) occurs not in the bulk but on the defect plane. Hence one can develop an effective theory on the defect which could be used to describe also stronger defect potentials. Since the defect couples only to the normal displacement $u_\perp(\mathbf{r}_D) = \mathbf{n}_D \mathbf{u}(\mathbf{r}_D)$ on the defect plane, we integrate out $u_\perp$ outside the defect and $\mathbf{u} - u_\perp \mathbf{n}_D$ across the entire sample. This integration is facilitated by the reasonable assumption of an effective Gaussian theory for the defect-free system at the Bragg glass fixed point [7]. We find that $u_\perp$ on the defect has long-ranged elasticity and is described by the effective Hamiltonian (compare [15,16] for a corresponding procedure in the clean case)

$$\mathcal{H}_{2\mathrm{D}} = \frac{K}{2} \int d^2\mathbf{q} |\mathbf{q}| |\varphi_{\mathbf{q}}|^2 + \int d^2 r_D \left\{ \frac{2\sqrt{\pi g} K}{\xi} \cos(\varphi - \alpha) + \frac{v_1}{L_a^2} \cos(\varphi) \right\}, \tag{7}$$

where $\varphi(\mathbf{r}_D) \equiv 2\pi u_\perp(\mathbf{r}_D)/l$ and $\mathbf{q}$ is the in-plane momentum. α is a random phase which is uncorrelated and uniformly distributed. The amplitude of the random potential has been chosen here as to reproduce the proper Gaussian replica theory of the Bragg glass for $v_1 = 0$. The elastic constant K depends on the bulk elastic moduli, the angle α and the direction of $\mathbf{q}$. The model of Eq. (7) shows a transition at $g = g_c(v_1)$ with $g_c(0) = 1$ in agreement with our previous considerations. In the present case $g = \eta(m_D^2 + m_D n_D + n_D^2)/2$ can only be changed in finite steps $(\eta/2, 3\eta/2, 7\eta/2, \ldots)$ by changing the orientation of the defect plane. Thus at small v_1 only the defects parallel to the three main crystallographic planes are relevant (with $m_D^2 + m_D n_D + n_D^2 = 1$). Because of the long-ranged elasticity in Eq. (7) we conclude from our analysis of a related model [21] that even strong isolated defects are relevant only for $g \leq 1$.

Density oscillations.—Next, we study the order of the flux lines in the vicinity of the defect. The potential of a relevant defect growth under renormalization and effectively decouples the two half-spaces. This allows us to determine the boundary induced modifications of the flux density via the method of images. We find that $\langle u_\perp^2(\mathbf{r}) \rangle = \frac{1}{2}\langle [u_\perp(\mathbf{r}) - u_\perp(\mathbf{r}_m)]^2 \rangle_0$, where $\mathbf{r}_m$ is the mirror image of $\mathbf{r}$ with respect to the defect plane. From this we obtain immediately that with increasing distance $L_\perp = |\mathbf{n}_D \mathbf{x} - \delta|$ from the defect, the slowest oscillations of the flux line density decay for a *relevant* defect plane ($g < 1$) as

$$\langle \rho(\mathbf{x}, z, \mathbf{u}) \rangle \sim \left(\frac{L_a}{L_\perp}\right)^g \cos[G_0(L_\perp \pm \delta)]. \tag{8}$$

Hence, a single relevant defect plane yields a long-ranged restoration of the order parameter. The oscillations of the density resemble Friedel oscillations observed in Luttinger liquids close to an isolated impurity [16]. This similarity is substantiated by considering the decay of the density oscillations if the defect plane is *irrelevant*, i.e., for $g > 1$. Then the defect potential decreases under renormalization and lowest order perturbation theory can be applied. Such an approach takes into account that the defect strength decreases as $v(L) \sim L^{1-g}$ and hence we obtain Eq. (8) with g replaced by $2g - 1 > 1$, in close analogy to the $(1 + 1)$-dimensional counterpart [13–15]. If the defect plane is *not* parallel to the applied magnetic field Friedel

oscillations occur as well. However, they decay exponentially beyond a characteristic distance $\xi = 1/(G_D|\sin\beta|)$ from the defect plane.

