Signatures for Generalized Macroscopic Superpositions

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We develop criteria sufficient to enable detection of macroscopic coherence where there are not just two macroscopically distinct outcomes for a pointer measurement, but rather a spread of outcomes over a macroscopic range. The criteria provide a means to distinguish a macroscopic quantum description from a microscopic one based on mixtures of microscopic superpositions of pointer-measurement eigenstates. The criteria are applied to Gaussian-squeezed and spin-entangled states.

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In his essay [1] of 1935, Schrödinger discussed the issue of quantum superpositions of macroscopically distinct states, and there is much interest in the possibility of generating such superpositions [2]. While there has been some progress [3,4], the experimental generation of these superpositions has been hindered by a sensitivity to decoherence caused by a coupling of the system to its environment. Caldeira and Leggett [5] have shown that, where losses are unavoidable, a superposition of two macroscopically different states ψ_+, ψ_- will rapidly decohere to a mixture so that the off-diagonal density matrix element $\langle\psi_+|\rho|\psi_-\rangle$ vanishes.

Yet there has been experimental confirmation [4,6,7] of other quantum features such as squeezing and entanglement in systems that might be described as macroscopic, in that they contain large numbers of particles. The quantum models [4,8,9] for these systems are more complex than those considered by Schrödinger, involving superpositions of the type $\psi_- + \psi_0 + \psi_+$ where only the ψ_- and ψ_+ provide macroscopically distinguishable outcomes for some measurement, which we will call the pointer measurement [10]. While these superpositions do not reflect the simple case discussed by Schrödinger, they do possess macroscopic coherence through the nonzero off-diagonal matrix element $\langle\psi_+|\rho|\psi_-\rangle$.

The extent however to which a quantum signature observed on a macroscopic system is actually due to an underlying macroscopic coherence needs careful analysis. The macroscopic spread in the outcomes of the pointer measurement could also be generated from mixtures of microscopic superpositions—that is, superpositions of pointer-measurement eigenstates that have only microscopic differences in their predictions for the pointer measurement. Decoherence effects are likely to degrade the system to such mixtures, where macroscopic coherence is lost.

In this Letter we address this issue by extending the concept of a signature for macroscopic coherence to situations that do not give only two macroscopically distinct outcomes. Specifically, we derive measurement criteria sufficient to confirm an intrinsic macroscopic off-diagonal matrix element of type $\langle\psi_+|\rho|\psi_-\rangle$. Equivalently, the criteria enable falsification of any quantum description involving only microscopic superpositions of pointer-measurement eigenstates.

The criteria can be applied to demonstrate such macroscopic coherence in realistic lossy systems based on Gaussian-squeezed states [9] and spin-entangled states [7,8]. These systems have a wide applicability. Continuous variable squeezing and entanglement have been experimentally observed using Gaussian states [6], and spin entanglement has been realized in multiparticle photonic systems [7], and between atomic ensembles [4]. We also discuss how the signatures allow for a demonstration of a macroscopic version of a type of Einstein-Podolsky-Rosen paradox [11].

We consider a macroscopic system A for which there is a pointer-measurement O giving outcomes x spread over a macroscopic range (Fig. 1). The domain for x is partitioned into three distinct regimes $I = -1, 0,$ and 1 corresponding to $x \le -S/2$, $-S/2 < x < S/2$, and $x \ge S/2$, that have probabilities P_-, P_0, P_+, respectively. The binned outcomes -1 and $+1$ are considered to be macroscopically distinct when S is macroscopic. We define ψ_+, ψ_0, and ψ_- to be quantum states certain to produce results only in the region $+1$, 0, and -1, respectively.

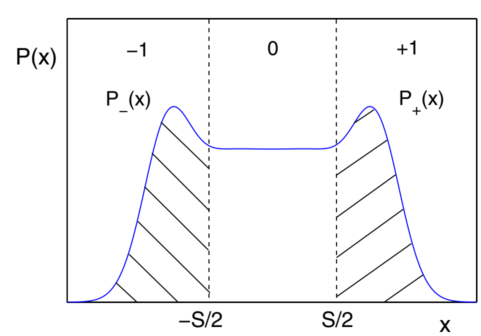

FIG. 1 (color online). Probability distribution for a measurement O which gives three distinct regions of outcome: 0, -1, and $+1$.

