Electronic Aharonov-Bohm Effect Induced by Quantum Vibrations

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Mechanical displacements of a nanoelectromechanical system shift the electron trajectories and hence perturb phase coherent charge transport through the device. We show theoretically that in the presence of a magnetic field such quantum-coherent displacements may give rise to an Aharonov-Bohm-type of effect. In particular, we demonstrate that quantum vibrations of a suspended carbon nanotube result in a positive nanotube magnetoresistance, which decreases slowly with the increase of temperature. This effect may enable one to detect quantum displacement fluctuations of a nanomechanical device.

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Following the ubiquitous trend of downsizing devices, micromechanical systems (MEMS) are today evolving into nanoelectromechanical systems (NEMS) rapidly approaching the limits set by the laws of quantum mechanics [1]. The ultimate potential for nanomechanical devices is governed by the ability to detect the NEMS motional response to various external stimuli. In the quantum regime of operation optical and other sensing methods used in MEMS are not practical and one has turned instead to various mesocopic sensing devices. Much work still needs to be done. For example, even though one has recently been able to detect flexural vibrations of a SiN beam resonator with the amazing sensitivity of 10^{-13} m using a radio-frequency single-electron transistor [2], this is still about 6 times the quantum limit set by the amplitude of the zero-point oscillations of the beam.

In this Letter we propose a different approach to sensing ultrasmall quantum vibrations of a beam—we have a suspended carbon nanotube in mind—and show that coherent tube vibrations can induce an effectively multiconnected electron path through the tube. Through an Aharonov-Bohm-type effect this in turn gives rise to a negative magnetoconductance that can be detected. We propose that this is but one example of how employing quantum coherence in both the electronic and mechanical degrees of freedom may lead to new functionality and novel applications.

Fundamental research on NEMS in the quantum coherent regime can profit from analogies with mesoscopic phenomena in confined structures, where the conductance may depend on quantum interference between electron waves. It is, e.g., well known that the conductance of a mesoscopic sample changes if a single impurity is displaced a small distance. We will show that a similar effect can be achieved by the mechanical displacement corresponding to quantum vibrations of a suspended carbon nanotube in the presence of a magnetic field. To this end, consider first the structure shown in Fig. 1(b). Here a beam of electrons can pass through two openings in an otherwise opaque screen. Interference between the quantum amplitudes for going through one or the other of the two holes will determine the probability for electrons to hit the detector. Obviously no such interference effect is expected with only one hole, as in Fig. 1(a). Even if the position of the hole in the screen were to move, no interference would occur if the trajectory of the hole is classically well defined. In every instant, only one classical trajectory would be relevant. This situation changes qualitatively if the hole motion is in the quantum regime, where its position cannot

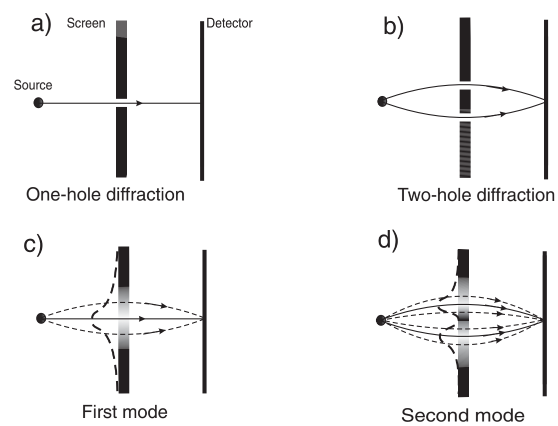

FIG. 1. Illustration of multiconnectivity in phase coherent electron transport by analogy with electron diffraction. In panel (b) electrons pass through two holes in a screen before hitting the detector. Unlike in panel (a), where the screen has only one hole, interference between the amplitudes of the two quasiclassical electron paths (solid arrows) determines the probability for electrons to hit the detector. In panels (c) and (d) electrons pass through a "quantum" hole, whose position is determined by a wave function with zero (c) and one (d) node. The fuzziness of the hole effectively creates different quantum electron paths of high (solid arrows) and low (dashed arrows) probability amplitude. In the text this analogy is shown to be valid for electrons tunneling through a suspended carbon nanotube in situations where only a few vibration states are allowed to couple to the electrons.

