Measuring Shear Viscosity Using Transverse Momentum Correlations in Relativistic Nuclear Collisions

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Elliptic flow measurements at the Brookhaven National Laboratory Relativistic Heavy Ion Collider suggest that quark-gluon fluid flows with very little viscosity compared to weak-coupling expectations, challenging theorists to explain why this fluid is so nearly "perfect." It is therefore vital to find quantitative experimental information on the viscosity of the fluid. We propose that measurements of transverse momentum fluctuations can be used to determine the shear viscosity. We use current data to estimate the viscosity-to-entropy ratio in the range from 0.08 to 0.3 and discuss how future measurements can reduce this uncertainty.

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Measurements of elliptic and radial flow at the Brookhaven National Laboratory Relativistic Heavy Ion Collider (RHIC) are described by viscosity-free hydrodynamics, indicating that the quark-gluon system produced in these collisions is a nearly perfect liquid [1–4]. In particular, the strong suppression of flow due to shear viscosity predicted by weak-coupling transport calculations is not observed [3]. This result is exciting, because a small viscosity relative to the entropy density of the system may indicate that the system is more strongly coupled than expected: The collisional shear viscosity is proportional to the mean free path, which is shorter when the coupling is stronger. But is the viscosity really small? Hirano *et al.* point out that color glass condensate formation may produce more elliptic flow than considered in Refs. [2,3], requiring a larger viscosity for agreement with data [5].

We seek an experimental probe of viscosity that is independent of elliptic flow. To that end, we propose that transverse momentum correlation measurements can be used to extract information on the kinematic viscosity

$$\nu = \eta/Ts, \qquad (1)$$

where η is the shear viscosity, s is the entropy density, and T is the temperature. This ratio characterizes the strength of the viscous force relative to the fluid's inertia and, consequently, determines the effect of η on the flow [4]. We argue that viscous diffusion broadens the rapidity dependence of transverse momentum correlations and then show how these correlations can be extracted from measurements of event-by-event p_t fluctuations.

A number of experiments have studied transverse momentum fluctuations at CERN Super Proton Synchrotron and RHIC [6,7]. Interestingly, the STAR Collaboration reports a 60% increase of the relative rapidity width for p_t fluctuations when centrality is increased [8]. While the STAR analysis differs from the one we propose, model assumptions provide a tantalizing hint that the viscosity is small.

Any experimental information on the kinematic viscosity of high energy density matter is vital for understanding the strongly interacting quark-gluon plasma. Theorists had long anticipated a large collisional viscosity based on weak-coupling QCD [9] and hadronic computations [10], with values of η/s roughly of order unity for both phases near the crossover temperature $\sim$170 MeV. Supersymmetic Yang-Mills calculations give the significantly smaller ratio $\eta/s = 1/4\pi$ in the strong coupling limit [11]. Lattice QCD calculations of the shear viscosity will eventually settle the question of the size of the viscosity near equilibrium [12]. However, the effective viscosity in the nonequilibrium ion-collision system may differ from these calculations. In particular, plasma-instability contributions can also explain the small viscosities in nuclear collisions [13].

We begin by formulating a simple model to illustrate how shear viscosity attenuates correlations due to fluctuations of the radial flow. Next, we show how transverse momentum fluctuations can be used to measure these correlations. We then demonstrate the impact of viscosity on the rapidity distribution of fluctuations. Finally, we explore the implications of current fluctuation data.

Before wading into the quark-gluon liquid, it is useful to recall how shear viscosity affects the flow of more common fluids. In a classic example of shear flow, a liquid is trapped between two parallel plates in the xy plane, while one plate moves at constant speed in the x direction. The fluid is pulled along with the plate, so that v_x varies with the normal distance z. In this case,

$$T_{zx} = -\eta \partial v_x/\partial z \qquad (2)$$

is the viscous contribution to the stress energy tensor.