We define a generalized macroscopic superposition

$$c_+\psi_+ + c_0\psi_0 + c_-\psi_-, \tag{1}$$

where $c_\pm$, c_0 are probability amplitudes but with $c_+, c_- \neq 0$, and where the minimum separation S between the outcomes for ψ_+ and ψ_- is macroscopic. These macroscopic superpositions [4,6–9] possess a macroscopic coherence in the sense of a nonzero matrix element $\langle\psi_-|\rho|\psi_+\rangle$, where ρ is the system density operator. As such, ρ cannot be constructed as a mixture of only microscopic superpositions which superpose states with predictions for O only microscopically distinct.

Most generally, the system is a mixed state

$$\rho = \sum_r P_r|\psi_r\rangle\langle\psi_r|, \tag{2}$$

where the $|\psi_r\rangle$ are pure states. In this context, we define the existence of the generalized macroscopic superposition (1) to mean that there must exist, in any expansion of ρ, a nonzero probability P_r for a state $|\psi_r\rangle$ of type (1).

Now in all cases where the macroscopic superposition does not exist, so that (2) can be written without (1), the $|\psi_r\rangle$ of (2) can only be superpositions of states with outcomes x lying within two adjacent regions I, $I+1$. The density operator then assumes the following form.

$$\rho_{\mathrm{mix}} = P_L\rho_L + P_R\rho_R. \tag{3}$$

Here ρ_R is a quantum density operator constrained only by the condition that it predicts for O a result $I = 1$ or 0, so that $x > -S/2$; similarly ρ_L always predicts either $I = -1$ or 0, so that $x < S/2$. P_L and P_R are arbitrary probabilities for these left and right sides of the outcome domain, so that $P_L + P_R = 1$.

The mixtures (3), that can incorporate all superpositions bar the macroscopic one (1), are constrained to satisfy measurable minimum uncertainty relations (inequalities) that form the key results, given as theorems, of this Letter. Violation of any one of these uncertainty relations thus acts as a signature of the existence of the macroscopic superposition (1).

The origin of this signature can be understood by noting that for ρ_{mix} the Heisenberg uncertainty relation $\Delta^2x\Delta^2p \geq 1$ for results x and p of complementary observables O and P applies to each of ρ_R and ρ_L, so that

$$\Delta_L^2x\Delta_L^2p \geq 1, \qquad \Delta_R^2x\Delta_R^2p \geq 1 \tag{4}$$

($\Delta_{L/R}^2x$ and $\Delta_{L/R}^2p$ are the variances for $\rho_{L/R}$). In addition, each of these density operators, being restricted to a smaller domain, has an upper limit to its variance for x that does not apply to the macroscopic superposition (1) which would describe the whole statistics. This imposes a minimum fuzziness in p for each of ρ_R and ρ_L, and hence for the mixture (3), which must satisfy [12]

$$\Delta^2p \geq P_L\Delta_L^2p + P_R\Delta_R^2p. \tag{5}$$

Superpositions (1) that have a reduced (or squeezed) variance in p, so that $\Delta^2p \to 0$, are able to violate the constraint that is thus placed on Δ^2p.

We derive a particular form for the limit of precision specified for the mixture (3) by combining (4) and (5) and using the Cauchy-Schwarz inequality.

$$\begin{aligned}(P_L\Delta_L^2x + P_R\Delta_R^2x)\Delta^2p &\geq \Big[\sum_{i=L,R}P_i\Delta_i^2x\Big]\Big[\sum_{i=L,R}P_i\Delta_i^2p\Big] \\ &\geq \Big[\sum_{i=L,R}P_i\Delta_ix\Delta_ip\Big]^2 \geq 1. \end{aligned}\tag{6}$$