be precisely defined and ''quantum alternative'' paths for the electrons through the screen appear. Figures 1(c) and 1(d) show two examples of such a ''quantum geometry'', giving rise to electron paths through the hole that are multiply connected. Such an intimate, coherent nanoelectromechanical coupling between electronic and mechanical degrees of freedom leads to characteristic quantum interference among the electron paths and, in particular, to an Aharonov-Bohm-type of effect in the presence of an external magnetic field.

Because of their low mass and unique mechanical and electronic properties, single-wall carbon nanotubes (SWNTs) offer perhaps the best possibility for studying quantum nanoelectromechanical phenomena [3,4]. Figure 2 shows a sketch of the system we have in mind to achieve coherent coupling between quantum electronic transport and quantum flexural vibrations of a nanotube: a free-hanging SWNT, doubly clamped to two metallic leads and subject to a transverse magnetic field, H. This system is described by the Hamiltonian,

$$\hat{H} = \hat{H}_{\rm leads} + \hat{H}_{\rm el} + \hat{H}_{\rm mech} + \hat{H}_{\rm tunn}, \quad (1)$$

where the first term,

$$\hat{H}_{\rm leads} = \sum_k \varepsilon_{l,k} \hat{a}^\dagger_{l,k}\hat{a}_{l,k} + \sum_k \varepsilon_{r,k}\hat{a}^\dagger_{r,k}\hat{a}_{r,k}, \quad (2)$$

models electrons in states k in the left (l) and right (r) leads and $\hat{a}^\dagger_{l/r,k}$ [$\hat{a}_{l/r,k}$] is the corresponding creation [annihilation] operator. The second term,

$$\hat{H}_{\rm el} = \int d^3\vec{r}\Big\{-\frac{\hbar^2}{2m}\hat{\psi}^\dagger(\vec{r})\Big(\frac{\partial}{\partial\vec{r}} - \frac{ie}{c\hbar}\vec{A}(\vec{r})\Big)^2\hat{\psi}(\vec{r}) + U(y-\hat{u}(x), z)\hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r})\Big\}, \quad (3)$$

describes the SWNT electrons, confined in the transverse direction by a potential $U(y, z)$ that depends on the deflection $u(x)$ of the tube (in the y direction). The operator $\hat{\psi}^\dagger(\vec{r})$ [$\hat{\psi}(\vec{r})$] creates [annihilates] an electron at $\vec{r} = (x, y, z)$; $\{\hat{\psi}^\dagger(\vec{r}), \hat{\psi}(\vec{r}')\} = \delta(\vec{r}-\vec{r}')$ and $\vec{A}(\vec{r}) = (-Hy, 0, 0)$.

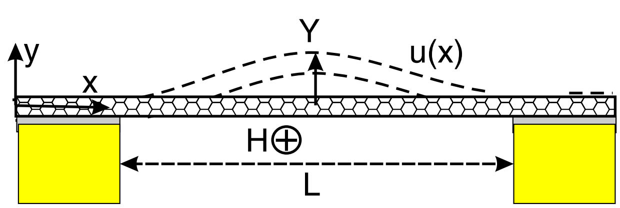

FIG. 2 (color online). Nanoelectromechanical system proposed to show the coherent coupling between quantum electron transport and quantum flexural vibrations discussed in the text. Electrons tunneling through a doubly clamped SWNT excite quantized vibrations of the SWNT in the presence of a magnetic field, H. The resulting effective multiconnectivity of the system leads to a negative magnetoconductance (see text).