Central nuclear collisions produce a high energy density fluid that flows outward with an average radial velocity v_r. In the hydrodynamic description of these collisions, we typically assume that v_r varies smoothly with spacetime $(t, \mathbf{x})$ and is the same for all collisions of a fixed impact parameter. For central collisions, v_r is cylindrically symmetric. More realistically, small deviations $\mathbf{u}(\mathbf{x})$ of the radial flow occur throughout the fluid, varying with each ion-collision event. Such deviations occur, e.g., because the number and location of nucleon-nucleon subcollisions vary in each event.

Viscous friction arises as neighboring fluid elements flow past each other. This friction reduces $\mathbf{u}$, driving the velocity toward the local average v_r. The final size of the velocity increment $\mathbf{u}$ depends on the magnitude of the viscosity and the lifetime of the fluid.

In order to illustrate how the damping of radial flow fluctuations depends on the viscosity of the fluid, we introduce a velocity increment in the radial direction u that depends only on the longitudinal coordinate z and t. Our aim is to determine the linear response of the fluid to this perturbation. For simplicity, we take the unperturbed flow as slowly varying and work in a comoving frame where v_r locally vanishes. As in (2), the flow of neighboring fluid elements at different radial speeds $u(z)$ produces a shear stress

$$T_{zr} = -\eta \partial u/\partial z. \tag{3}$$

This stress changes the radial momentum current of the fluid, which is generally $T_{0r} = \gamma^2(\epsilon + p)v_r$ for energy density ϵ, pressure p, and $\gamma = (1 - v^2)^{-1/2}$. The perturbation u results in the change $g_t(\mathbf{x}) = \delta T_{0r} \approx (\epsilon + p)u$ in the comoving frame, while energy-momentum conservation $\partial_\mu T^{\mu\nu} = 0$ implies $\partial g_t/\partial t = -\partial T_{zr}/\partial z$.

We combine these results to obtain a diffusion equation for the momentum current:

$$\partial g_t/\partial t = \nu \nabla^2 g_t \tag{4}$$

to linear order, where the kinematic viscosity is given by (1), since $\epsilon + p \approx Ts$ for small net baryochemical potential $\mu \approx 0$. Observe that (4) applies for any fluctuation $\mathbf{g}_t$ for which $\nabla \cdot \mathbf{g}_t = 0$; our physically motivated radial $g_t(z, t)$ is a specific instance of such a flow. Such shear modes are related to sound waves (compression modes) but diffuse rather than propagate.

Viscosity tends to reduce fluctuations by distributing the excess momentum density g_t over the collision volume. This effect broadens the rapidity profile of fluctuations. We write (4) in terms of the spatial rapidity $y = 1/2 \ln(t + z)/(t - z)$ and proper time $\tau = (t^2 - z^2)^{1/2}$ to find $\partial g_t/\partial \tau = (\nu/\tau^2)\partial^2 g_t/\partial y^2$. A similar equation is used to study net charge diffusion in Ref. [14], and we can translate many of those results to the present context. Defining $V \equiv \langle (y - \langle y \rangle)^2 \rangle = \int y^2 g_t dy / \int g_t dy$ for $\langle y \rangle = 0$, we compute the rapidity broadening

$$\Delta V = \frac{2\nu}{\tau_0}\left(1 - \frac{\tau_0}{\tau}\right), \tag{5}$$

where $\Delta V \equiv V - V(\tau_0)$ for τ_0 the formation time.

To address an ensemble of more general fluctuations, we consider the correlation function

$$r_g = \langle g_t(\mathbf{x}_1) g_t(\mathbf{x}_2) \rangle - \langle g_t(\mathbf{x}_1) \rangle \langle g_t(\mathbf{x}_2) \rangle. \tag{6}$$

In local equilibrium, r_g has the value $r_{g,\mathrm{eq}}$. The spatial rapidity dependence of $\Delta r_g \equiv r_g - r_{g,\mathrm{eq}}$ is broadened by momentum diffusion. If the rapidity width of the one-body density follows (5), then the width of Δr_g in the relative rapidity $y_r = y_1 - y_2$ grows from an initial value σ_o following

$$\sigma^2 = \sigma_o^2 + 2\Delta V(\tau_f), \tag{7}$$

where τ_f is the proper time at which freeze-out occurs. This equation is entirely plausible, since diffusion spreads the rapidity of each particle in a given pair with a variance ΔV. We then take

$$\Delta r_g(y_r, y_a) \propto e^{-y_r^2/2\sigma^2 - y_a^2/2\Sigma^2}, \tag{8}$$

where (7) gives the width in relative rapidity and the width in average rapidity $y_a = (y_1 + y_2)/2$ is Σ. We assume $\Sigma \gg \sigma$ [14]. Observe that (7) and (8) are exact for our diffusion model [14].

