Anomalous Transparency of Water-Air Interface for Low-Frequency Sound

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Sound transmission through a water-air interface is normally weak because of a strong mass density contrast. We show that the transparency of the interface increases dramatically at low frequencies. Almost all acoustic energy emitted by a sufficiently shallow monopole source under water is predicted to be radiated into air. Increased transparency at lower frequencies is due to the increasing role of inhomogeneous waves. For sources symmetric with respect to a horizontal plane, transparency is further increased by a destructive interference of direct and surface-reflected waves under water. The phenomenon of anomalous transparency has significant geophysical and biological implications.

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Because of the stark mass density contrast between air and water, a water-air interface is normally considered as a perfectly reflecting, pressure-release boundary in underwater acoustics or a rigid boundary in atmospheric acoustics [1]. Ray-theoretical calculations predict weak coupling between sound fields in air and water, with an energy transmission coefficient on the order of the ratio of acoustic impedances of air and water [2,3]. However, at infrasonic frequencies, underwater sources are typically located within a fraction of the wavelength from the interface, and ray calculations cease to be applicable. In this Letter, we show that the water-air interface is anomalously transparent for sound radiated by shallow sources, and almost all of the energy emitted under water can be radiated into the air. For a monopole source at a depth which is small compared to the acoustic wavelength, the ratio of energy radiated into the air to the total emitted energy is larger by a factor of up to 3400 under normal conditions than it is for the same source located a wavelength or more from the interface. We also show that the increase in transparency of the interface and the absolute value of the acoustic power radiated into the air are sensitive to the type of the underwater acoustic source.

Instead of being an almost perfect mirror, as previously believed, a water-air interface can be a good conduit of low-frequency underwater sound into the atmosphere. The anomalous transparency of the water-air interface may have significant implications in problems that range from generation of low-frequency ambient noise in the air by bubbles collapsing under water and heating of the upper atmosphere due to absorption of infrasound to understanding the role of hearing in avian predation of aquatic animals and acoustic monitoring and detection of powerful underwater explosions for the purposes of the Comprehensive Nuclear-Test-Ban Treaty.

Previous theoretical [4–12] and experimental [7,9,10,12–15] studies of sound transmission through the air-water interface concentrated on acoustic fields under water due to airborne sources, primarily because of the existence of powerful noise sources in the atmosphere (such as helicopters [14], propeller-driven aircraft [10], and supersonic transport with their attending sonic booms [9,11,12,15]) and possible effects of these manmade sources on marine life [9,14].

We apply a theory of acoustic fields in layered media [3,16] to develop a full-wave description of sound transmission from water to air. Penetration of sound through the interface will be characterized by acoustic transparency, defined as the ratio of acoustic power radiated into air to the total acoustic power emitted by an underwater continuous wave source. Introduce Cartesian coordinate system $\mathbf{r} = (x, y, 0)$ with a vertical coordinate z increasing downward. Plane $z = 0$ separates homogeneous half-spaces $z > 0$ (water) and $z < 0$ (air), where sound speeds and mass densities are c_1, ρ_1 and $c_2 = c_1/n$, $\rho_2 = m\rho_1$, respectively. Representative values of the refraction index n and mass density ratio are $n = 4.5$, $m = 1.3 \cdot 10^{-3}$. It is the smallness of m and n^{-2} compared to unity that is responsible for peculiarities of sound transmission through the water-air interface.

Let a point source be situated at a point $(0, 0, z_0)$, $z_0 > 0$ in water. Acoustic pressure p_1 due to the source in the absence of an interface, wave p_2 reflected from the interface into water, and wave p_3 refracted into air are given by integrals over plane waves [16]:

$$p_j(\mathbf{r}) = (2\pi)^{-1} i \int d^2\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{r} + i\nu_1 z_0} \nu_1^{-1} Q_j(\mathbf{q}), \qquad j = 1, 2, 3, \tag{1}$$

$$\begin{aligned} Q_1 &= S_1(\mathbf{q}) e^{i\nu_1(z-2z_0)}, && z > z_0; \\ Q_1 &= S_2(\mathbf{q}) e^{-i\nu_1 z}, && z < z_0, \end{aligned} \tag{2}$$