The bending of the tube is modeled by the third term in the Hamiltonian (1) as

$$\hat{H}_{\rm mech} = \int_{-L/2}^{L/2} dx\Big\{\frac{1}{2\rho}\hat{\pi}^2(x) + \frac{EI}{2}\Big(\frac{\partial^2\hat{u}(x)}{\partial x^2}\Big)^2\Big\}. \quad (4)$$

Here $\hat{\pi}(x)$ is the momentum density operator conjugate with the deflection field operator $\hat{u}(x)$, i.e. $[\hat{u}(x), \hat{\pi}(x')] = i\hbar\delta(x-x')$, ρ is the linear mass density of the SWNT, I is its area moment of inertia and E is the Young's modulus. The tube is doubly clamped, which gives the boundary conditions $u(x) = 0$ and $u'(x) = 0$ for $|x| \geq L/2$.

The tunneling Hamiltonian, $\hat{H}_{\rm tunn} = \hat{T}_l + \hat{T}_r$, where

$$\hat{T}_{l/r} = \sum_k \int d\vec{r}\mathcal{T}_{l/r}(\vec{r}, k)\hat{\psi}^\dagger(\vec{r})\hat{a}_{l/r,k} + {\rm H.c.}, \quad (5)$$

and $\mathcal{T}_{l/r}(\vec{r}, k)$ are overlap integrals, describes how electron tunneling couples the SWNT and the two leads.

In order to proceed it is convenient to make the unitary transformation $e^{i\hat{S}}\hat{H}e^{-i\hat{S}}$, with

$$\hat{S} = -i\int d^3\vec{r}\Big\{\hat{u}(x)\hat{\psi}^\dagger(\vec{r})\frac{\partial\hat{\psi}(\vec{r})}{\partial y} + i\frac{eH}{\hbar c}\Big(\int_0^x dx'\hat{u}(x')\Big)\hat{\psi}^\dagger(\vec{r})\hat{\psi}(\vec{r})\Big\}.$$

Here the first term produces a coordinate transformation to the nanotube reference frame, while the second generates a gauge transformation that eliminates the vector potential from the Hamiltonian $\hat{H}_{\rm el}$. Furthermore, since the transverse electron motion in the SWNT is strongly quantized it may be decoupled from the longitudinal motion by letting $\hat{\psi}(\vec{r}) = \Psi(y, z)\hat{\psi}(x)$. Here $\Psi(y, z) = \Psi(\vec{r}_t)$ is the wave function corresponding to a transverse quantized energy level E_t. As a result the terms $\hat{H}_{\rm el}$, $\hat{H}_{\rm mech}$ and $\hat{T}_{l/r}$ in the Hamiltonian (1) simplify to

$$\begin{aligned}\hat{H}_{\rm el} &= \int dx\hat{\psi}^\dagger(x)\Big(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \varepsilon_t\Big)\hat{\psi}(x),\\ \hat{H}_{\rm mech} &= \int_{-L/2}^{L/2} dx\Big\{\frac{1}{2\rho}\Big(\hat{\pi}(x) - \frac{eH}{c}\int_0^x dx'\hat{\psi}^\dagger(x)\hat{\psi}(x)\Big)^2 + \frac{EI}{2}\Big(\frac{\partial^2\hat{u}(x)}{\partial x^2}\Big)^2\Big\},\\ \hat{T}_{l/r} &= \exp\Big[i\frac{eH}{\hbar c}\int_0^{\mp L/2} dx\hat{u}(x)\Big] \times \sum_k \int dx T_{l/r}(x, k)\hat{\psi}^\dagger(x)\hat{a}_{l/r,k} + {\rm H.c},\end{aligned} \quad (6)$$

where $T_{l/r}(x, k) = \int d\vec{r}_t\mathcal{T}_{l/r}(\vec{r}, k)|\Psi(\vec{r}_t)|^2$ and [5] $\varepsilon_t = E_t + (e^2H^2/2mc^2)\iint d\vec{r}_t y^2|\Psi(\vec{r}_t)|^2$.