Creep.—To find the current-voltage relation for a relevant defect we consider the most interesting case where the current J is parallel to the defect and normal to the magnetic field $\mathbf{B}$. Flux creep in the presence of pinning forces arises via formation of critical droplets [2]. In the present case of a defect plane the droplet is characterized by $G_D u_\perp(\mathbf{r}_D) = 0$ and 2π outside and inside of the droplet, respectively. Since on scale L_a typical distortions in the Bragg glass phase are of the order a, the droplet volume is $\sim L^2 L_a$ and the volume energy gain in the droplet is of the order $\sim -JBL^2L_a$ which has to be compared with the surface energy loss of the order $\sim \upsilon_1^{1/2(1-g)} L L_a^{1/2}$, where we have included the renormalization of υ_1. From the balance of the two terms follows a critical droplet size $L_J \sim \upsilon_1^{1/2(1-g)}(L_a JB)^{-1}$. Droplets of this size have a free energy $\sim \upsilon_1^{1/(1-g)}(JB)^{-1}$ which determines via the Arrhenius law the creep velocity due to thermal activation, yielding the voltage drop normal to the defect plane

$$V \sim \exp\{-[C_0/(TJ)]\}, \qquad C_0 \sim \upsilon_1^{1/(1-g)} B^{-1}. \tag{9}$$

Thus the flux creep across the defect is much slower than in the Bragg glass phase. If for weak defects $g \to 1-$, C_0 becomes small, hence reducing the applicability of this formula to extremely small currents.

Many defects.—Crystals usually contain either two orthogonal families (''colonies'') of parallel twin boundaries or a *single* family of parallel twin planes [2,22]. The latter case has been studied theoretically for impure samples so far only in $(1+1)$ dimensions [23]. Here we consider the more general situation of planes with random positions δ_j and/or orientations $\mathbf{n}_{D,j}$ that are aligned with the applied field. Each plane contributes $\mathcal{H}_D(\delta_j, \mathbf{n}_{D,j})$ of Eq. (3) with δ_j, $\mathbf{n}_{D,j}$ replacing δ, $\mathbf{n}_D$. For the averages over the δ_j and $\mathbf{n}_{D,j}$ one finds $\overline{\langle \sum_j \mathcal{H}_D(\delta_j)\rangle} = 0$, but $\overline{\langle \sum_j \mathcal{H}_D^2(\delta_j)\rangle} \sim \sum_j \varrho_j L^{5-2g_j}$ from the term with $k=1$, where ϱ_j is the mean density of planes with an orientation characterized by g_j, see Eq. (4). Since $\mathcal{H}_0^2 \sim L^2$, *weak* defect planes are relevant and destroy the Bragg glass if there is a finite $\varrho_j > 0$ with $g_j < \tilde{g}_c = 3/2$, i.e., if there is a finite density of planes that are oriented with a main plane of the flux lattice. Contrary to a single defect plane, we expect that a larger defect strength yields an increased $\tilde{g}_c > 3/2$ rendering additional defect orientations relevant. However, a description of the transition at strong coupling and the localized fixed point describing relevant defect planes is not available at present [24]. Only for defect planes of equal distance we expect long range order in the direction perpendicular to the planes. Flux creep in samples with many defects of the same orientation is anisotropic. For a current parallel to the defect planes Eq. (9) applies (for defect separations larger than L_a). If the current is perpendicular to the planes, the original Bragg glass creep law holds. For randomly oriented planes, the creep is for all current orientations described again by Eq. (9).

We acknowledge useful comments by L. Radzihovsky, E. Zeldov, and, in particular, by D. R. Nelson who brought the problem to our attention, and T. N. thanks R. Woerdenweber for conversations. This work was supported by DFG through Emmy Noether Grant No. EM70/2 (TE), and SFB 608 (T. E. and T. N.).

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