Gyulassy and Hirano surveyed computations of the ratio of the shear viscosity to the entropy and found that both the hadron gas and the perturbative quark-gluon plasma have $\eta/s \sim 1$, if one naively extrapolates these calculations near T_C [4]. These values correspond to $\nu = \eta/Ts$ roughly of order 1 fm for $T_C = 170$ MeV. On the other hand, they argue that the entropy increase near T_C reduces η/s for a strongly interacting plasma, perhaps to the supersymmetric Yang-Mills value $\eta/s = 1/4\pi$.

Motivated by these estimates, we show in Fig. 1 the increase of σ given by (5) and (7) as a function of τ. Calculations for two values $\nu/\tau_0 \sim 0.1$ and 1 schematically exhibit the likely range of viscous broadening. For $\tau_0 \sim 1$ fm, these values, respectively, correspond to $\eta/s \sim 1/4\pi$ and 1. We provide these calculations as benchmarks; more realistically, ν would effectively increase with τ depending on the state of the fluid.

We stress that the rapidity width depends on the viscous diffusion coefficient integrated over the collision lifetime. Comparing the viscous and perfect scenarios in Fig. 1, we see that the largest contribution to this width comes from

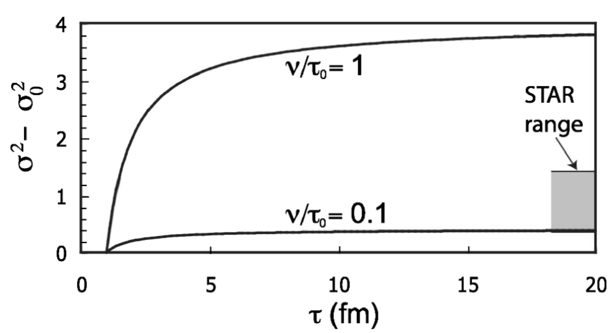

FIG. 1. Rapidity spread vs time for momentum diffusion from (5) and (7) for two viscosity values. The gray area marks the range extrapolated from data in Ref. [8] using (14).

the earliest times. Consequently, we expect measurements of this width to yield information on the viscosity when the evolution is dominated by partons.

Variation of the radial fluid velocity over the collision volume induces correlations in the transverse momenta p_t of particles [15]. To describe such correlations, we divide the inhomogeneous fluid into cells small enough to be uniform. Particles emerging from cells of different radial velocity v_r are more likely to have different p_t than particles from the same cell. The number of particles of momentum $\mathbf{p}$ in a cell at position $\mathbf{x}$ at the instant of freeze-out is $dn = f(\mathbf{x}, \mathbf{p})dpdx$, where $dp \equiv d^3p/(2\pi)^3$ and $dx \equiv d^3x$. We take $f(\mathbf{x}, \mathbf{p})$ to be a Boltzmann distribution corresponding to a fluid velocity $\mathbf{v}(\mathbf{x})$ and a temperature $T(\mathbf{x})$ that vary with each event. A similar formulation is used in Ref. [16] to compute nonequilibrium p_t fluctuations. Here we focus on central collisions where local equilibrium is likely achieved.

To characterize the dynamic correlations of p_t, we use the transverse momentum covariance

$$\mathcal{C} = \langle N\rangle^{-2}\left\langle\sum_{i\neq j} p_{ti}p_{tj}\right\rangle - \langle p_t\rangle^2, \tag{9}$$

where i labels particles from each event and the brackets represent the event average. The average transverse momentum is $\langle p_t\rangle \equiv \langle\sum p_{ti}\rangle/\langle N\rangle$. This covariance vanishes in local equilibrium, where the momenta are uncorrelated and number fluctuations satisfy Poisson statistics.