$$\begin{aligned} Q_2 &= S_2(\mathbf{q}) V(q) e^{i\nu_1 z}, && z > 0; \\ Q_3 &= S_2(\mathbf{q}) W(q) e^{-i\nu_2 z}, && z < 0, \end{aligned} \tag{3}$$

where $\mathbf{q} = (q_1, q_2, 0)$, $q \equiv |\mathbf{q}|$; $\nu_s = (k_s^2 - q^2)^{1/2}$, $\mathrm{Im}\nu_s \geq 0$; $k_s = \omega/c_s$, $s = 1, 2$; ω is sound frequency, and

$$\begin{aligned} V &= (m\nu_1 - \nu_2)/(m\nu_1 + \nu_2), \\ W &= 2m\nu_1/(m\nu_1 + \nu_2) \end{aligned} \tag{4}$$

are Fresnel reflection and transmission coefficients [3] for an incident plane wave with the wave vector $(q_1, q_2, -\nu_1)$. Functions $S_1(\mathbf{q})$ and $S_2(\mathbf{q})$ are plane-wave spectra of the field emitted by the source downward and upward, respectively. These functions determine the source type. In particular, if $S_1 = S_2 = 1$, we have a monopole sound source with $p_1 = p_0$, $p_0 = R^{-1}\exp(ik_1R)$, $R = [x^2 + y^2 + (z - z_0)^2]^{1/2}$ [16]. When $S_1 = -S_2 = i\nu_1/k_1$, we have a vertically oriented dipole source with $p_1 = k_1^{-1}\partial p_0/\partial z$. The spectra $S_1 = S_2 = iq_1/k_1$ correspond to a horizontal dipole source with $p_1 = k_1^{-1}\partial p_0/\partial x$.

Wave vectors of reflected and refracted plane waves are (q_1, q_2, ν_1) and $(q_1, q_2, -\nu_2)$. According to Snell's law [3], the horizontal components of a wave vector do not change at reflection and refraction. Plane waves with $0 \le q \le k_1$ are homogeneous in water (i.e., $\mathrm{Im}\nu_1 = 0$) and give homogeneous refracted waves in air with refraction angles $0 \le \theta_2 \le \delta$, $\delta \equiv \arcsin n^{-1}$. Plane waves with $k_1 < q \le k_2$, which are inhomogeneous (evanescent) in water (i.e., $\mathrm{Im}\nu_1 > 0$), give homogeneous refracted waves in air with refraction angles $\delta < \theta_2 \le \pi/2$. When $q > k_2$, both incident and refracted waves are evanescent. Under normal conditions, the critical angle $\delta \approx 13°$.

The acoustic power flux J_a into air can be calculated by integrating the normal component of the acoustic power flux density $(2\omega\rho)^{-1}\,\mathrm{Im}(p^*\nabla p)$ [3] over the interface $z = 0$. Here the asterisk denotes complex conjugation. Using Eqs. (1)–(4), we obtain

$$J_a = \frac{J_0}{4\pi k_1}\int_{q<k_2} d^2\mathbf{q}|S_2(\mathbf{q})|^2 \exp(-2z_0\,\mathrm{Im}\nu_1)\,\mathrm{Re}\left(\frac{1 - |V|^2 + 2i\,\mathrm{Im}V}{\nu_1}\right), \tag{5}$$

where $J_0 = 2\pi/\rho_1 c_1$ is the acoustic power radiated by the waterborne monopole source in the absence of the interface. The dipole sources defined above radiate acoustic power $J_d = J_0/3$ in unbounded water.

The total power output of the generic source $J_t = J_a + J_w$ includes the acoustic power flux

$$J_w = \frac{J_0}{4\pi k_1}\int_{q<k_1} \frac{d^2\mathbf{q}}{\nu_1}|S_1(\mathbf{q}) + V(q)S_2(\mathbf{q})e^{2i\nu_1 z_0}|^2, \tag{6}$$

which is carried to infinity within water [17,18]. With Eqs. (5) and (6), the power fluxes J_a and J_w can be readily calculated numerically or evaluated analytically using the small parameters m and n^{-2} of the problem.

