Effect of Suddenly Turning on Interactions in the Luttinger Model

M. A. Cazalilla

Donostia International Physics Center (DIPC), Manuel de Lardizabal 4, 20018-Donostia, Spain

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The evolution of correlations in the *exactly* solvable Luttinger model (a model of interacting fermions in one dimension) after a suddenly switched-on interaction is *analytically* studied. When the model is defined on a finite-size ring, zero-temperature correlations are periodic in time. However, in the thermodynamic limit, the system relaxes algebraically towards a stationary state which is well described, at least for some simple correlation functions, by the generalized Gibbs ensemble recently introduced by Rigol *et al.* (cond-mat/0604476). The critical exponent that characterizes the decay of the one-particle correlation function is different from the known equilibrium exponents. Experiments for which these results can be relevant are also discussed.

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Experiments with cold atomic gases are motivating research into problems that previously would have looked highly academic. One such problem concerns the evolution of a quantum many-body system where interactions (or other parameters of the system) are time dependent. An example is an interaction *quench*: an experiment where the strength of interactions is suddenly changed. This type of experiment is feasible nowadays thanks to the phenomenon known as Feshbach resonance [1], which allows us to tune the strength and sign of interactions in a cold atomic gas by means of a magnetic field. If the applied magnetic field is time dependent, the interactions become time dependent. Alternatively, in optical lattices [2], it is possible to change the lattice parameters in a time-dependent fashion, which effectively amounts to varying the ratio of the interaction to the kinetic energy in time. On the theory side, the recent development of time-dependent extensions of the density-matrix renormalization-group algorithm [3] has spurred the interest in understanding the properties of quantum many-body systems out of equilibrium and, in particular, in the dynamics following a quench.

Because of these new possibilities, the evolution of observables and correlations following a sudden change of the system parameters is attracting much theoretical interest [4–13]. One interesting question that has been raised by a recent experiment in an array of 1D cold atomic gases [14] is whether, after a quench, a system possessing an infinite number of integrals of motion can exhibit relaxation towards a steady state or not. This question has been analyzed by the authors of Ref. [12], who have *numerically* shown that the steady state of an integrable gas of hard-core bosons is described by a generalized Gibbs distribution that maximizes the entropy with all possible constraints imposed by the existence of the (infinite number of) integrals of motion. Here the effect of suddenly turning on the interactions in the Luttinger model is *analytically* studied. It is shown that, when the model is defined on a finite-size ring, the asymptotic form of the two-point one-body and density correlations at zero temperature is periodic in time, and therefore the system exhibits no relaxation to a steady state with time-independent properties. In the thermodynamic limit, however, the same correlation functions relax to a steady state, whose properties are different from those of the ground state. Indeed, the decay of the one-particle correlations with distance is governed by a critical exponent which is different from the known equilibrium exponents. Interestingly, one-particle and density correlations in the steady state can be obtained using the generalized Gibbs ensemble introduced by Rigol *et al.* in Ref. [12].

The Luttinger model (LM) describes a system of interacting fermions in one dimension. It was introduced by Luttinger [15] in 1963, but the correct exact solution was found in 1965 by Lieb and Mattis [16]. Asymptotic forms of one- and two-particle correlations in equilibrium were obtained by Luther and Peschel [17]. Later, Haldane [18–20] proposed that this model describes the low-energy properties of systems in the Tomonaga-Luttinger liquid (TLL) universality class [18,21,22].

The Hamiltonian of the LM is $H_{\rm LM} = H_0 + H_2 + H_4$, where $H_0 = \sum_{p,\alpha} \hbar v_F p \; :\psi^\dagger_\alpha(p)\psi_\alpha(p):$ is the free-fermion Hamiltonian, and interactions are described by $H_2 = \frac{2\hbar\pi}{L}\sum_q g_2(q) J_R(q) J_L(q)$ and $H_4 = \frac{\hbar\pi}{L}\sum_{q,\alpha} g_4(q) :J_\alpha(q)J_\alpha(-q):$. The Fermi operators $\{\psi_\alpha(p), \psi^\dagger_\beta(p')\} = \delta_{p,p'}\delta_{\alpha,\beta}$ $(\alpha, \beta = L, R)$ and anticommute otherwise. To avoid a degenerate ground state, antiperiodic boundary conditions are chosen: $\psi_\alpha(x+L) = -\psi_\alpha(x)$ [$\psi_\alpha(x) = \sum_p e^{is_\alpha p x}\psi_\alpha(p)/\sqrt{L}$ is the Fermi field operator, $s_R = -s_L = +1$, and L is the length of the system] so that $p = 2\pi(n - \frac{1}{2})/L$, and n is an integer. The "current" operators $J_\alpha(q) = \sum_q :\psi^\dagger_\alpha(p+q)\psi_\alpha(p):$, where $q = 2\pi m/L$, m being an integer; $:\cdots:$ stands for the normal order prescription [19]. Thus, the above model describes a system of fermions interacting via the four-fermion terms H_2 and H_4. Fermions come in two chiralities, R and L standing for right-moving and left-moving particles, respectively. The dispersion is linear and therefore it is not bounded from below. To define a stable ground state for H_0 all single-particle levels with $p < 0$ are filled up for both chiralities, which yields a Dirac sea which will be denoted as $|0\rangle$. The coupling functions $g_2(q)$ and $g_4(q)$ are assumed to be finite for $q = 0$. Moreover, to ensure that the Hilbert space of $H_{\rm LM}$ and H_0 remain the same and, in particular, that their ground states have a finite overlap at finite L, $g_2(q)/[v_F + g_4(q)] \to 0$ faster than $|q|^{-1/2}$ as $|q| \to \infty$ and $|g_2(q)| < |v_F + g_4(q)|$ for all q [19].