By analogy one may think of the elementary excitations created by $\hat{\psi}^\dagger(x)$ in the transformed Hamiltonian (6) as polarons. It is important that due to the quantum vibrations of the nanotube the wave function of this polaronic state is extended in the direction perpendicular to the tube axis.

These "swinging states" serve as intermediate states for electrons tunneling through the vibrating SWNT. The quantum phase acquired by the tunneling electrons depends on magnetic field and below we will show that a nanomechanically induced magnetoconductance may follow.

So far our approach has been quite general, but here we will make some approximations that allow us to find an analytical solution. First, we consider the weak coupling limit, where the spacing $\delta\varepsilon \simeq hv_F/4L_0$ of quantized energies associated with the *longitudinal* motion of electrons along a SWNT of total length L_0 is much larger than the level broadening due to coupling to the continuous energy spectra in the leads (h is Planck's constant, v_F the Fermi velocity). In this case only a few longitudinal states with energy ε_n are relevant. Further simplifications arise if one restricts the mechanical SWNT dynamics to the fundamental bending mode, which gives the most important contribution, and expresses the corresponding deflection operator as $\hat{u}(x) = Y_0 u_0(x/L)(\hat{b}^\dagger + \hat{b})/\sqrt{2}$, where $u_0(\xi)$ is the normalized profile of the fundamental bending mode, $Y_0 = (\hbar^2 L^2/\beta_0 \rho EI)^{1/4}$ is the amplitude of zero-point fluctuations in the fundamental mode, β_0 is the dimensionless eigenvalue associated with this mode and $\hat{b}^\dagger$ [$\hat{b}$] is a boson operator that creates [annihilates] one vibration quantum.

Next we consider a nonresonant case where no electron energy level in the SWNT is close enough to the chemical potential μ in the leads to be involved in even inelastic electron tunneling via a real state on the tube. This requires the criteria $|\varepsilon_n + \varepsilon_t - \mu| \gg \nu |T_{l/r}|^2, \hbar\omega, eV$ to be fulfilled, where ν is the electron density of states in the leads, $\omega = (\beta_0 IE/\rho)^{1/2}/L^2$ is the eigenfrequency of the fundamental mode, and V is the applied bias voltage. Under such conditions the coupling between electronic states in the left and right leads occurs via virtual states on the tube and may be described by an overlap integral. As a result one comes to a tunneling Hamiltonian,

$$\hat{H}_{\text{eff}} = \sum_{k;\sigma=l,r} \varepsilon_{\sigma,k} \hat{a}^\dagger_{\sigma,k} \hat{a}_{\sigma,k} + \hbar\omega \hat{b}^\dagger \hat{b} + e^{i\phi(\hat{b}^\dagger + \hat{b})} \sum_{k,k'} T_{\text{eff}}(k,k') \hat{a}^\dagger_{r,k} \hat{a}_{l,k'} + \text{H.c.}, \tag{7}$$

that describes nonresonant charge transfer through the suspended SWNT. In this equation $\phi = g4\pi Y_0 LH/\Phi_0$ is a dimensionless magnetic flux, $\Phi_0 = hc/e$ is the flux quantum, $g = (\pi/\sqrt{2}) \int d\xi u_0(\xi)$ is a geometric factor of order unity, and $T_{\text{eff}}(k,k') = \sum_n \tilde{T}_{n,l}(k) \tilde{T}^*_{n,r}(k')/(\mu - \varepsilon_t - \varepsilon_n)$ is the effective overlap integral [$\tilde{T}_{n,l/r}(k) = \int dx T_{l/r}(x,k) |\psi_n(x)|^2$]. Below we will use Eq. (7) to analyze the magnetoconductance of the system shown in Fig. 2. In doing so we assume that the overlap integral $T_{\text{eff}}(k,k')$ does not depend on the momenta k and k'.