When $m \ll 1$, $V \approx -1$ except for plane waves with $k_2 - q = O(m^2)$. The power output of sound sources (Fig. 1) is very close to that in the case of a pressure-release boundary, where $V = -1$, with a possible exception for source depths z_0, which are small compared to the acoustic wavelength in water. For sources with antisymmetric plane-wave spectra ($S_1 = -S_2$), power output J_t nearly doubles at $z_0 \to 0$ compared to its value at large z_0, because of the constructive interference of incident (p_1) and reflected (p_2) waves. For sources with symmetrical spectra ($S_1 = S_2$) near the pressure-release boundary, the power output vanishes at $z_0 \to 0$ because of the destructive interference of incident and reflected waves. When $0 < m \ll 1$, J_t remains finite for all source depths and has a deep minimum (Fig. 1).

The directivity of radiation in the air is characterized by angular density D of the acoustic power flux: $J_a = \int_0^{\pi/2} D(\theta_2)d\theta_2$. D has a meaning of the source directivity factor averaged over the azimuthal angle. The angular density D is shown in Fig. 2. For shallow sources, the bulk of the radiation occurs at refraction angles $\theta_2 > \delta$. Radiation in such directions rapidly decreases with the

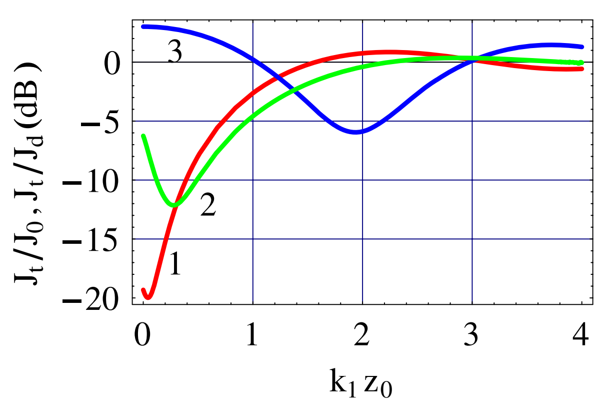

FIG. 1 (color online). Power output of (1) monopole, (2) horizontal dipole, and (3) vertical dipole sound sources. The power output J_t is normalized by its value in the absence of the interface; k_1 and z_0 are the acoustic wave number in water and the source depth, respectively.

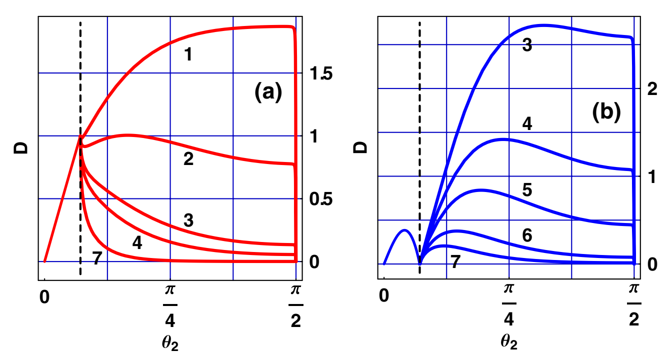

FIG. 2 (color online). Directivity D of sound radiated into air by a waterborne (a) monopole and (b) vertical dipole sources. The dotted line shows the critical angle $\delta \equiv \arcsin n^{-1}$. Radiated power is normalized by the power radiated by the same source in unbounded water. The refraction index and mass density ratio are $n = 4.5$, $m = 1.3 \cdot 10^{-3}$. Nondimensional source depth $k_1 z_0$ is (1) 0.1, (2) 0.2, (3) 0.4, (4) 0.5, (5) 0.6, (6) 0.8, and (7) 1.0.