This covariance is related to the spatial correlations of the momentum current (6) by

$$\mathcal{C} = \langle N\rangle^{-2}\int \Delta r_g(\mathbf{x}_1, \mathbf{x}_2)dx_1dx_2. \tag{10}$$

To obtain this result, observe that near local equilibrium $f(\mathbf{x}, \mathbf{p}) = \langle f\rangle + \delta f$, where the average distribution is $\langle f(\mathbf{x}, \mathbf{p})\rangle$ and the eventwise deviation δf is necessarily small. Then $\langle N\rangle\langle p_t\rangle = \langle\int p_t dn\rangle = \int p_t\langle f\rangle dpdx + \int\langle g_t(\mathbf{x})\rangle dx \equiv \int p_t\langle f\rangle dpdx$. The contribution of fluctuations to the momentum current

$$g_t(\mathbf{x}) = \int \delta f(\mathbf{x}, \mathbf{p})p_t dp \tag{11}$$

vanishes on event averaging. Similarly, the unrestricted sum is $\langle\sum p_{ti}p_{tj}\rangle = \langle\int p_{t1}p_{t2}dn_1dn_2\rangle = \langle N\rangle^2\langle p_t\rangle^2 + \int\langle g_t(\mathbf{x}_1)g_t(\mathbf{x}_2)\rangle dx_1dx_2$. We find

$$\int r_g dx_1dx_2 = \left\langle\sum_{\text{all } i,j} p_{ti}p_{tj}\right\rangle - \langle N\rangle^2\langle p_t\rangle^2$$
$$= \langle N\rangle^2\mathcal{C} + \left\langle\sum p_{ti}^2\right\rangle; \tag{12}$$

the second equality follows from (9). In local equilibrium, $\mathcal{C} \equiv 0$ implies $\int r_{g,\text{eq}}dx_1dx_2 = \langle\sum p_{ti}^2\rangle$. Subtracting this term from (12) gives (10).

The correlation information probed by $\mathcal{C}$ differs from that found in the multiplicity variance $R = (\langle N^2\rangle - \langle N\rangle^2 - \langle N\rangle)/\langle N\rangle^2$. As before, we write $R = \langle N\rangle^{-2} \times \int\Delta r_n dx_1dx_2$, where $\Delta r_n = r_n - r_{n,\text{eq}}$ and

$$r_n = \langle n(\mathbf{x}_1)n(\mathbf{x}_2)\rangle - \langle n(\mathbf{x}_1)\rangle\langle n(\mathbf{x}_2)\rangle. \tag{13}$$

The density correlation function (13) carries different information than (6) because particle number is not conserved. Density fluctuations evolve by the full hydrodynamic equations, while g_t follows diffusion.

Viscosity information can be obtained from $\mathcal{C}$ as follows. For simplicity, we identify spatial and momentum space rapidity. The broadening in rapidity of Δr_g depends on the shear viscosity via (7). Equation (10) implies that the rapidity dependence of Δr_g can be measured by studying the dependence of (9) on the rapidity window in which particles are measured. We illustrate this acceptance dependence in Fig. 2 for the ν/τ_0 values from Fig. 1 by integrating (8) over the interval $-\Delta/2 \leq y_1, y_2 \leq \Delta/2$; $\langle N\rangle\mathcal{C}_\infty$ is the value for the full rapidity range. We assume $\tau_f/\tau_0 \sim 20$.

The STAR analysis in Ref. [8] incorporates some of these ideas and, intriguingly, finds a broadening in rapidity together with a narrowing in azimuth for p_t correlations in central compared to peripheral collisions. We will use the rapidity information to estimate the viscosity. However, the measured quantities differ sufficiently from $\mathcal{C}$ that this estimate requires significant model assumptions. We therefore regard the result only as a signal of our method's promise.