It is clear from the effective Hamiltonian (7) that in the presence of a magnetic field electronic transport through the SWNT may be accompanied by inelastic processes. Although in principle these may drive the vibration modes out of thermal equilibrium, we will assume here that the coupling to the thermal bath is strong enough for this not to happen. Under this condition the current I through the SWNT is to leading order in the tunneling coupling given by the expression

$$I = G_0 \sum_{n=0}^{\infty} \sum_{l=-n}^{\infty} P(n) |\langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n+l\rangle|^2 \times \int d\varepsilon [f_l(\varepsilon)(1 - f_r(\varepsilon - l\hbar\omega)) - f_r(\varepsilon)(1 - f_l(\varepsilon - l\hbar\omega))]. \tag{8}$$

Here, $G_0 = (2e/\hbar)\nu |T_{\text{eff}}|^2$ is the zero-field conductance, $P(n) = (1 - e^{-\beta\hbar\omega}) e^{-n\beta\hbar\omega}$, where $\beta \equiv 1/k_B T$, is the probability that the fundamental mode is in state $|n\rangle$ with energy $n\hbar\omega$, and $f_{l/r}(\epsilon) \equiv (1 + e^{\beta(\epsilon - \mu_{l/r})})^{-1}$ are Fermi distribution functions in the leads. The restrictions on the tunneling channels imposed by the Pauli principle are crucial; it easy to see from Eq. (8) that if all electron states in the right lead were empty it would follow from the completeness of the set of vibronic states that there is no effect of the magnetic field.

Finally, after integrating over the electron energy ε in Eq. (8) and using the completeness of the set of vibronic states, i.e., $\sum_{n'} |\langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n'\rangle|^2 = 1$, one arrives at the final result for the linear conductance G,

$$\frac{G}{G_0} = \sum_{n=0}^{\infty} P(n) |\langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n\rangle|^2 + \sum_{n=0}^{\infty} \sum_{l=1}^{\infty} P(n) |\langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n+l\rangle|^2 \frac{2l\beta\hbar\omega}{e^{l\beta\hbar\omega} - 1}. \tag{9}$$

The first term in this expression gives the contribution to the conductance from the elastic tunneling processes. The second term is due to the inelastic processes, which contribute at finite temperature.

Equation (9) has to be evaluated numerically, but the asymptotic forms in the limits of high and low temperature,

$$\frac{G}{G_0} = \begin{cases} 1 - \frac{1}{6} \frac{\hbar\omega}{k_B T} \left(\frac{g4\pi Y_0 LH}{\Phi_0} \right)^2, & \frac{\hbar\omega}{k_B T} \ll 1, \quad \phi \lesssim 1 \\ e^{-\frac{1}{2}(g4\pi Y_0 LH/\Phi_0)^2}, & \frac{\hbar\omega}{k_B T} \gg 1 \end{cases} \tag{10}$$

follow readily from the identities $\langle 0| e^{i\phi(\hat{b}^\dagger + \hat{b})} |0\rangle = e^{-\phi^2/2}$ and $\sum_{n=0}^{\infty} (n' - n) |\langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n'\rangle|^2 = \phi^2$, respectively.

It is instructive to express the probability amplitude $A_n = \langle n| e^{i\phi(\hat{b}^\dagger + \hat{b})} |n\rangle$ for elastic transitions involving the nth vibration mode in the coordinate representation as