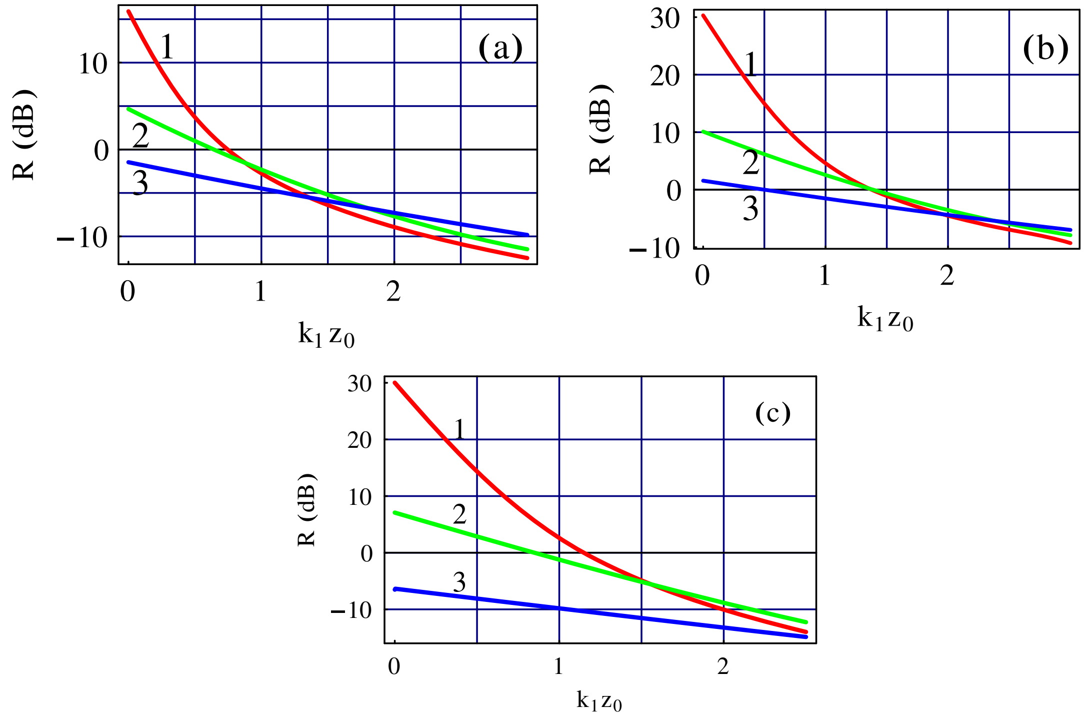

FIG. 3 (color online). Significance of inhomogeneous waves in sound transmission through a water-air interface. The ratio R of acoustic power fluxes into air due to inhomogeneous (evanescent) and homogeneous incident plane waves is shown as a function of nondimensional source depth $k_1 z_0$ for (a) monopole, (b) horizontal dipole, and (c) vertical dipole sources located under an interface with strong density contrast ($m \ll 1$) and three values of the refraction index n: (1) 1.1, (2) 1.5, and (3) 4.5.

depth increase, because of a decrease in amplitude of incident evanescent waves at the interface $z = 0$, as described by the exponential factor in Eq. (5). Ray theory accounts only for radiation in the directions $\theta_2 \leq \delta$. This radiation is due to incident homogeneous plane waves and is independent of the source depth.

The relative contribution of inhomogeneous waves to sound transmission through the interface is also sensitive to refraction index and source type (Fig. 3) but is insensitive to the density ratio m as long as the latter is small. Inhomogeneous waves play a dominant role when acoustic frequency is low and refraction index is large. According to Eqs. (4) and (5), for the water-air interface, the contribution of inhomogeneous waves exceeds that of homogeneous waves by the factors $2n^2[1 + O(n^{-2})] \approx 40$ and $(8n^4/3) \times [1 + O(n^{-2})] \approx 1100$ for very shallow monopole and dipole sources, respectively. As long as the contribution of homogeneous waves is independent of source depth and frequency, these factors also determine the increase in the absolute value of the acoustic power transmitted into air when a source depth and/or frequency decrease, so that the nondimensional source depth $k_1 z_0$ changes from $k_1 z_0 \gg 1$ to $k_1 z_0 \ll 1$. The role of inhomogeneous waves and, consequently, the increase in sound transmission into air are even greater for higher-order multipole sources due to the greater weight of plane waves with $1 < q < n$ in their spectra.