STAR employs the transverse momentum fluctuation observable $\Delta\sigma^2_{p_t:n}$ to construct a correlation function as a function of rapidity and azimuthal angle. They find that nearside correlations in azimuth are broadened in relative rapidity, with a rapidity width σ_* that increases from roughly 0.45 in the most peripheral collisions to 0.75 in central ones [8]. In our terms, $\langle N\rangle\Delta\sigma^2_{p_t:n} = \langle\sum_{i\neq j}(p_{ti} - \langle p_t\rangle)(p_{tj} - \langle p_t\rangle)\rangle$, so that $\Delta\sigma^2_{p_t:n}/\langle N\rangle = \mathcal{C} - \langle p_t\rangle^2R$. This quantity therefore depends on both momentum current and density correlation functions (6) and (13),

$$\Delta\sigma^2_{p_t:n} = \langle N\rangle^{-1}\int\{\Delta r_g - \langle p_t\rangle^2\Delta r_n\}dy_1dy_2. \tag{14}$$

We can directly compare σ_* to σ in Fig. 1 if Δr_g and Δr_n have the same widths. Equation (7) then implies that the

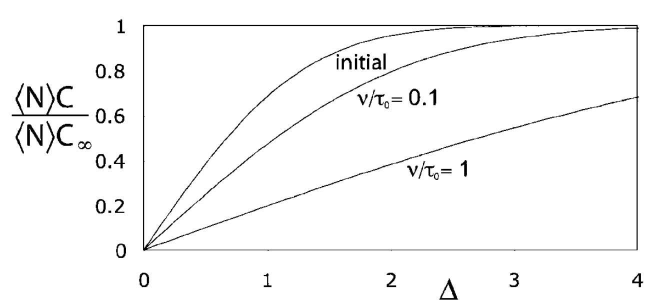

FIG. 2. Rapidity dependence of the p_t covariance (9) for $\tau_f/\tau_0 \sim 20$. The initial distribution has $\sigma_o \sim 0.5$.

widths in central and peripheral collisions satisfy $\sigma_c^2 - \sigma_p^2 = 4\nu(\tau_{f,p}^{-1} - \tau_{f,c}^{-1})$. Observe that the dependence on τ_0 cancels. Taking the freeze-out times in central and peripheral collisions to be $\tau_{f,c} \sim 20$ fm and $\tau_{f,p} \sim 1$ fm, respectively, we then find $\nu \sim 0.09$ fm. The value $\tau_{f,p} \sim 1$ fm is reasonable, since Ref. [8] argues that the average participant path length is about 1 fm for these peripheral collisions. We use (1) to find $\eta/s \sim 0.08$.

This result is remarkably close to the supersymmetric Yang-Mills value $1/4\pi$ and is consistent with some hydrodynamic comparisons to elliptic flow data [3]. However, we must be cautious: If Δr_g and Δr_n have different rapidity widths σ and σ_n, then their relation to σ_* depends on the relative strength of these contributions. The data in Ref. [17] may indicate that σ_n is roughly twice σ_*. Generally, σ is bounded by σ_n and σ_*, since (14) implies $\sigma_*^2 \approx \sigma^2 + \beta(\sigma^2 - \sigma_n^2)$. Although β is not measured, the width cannot exceed $\sigma_n \sim 2\sigma_*$. At the maximum value $\sigma = 2\sigma_*$, our dynamic assumptions yield $\eta/s = 0.3$. Together, our estimates constitute an uncertainty range for the viscosity-to-entropy ratio $0.08 < \eta/s < 0.3$. We also indicate the range of $\sigma_c^2 - \sigma_p^2$ implied by the STAR data in Fig. 1 as a gray band corresponding to $\sigma_* < \sigma < 2\sigma_*$.