To express (6) in terms of variances that are measurable, we derive the constraints on $\Delta_{R/L}^2x$. We partition the probability distribution $P_R(x)$, for a result x given ρ_R, according to its outcome domains $I = 0, +1$. Thus,

$$P_R(x) = P_{R0}P_{R0}(x) + P_{R+}P_+(x), \tag{7}$$

where $P_{R0}(x) \equiv P_R(x|x < S/2)$ and $P_+(x) \equiv P_R(x|x \geq S/2)$ are the normalized distributions for a result x in region $I = 0$ or $I = +1$, respectively. We use [12] to write $\Delta_R^2x = P_{R0}\Delta_{R0}^2x + P_{R+}\Delta_+^2x + P_{R0}P_{R+}(\mu_+ - \mu_{R0})^2$, where μ_+ (Δ_+^2x) and μ_{R0} (Δ_{R0}^2x) are the averages (variances) of $P_+(x)$ and $P_{R0}(x)$, respectively. Using $P_{R0} \leq P_0/(P_0 + P_+)$, $\Delta_{R0}^2x \leq S^2/4$, $P_{R+} \leq 1$, and $0 \leq \mu_+ - \mu_{R0} \leq \mu_+ + S/2$, we obtain

$$\Delta_R^2x \leq \Delta_+^2x + \frac{P_0}{P_0 + P_+}[(S/2)^2 + (\mu_+ + S/2)^2] \tag{8}$$

and, similarly, $\Delta_L^2x \leq \Delta_-^2x + \frac{P_0}{P_0+P_-}[(S/2)^2 + (\mu_- - S/2)^2]$, where $\mu_\pm$ and $\Delta_\pm^2x$ are the mean and variance of $P_\pm(x)$, defined (Fig. 1) as the normalized + and − parts of $P(x)$ [$P_+(x) = P(x|x \geq S/2)$ and $P_-(x) = P(x|x \leq -S/2)$]. We substitute (8) in (6), and use $P_0 + P_+ \geq P_R$ and $P_0 + P_- \geq P_L$ to derive the following theorem which is the main result of this Letter.

Theorem 1: The mixture (3) implies

$$(\Delta_{\mathrm{ave}}^2x + P_0\delta)\Delta^2p \geq 1, \tag{9}$$

where we define $\Delta_{\mathrm{ave}}^2x = P_+\Delta_+^2x + P_-\Delta_-^2x$ and $\delta \equiv \{(\mu_+ + S/2)^2 + (\mu_- - S/2)^2 + S/2\} + \Delta_+^2x + \Delta_-^2x$. Measurements of the probability distributions for x and p are all that is needed to determine all the terms in this inequality. Given those distributions, one can search for the maximum value of S for which there is a violation.

Theorem 2: Where we have a system comprised of subsystems A and B, the mixture (3) implies

$$(\Delta_{\mathrm{ave}}^2x + P_0\delta)\Delta_{\mathrm{inf}}^2p \geq 1. \tag{10}$$

In this case the ρ_L and ρ_R of (3) are density operators for the composite system. We define $\Delta_{\mathrm{inf}}^2p = \Delta^2\tilde{p}$, where $\tilde{p} = p - gp^B$ and g is a constant. The $\Delta_{\mathrm{inf}}p$ can be interpreted as the error in the inference of p based on a result p^B of a measurement on B, if we infer p to be gp^B [13], and has been measured in experiments concerned with realization of the EPR paradox [6]. To optimize violation of the inequality, we would, given the joint measurement of p and p^B, choose g to minimize $\Delta^2_{\rm inf}p$. The ideal case of $\Delta^2_{\rm inf}p = 0$ reflects a maximum correlation between measurements p and p^B. The proof of Theorem 2 follows similarly to that of Theorem 1, except that we use the uncertainty relation $\Delta^2 x\Delta^2\tilde{p} \geq 1$.

Theorem 3: Suppose the spin measurement J_z on a system A gives outcome m with a probability distribution $P(m)$ that indicates $I = +1$, 0, and -1, respectively, for $m \geq S$, $S > m > -S$, and $m \leq -S$. The assumption of any mixture that excludes (1) will always imply

$$\Delta J_x \Delta_{\rm inf} J_y \geq \frac{1}{2}\sum_{I=\pm 1} P_I^2 |\langle J_z\rangle_I|/(P_I + P_{0,I}). \qquad (11)$$

Here $\langle J_z\rangle_I$ is the mean of $P_I(m)$, the distribution conditional on m satisfying either $I = +1$ or $I = -1$. The $\Delta_{\rm inf}J_y$ is defined similarly to $\Delta_{\rm inf}p$, to be $\Delta\tilde{J}_y$, where $\tilde{J}_y = J_y - gJ_y^B$, and J_y^B is a measurement on a second system B. J_x and J_y refer to spin measurements made on subsystem A. Here $P_{0,+}$ ($P_{0,-}$) is the probability that the result m of J_z satisfies $0 \leq m < S$ ($-S < m < 0$), and the P_+ (P_-) in this case is the probability for $m \geq S$ ($m \leq -S$). The proof [14] follows that of Theorem 1, but results are based on the spin uncertainty relations.