Because of the contribution of inhomogeneous waves, the acoustic transparency J_a/J_t of the water-air interface rapidly grows with diminishing source depth, when $k_1 z_0 < 1$, from its ray-theoretical value $O(m/n)$ at $k_1 z_0 \gg 1$ (Fig. 4). For sound sources with symmetrical spectra ($S_1 = S_2$), the transparency closely approaches unity: $J_w = J_a O(m)$ at $z_0 \to 0$; i.e., almost all emitted energy is radiated into the air (Fig. 4). Although counterintuitive, this phenomenon is easy to understand. Indeed, when such a

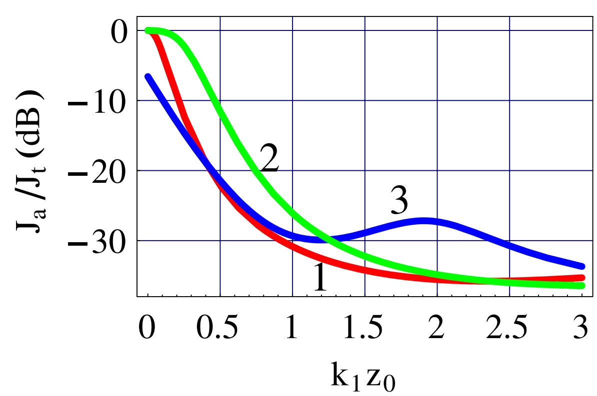

FIG. 4 (color online). Acoustic transparency of a water-air interface as a function of source depth for waterborne (1) monopole, (2) horizontal, or (3) vertical dipole sources. The refraction index and mass density ratio are $n = 4.5$, $m = 1.3 \cdot 10^{-3}$.

source is on the interface, acoustic pressure in both air (p_3) and water ($p_1 + p_2$) is proportional to the small parameter m [see Eqs. (1)–(4)]. Then, because of the different mass densities of the two media, the acoustic power flux in air is $J_0 O(m)$, while the acoustic power flux in water is $J_0 O(m^2)$.

In summary, contrary to the conventional wisdom based on ray-theoretical predictions and observations at higher frequencies, infrasonic energy from localized waterborne sources can be effectively transmitted into air. We have demonstrated theoretically that a water-air interface is anomalously transparent to low-frequency acoustic waves. The phenomenon of anomalous transparency occurs when a sound source is located at a shallow depth, meaning that the depth is a fraction of the acoustic wavelength. For shallow sources, acoustic intensity in the air increases dramatically due to energy transfer from the source by evanescent waves under water. Furthermore, almost all emitted acoustic energy is channeled into the air when sound is generated by a shallow monopole source or any other localized source with a radiation pattern symmetric with respect to the horizontal plane. For such sources, an increased power flux into the air due to evanescent waves is accompanied by a decrease in downward acoustic power flux due to the destructive interference of direct and surface-reflected waves under water.

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[16] L. M. Brekhovskikh and O. A. Godin, *Acoustics of Layered Media. 2: Point Sources and Bounded Beams* (Springer, Berlin, 1999), 2nd ed.
[17] An actual water-air interface is a rough surface rather than a plane. For low-frequency sound, roughness elevation h is small compared to other relevant spatial scales. Assuming $k_2 h \ll 1$, $h/z_0 \ll 1$, it can be shown that the effect of surface roughness on sound transmission and sound source output is negligible.
[18] For monochromatic sound, as opposed to transient waves, plane-wave components of the field that are inhomogenous in both water and air do not carry acoustic energy from the source. Although the vertical component of the acoustic power flux density $(2\omega\rho)^{-1}\,\mathrm{Im}(p^* \partial p/\partial z)$ is zero in a monochromatic inhomogeneous wave with the wave vector $(q_1, q_2, \pm i|\nu_1|)$, superposition of such inhomogeneous plane waves with opposite signs of the vertical component of the wave vector results in a nonzero power flux in the vertical direction, provided there is a phase shift between the two waves. In the water, inhomogeneous waves carry energy from the source to the air-water interface as a result of interference of incident and reflected inhomogeneous waves.