In summary, we find that shear viscosity can broaden the rapidity correlations of the momentum current. This broadening can be observed by measuring the transverse momentum covariance (9) as a function of rapidity acceptance. Our rough estimate from current data $\eta/s \sim$ 0.08–0.3 is small compared to perturbative computations [4]. To reduce the experimental uncertainty, we suggest measuring $\mathcal{C}$ to allow more direct access to the momentum density correlation function. That said, we stress that there is additional theoretical uncertainty in this estimate, mainly due to our freeze-out model. In practice, $\sigma_c^2 - \sigma_p^2 \approx 4\nu\tau_{f,p}^{-1}$, since $\tau_{f,p} \ll \tau_{f,c}$. The freeze-out time in peripheral collisions $\tau_{f,p}$ is not plausibly smaller than our value 1 fm (the nucleon radius) but may be twice as large. This would double the upper limit of our uncertainty band. That added uncertainty can eventually be reduced by measuring $\tau_{f,p}$ as in Ref. [18]. Identical particle and resonance effects omitted here may contribute only at the 10% and 15% levels, respectively [7]. Minijets, color glass, and other particle production effects modify σ_o in (7). We assume that any modification cancels in studying the centrality dependence at fixed beam energy. Additionally, our linearized diffusion model of flow fluctuations is physically reasonable but highly idealized. A more general hydrodynamic description will be necessary to confront the measurements we suggest.

How can we reduce the theoretical uncertainty? The viscosity of a common fluid can be measured by applying a known pressure and observing the resulting flow in a fixed geometry, e.g., a pipe. Alternatively, one can study the attenuation of high frequency sound waves from a calibrated source. Efforts to compare flow measurements to viscous hydrodynamic calculations are analogous to the first method [3]. Our observable $\mathcal{C}$ is in the spirit of ultrasonic attenuation. The early dynamics produces a spectrum of fluctuations analogous to sound waves that are attenuated by viscosity. We suggest that experimenters pursue both approaches, since the initial conditions and model parameters are all unknown.

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[1] BRAHMS, PHENIX, PHOBOS, and STAR White Papers, Nucl. Phys. **A757**, 1 (2005).
[2] P. Huovinen and P. V. Ruuskanen, nucl-th/0605008.
[3] D. Teaney, Phys. Rev. C **68**, 034913 (2003); J. Phys. G **30**, S1247 (2004).
[4] T. Hirano and M. Gyulassy, Nucl. Phys. **A769**, 71 (2006).
[5] T. Hirano, U.W. Heinz, D. Kharzeev, R. Lacey, and Y. Nara, Phys. Lett. B **636**, 299 (2006).
[6] C. Pruneau, Proceedings of Quark Matter 2005 (to be published); J. T. Mitchell, J. Phys. G **30**, S819 (2004).
[7] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **72**, 044902 (2005).
[8] J. Adams *et al.* (STAR Collaboration), J. Phys. G **32**, L37 (2006).
[9] A. Hosoya and K. Kajantie, Nucl. Phys. **B250**, 666 (1985); P. Danielewicz and M. Gyulassy, Phys. Rev. D **31**, 53 (1985); P. Arnold, G. D. Moore, and L. G. Yaffe, J. High Energy Phys. 11 (2000) 001; 05 (2003) 051.
[10] S. Gavin, Nucl. Phys. **A435**, 826 (1985); A. Muronga, Phys. Rev. C **69**, 044901 (2004); S. Muroya and N. Sasaki, Prog. Theor. Phys. **113**, 457 (2005).
[11] P. Kovtun, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **94**, 111601 (2005); G. Policastro, D. T. Son, and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001).
[12] A. Nakamura and S. Sakai, Nucl. Phys. **A774**, 775 (2006).
[13] M. Asakawa, S. A. Bass, and B. Muller, Phys. Rev. Lett. **96**, 252301 (2006).
[14] M. Abdel-Aziz and S. Gavin, Phys. Rev. C **70**, 034905 (2004).
[15] S. A. Voloshin, Phys. Lett. B **632**, 490 (2006).
[16] S. Gavin, Phys. Rev. Lett. **92**, 162301 (2004).
[17] J. Adams *et al.* (STAR Collaboration), Phys. Rev. C **73**, 064907 (2006).
[18] M. Lisa, Acta Phys. Pol. B **35**, 37 (2004); C. Markert, J. Phys. G **31**, S169 (2005).