Violation of inequalities (9) or (10) or (11) would provide confirmation of a superposition (1) with separations between ψ_- and ψ_+ of at least S. Such confirmation (for macroscopic S) holds interest in relation to Schrödinger's 1935 essay, in that it is demonstrated that microscopic superpositions alone, or mixtures of them, cannot explain the observed statistics. An appropriate extension of Schrödinger's description of the cat is given in footnote [15].

The inequalities are not violated by all macroscopic superpositions. Nevertheless we present two important practical examples of generalized macroscopic superpositions (1) that predict violations. First, we consider the entangled spin superposition state [8,16]

$$|\psi\rangle = \frac{1}{\sqrt{2j+1}}\sum_{m=-j}^{j}|j,m\rangle_A|j,m\rangle_B, \qquad (12)$$

where j is large. Such states for lower values of j have been realized in systems based on parametric amplification [7]. Here $|j,m\rangle_A$ are the J^2, J_z spin eigenstates for a subsystem A ($|j,m\rangle_B$ are spin eigenstates of subsystem B). Denoting $|j,m\rangle_A|j,m\rangle_B = |m,m\rangle$, the state (12) is a superposition of states $|-j,-j\rangle,\ldots,|j,j\rangle$ having a macroscopic range of $2j$ for outcomes of J_z. It thus possesses a nonzero coherence $\langle -j,-j|\rho|j,j\rangle$. The experimental criterion (11) provides a means to distinguish the macroscopic quantum description (12) from a microscopic one based only on superpositions, like $|\psi_r\rangle = (|j,j\rangle + |j-1,j-1\rangle)/\sqrt{2}$, which have $\langle -j,-j|\rho|j,j\rangle = 0$. Calculations show maximum correlation between J_y and J_y^B, so $\Delta_{\rm inf}J_y = 0$. State (12) predicts violations of (11) for all S up to j, to confirm a superposition of type (1).

Second, we consider single- and two-mode momentum-squeezed states $S(r)|0\rangle = e^{r(a^2 - a^{\dagger 2})}|0\rangle$ and $e^{r(ab - a^\dagger b^\dagger)}|0\rangle$ [9]. Here a, b are boson operators for fields A, B respectively; $|0\rangle$ is the vacuum state. We define quadrature phase amplitude measurements $X = a + a^\dagger$, $P = (a - a^\dagger)/i$, $X_B = b + b^\dagger$, $P_B = (b - b^\dagger)/i$; outcomes of X and P ($\Delta X\Delta P \geq 1$) are denoted x and p, respectively. These states for large r are generalized macroscopic superpositions (1) of the continuous set of eigenstates $|x\rangle$ of the pointer-measurement X. The wave function is

$$\psi(x) = \exp[-x^2/4\Delta^2 x]/(2\pi\Delta^2 x)^{1/4}, \qquad (13)$$

where $\Delta^2 x = e^{2r}$ and $\Delta^2 x = \cosh(2r)$, respectively, for the single- and two-mode states. The probability distribution of p in the single-mode case is Gaussian with variance $\Delta^2 p = 1/\Delta^2 x$, indicating a "squeezing" of noise below the quantum limit of 1. The two-mode state has squeezing in the momenta sum and $\Delta^2_{\rm inf}p = 1/\Delta^2 x$ is obtained for the choice $g = \langle PP_B\rangle/\langle P_B P_B\rangle$ which minimizes $\Delta^2_{\rm inf}p$ [13]. The Gaussian distribution $P(x) = \exp[-x^2/2\Delta^2 x]/(\sqrt{2\pi}\Delta x)$ for X implies a macroscopic range of values x in the highly squeezed limit.