$$A_n = \int dy |\Phi_n(y)|^2 e^{i\sqrt{2}\phi y},$$

where $\Phi_n(y)$ is the wave function of the state $|n\rangle$. From this

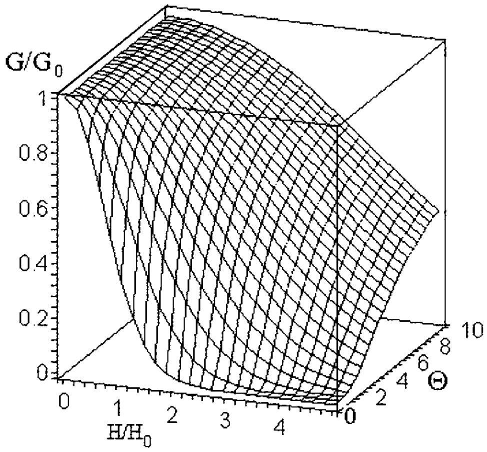

FIG. 3. Magnetoconductance of the doubly clamped SWNT shown in Fig. 2 as a function of normalized magnetic field and temperature calculated from Eq. (9) ($H_0 = \Phi_0/g2\pi LY_0$ and $\Theta = k_BT/\hbar\omega$). The negative magnetoconductance is due to a strong coupling between the coherent electrons and quantized flexural vibrations of the tube. This coupling leads to an effective multiconnectivity in the phase coherent electron transport and in a transverse magnetic field to destructive interference between different quantum electron paths.

expression it is evident that the main contribution to the elastic tunneling amplitude corresponds to quasiclassical trajectories passing through points where the probability to "find" the SWNT is maximal, while electronic trajectories passing through nodes of the SWNT wave functions do not contribute. This can be interpreted as an effective multiconnectivity of the system. We note, however, that this interpretation is strictly valid only in the limit of vanishing temperature and bias voltage.

Figure 3 shows that quantum oscillations of a suspended SWNT result in a positive magnetoresistance. The most striking feature of this electronic conduction effect is its temperature dependence. It comes from the dynamics of the entire nanotube, rather than from the electron dynamics. Consequently, the characteristic temperature scale of the resulting magnetoresistance—which decays as T^{-1} for $k_BT \gtrsim \hbar\omega$, where ω is the swinging frequency of the SWNT—is anomalously small, see Eq. (10). A complete analysis has to account for the strong nonlinearity of the magnetoconductance. As more and more inelastic channels are switched on with increasing bias voltage V, the influence of the magnetic field is suppressed in the same way as by temperature. As a result the change in current due to a magnetic field H does not exceed $|G - G_0|\hbar\omega/e$ as eV becomes larger than the vibration quantum $\hbar\omega$ [6]. Hence, in order to easily detect the magnetoconductance effect discussed here, one needs to be in the ballistic transport regime [7], where $G \simeq G_Q \equiv 2e^2/h \sim 10^{-4}$ mho. For a 1 μm long SWNT (where $\delta\varepsilon/k_B \sim 10$ K) at $T = 30$ mK and $H \sim 20$–40 T we find from Eq. (10) a relative conductance change $|(G - G_0)/G_0|$ of about 1%–3%, which corresponds to a maximal magnetocurrent $|I(H) - I(0)|$ of 0.1–0.3 pA. Its characteristic dependence on temperature and transverse magnetic field makes this effect readily distinguishable from, e.g., the recently observed SWNT magnetoconductance in a longitudinal field [8].

In conclusion we have shown that quantum vibrations of a current-carrying suspended SWNT subject to a transverse magnetic field couple electronic and vibronic states – forming what we call swinging states—in such a way that a negative magnetoconductance can follow. This effect may be used to detect nanomechanical SWNT vibrations in the quantum regime. Other effects due to electron-phonon coupling can be expected to occur in mesoscopic rings and in superconducting Josephson junctions containing nanomechanical elements. A possible consequence of the coupling to the macroscopic phase of a superconductor is the prospect of using SQUIDs to detect nanomechanical displacements in the quantum limit.

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[6] Nonresonant tunneling requires V and T to be smaller than the level spacing $\delta\varepsilon$ in a metallic SWNT (or smaller than the energy gap Δ in a semiconducting nanotube). There is, however, still room for a nonlinear dependence of the current on V and T in the interval $\hbar\omega \ll V \ll \Delta$, $\delta\varepsilon$.
[7] Equation (10) was derived assuming $G \ll G_Q \equiv 2e^2/h$, but a more general analysis (to be published elsewhere) shows that Eq. (10) is still valid in the ballistic regime, $G \simeq G_Q$.
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