The squeezed state $S(r)|0\rangle$ with r large is a superposition possessing nonzero matrix elements $\langle x|\rho|x'\rangle$, where $x - x'$ is macroscopic. But whether or not such generalized macroscopic coherence is preserved in a real experiment given the sensitivity to loss is an open question. The inequalities (9) and (10) could be used to confirm the preservation of such macroscopic coherence. Violation of (9) and (10) is predicted (Fig. 2) for the ideal squeezed states to confirm superpositions (1) with $S = x' - x$ up to 0.5 of the standard deviation Δx of the Gaussian probability distribution $P(x)$. The observation of large squeezing ($\Delta^2 p = 1/\Delta^2 x \to 0$) for these minimum uncertainty squeezed states where $\Delta x\Delta p = 1$ will confirm a generalized macroscopic coherence (1) with $S \to \Delta x/2$.

However, while significant squeezing and Gaussian probability distributions have been measured [6,17], the states generated experimentally are not the ideal minimum uncertainty squeezed states defined by $S(r)|0\rangle$. Generally, we have $\Delta x\Delta p > 1$ (or $\Delta x\Delta_{\rm inf}p > 1$). For such Gaussian-squeezed states, the maximum S giving violation of (9) reduces from $0.5\Delta x$ to 0 as $\Delta x\Delta p$ (or $\Delta x\Delta_{\rm inf}p$) increases to $\sim$1.6 (Fig. 2). Tests of at least mesoscopic superpositions could be feasible though for well-squeezed systems that maintain a good approximation to the minimum uncertainty state.

To summarize, we have presented criteria for experimental confirmation of generalized macroscopic quantum superpositions. This is achieved by deriving inequalities that are experimentally satisfied if the system is describable as a mixture of underlying quantum states that exclude

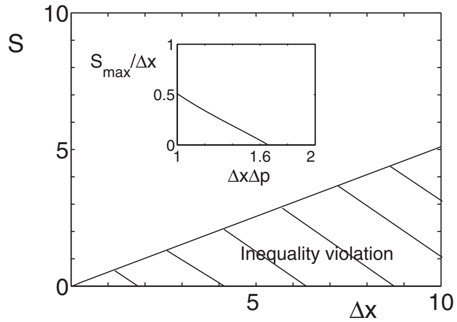

FIG. 2. Violation of (9) [and (10)] for single- (and two-mode) squeezed minimum uncertainty states. Inset shows behavior for general Gaussian-squeezed states. The maximum $S/\Delta x$ giving violation of (9) [and (10)] is plotted versus $\Delta x \Delta p$ (replace Δp with $\Delta_{\inf} p$ for the two-mode case).

these macroscopic superpositions. It is crucial to the derivation that these underlying states satisfy the Heisenberg uncertainty relations. Violations of the inequalities would therefore not rule out all hidden variable descriptions [18] compatible with a "macroscopic reality", such as those considered by Leggett and Garg [2] which allow for mixtures of hidden variable states. In this sense, the criteria cannot falsify all types of macroscopic realistic theories.

This point is nicely illustrated for the Gaussian-squeezed states which satisfy the criteria for generalized macroscopic superpositions. The quantum Wigner function $W(x, p)$ for $S(r)|0\rangle$ is positive, and it has been shown [18] that a hidden variable theory consistent with macroscopic reality reproduces the quantum predictions for X and P. In this hidden variable theory the system is defined to be in, with probability $W(x, p)$, a hidden variable state where variables x and p are defined simultaneously to be the outcomes of measurements X and P, respectively, should they be performed. There is no conflict with the system being in a quantum superposition because such a hidden variable state has a predetermined position and momentum specified more precisely than can be allowed by the uncertainty principle.

We note an analogy with the Einstein-Podolsky-Rosen paradox where it is shown that a consistency of the quantum predictions with a type of reality (in our case "macroscopic reality") is achieved if one invokes the use of hidden variables [11].

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[15] Schrödinger's premise is that the "cat" is a mixture of "dead" and "alive" states (so the cat cannot be both "dead and alive"). Where there is an outcome 0 ("coma") between dead and alive, we might permit a microscopic superposition (so the cat is "dead and coma" as allowed by ρ_L), but we expect the cat to be either dead and coma (ρ_R) or "alive and coma" (ρ_R). Superposition (1) defies this interpretation.
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