The currents obey a Kac-Moody algebra [16,19,21,22]: $[J_\alpha(q), J_\beta(q')] = \frac{qL}{2\pi}\delta_{q+q',0}\ \delta_{\alpha,\beta}$. This fact allows one to introduce, for $q \neq 0$, the following operators: $b_0(q) = -i(2\pi/|q|L)^{1/2}[\theta(q)J_R(-q) - \theta(-q)J_L(q)]$ and $b_0^\dagger(q) = i(2\pi/|q|L)^{1/2}[\theta(q)J_R(q) - \theta(-q)J_L(-q)]$, which obey the standard algebra of boson ("phonon") operators. Moreover, there are two conserved operators $\delta N = N_R + N_L$, i.e., the number operator referred to the ground state $|0\rangle$, and the total current $J = N_R - N_L$, where $N_\alpha = J_\alpha(0)$. For fermions, the physical states obey the selection rule $(-1)^{\delta N} = (-1)^J$. In terms of the boson operators $b_0^\dagger(q)$, $b_0(q)$ the Hamiltonian $H_{\rm LM}$ is quadratic but not diagonal. It can be diagonalized by means of a Bogoliubov ("squeezing") transformation [16]:

$$b(q) = \cosh\varphi(q) b_0(q) + \sinh\varphi(q) b_0^\dagger(-q), \quad (1)$$

$$b^\dagger(q) = \sinh\varphi(q) b_0(-q) + \cosh\varphi(q) b_0^\dagger(q). \quad (2)$$

To render $H_{\rm LM}$ diagonal, $\tanh 2\varphi(q) = g_2(q)/[v_F + g_4(q)]$. Thus the Hamiltonian becomes $H_{\rm LM} = \sum_{q\neq 0}\hbar v(q)|q| b^\dagger(q)b(q) + \hbar\pi v_N \delta N^2/2L + \hbar\pi v_J J^2/2L$ [19], where $v(q) = \{[v_F + g_4(q)]^2 - g_2^2(q)\}^{1/2}$, $v_N = ve^{2\varphi}$, and $v_J = ve^{-2\varphi}$ [19], being $v = v(0)$ and $\varphi = \varphi(0)$.

Let us now consider an interaction quench in the LM. Here I consider only the case where the coupling functions $g_2(q)$ and $g_4(q)$ are suddenly switched on at $t = 0$. Thus, the initial state of the system will be described by a thermal distribution determined by the noninteracting Hamiltonian H_0, $\rho(t=0) = \rho_0 = e^{-H_0/T}/Z_0$, where $Z_0 = {\rm Tr} e^{-H_0/T}$. However, for $t > 0$, the evolution is dictated by the full Hamiltonian $H_{\rm LM}$. A more general type of quench corresponds to a sudden switch between two different forms of $g_2(q)$ and $g_4(q)$. Whereas the results described below can be generalized to such a case, I believe a quench from the noninteracting limit is most interesting because the spectrum of H_0 contains free fermions whereas the spectrum of $H_{\rm LM}$ does not [16,17,19,21,22]. Thus, a sudden switch on of the interactions describes a time-dependent destruction of the characteristic discontinuity of the momentum distribution at the Fermi points where $p = 0$.

Equal time correlations of an operator $O(x)$ read

$$C_O(x,t) = \langle e^{iH_{\rm LM}t/\hbar} O^\dagger(x) O(0) e^{-iH_{\rm LM}t/\hbar}\rangle_0, \quad (3)$$

$$= {\rm Tr}\rho_0 e^{iH_{\rm LM}t/\hbar} O^\dagger(x) O(0) e^{-iH_{\rm LM}t/\hbar}. \quad (4)$$

Note that since $[H_0, H_{\rm LM}] \neq 0$, $C_O(x,t)$ is explicitly time dependent. Indeed, in the LM model time dependence stems from H_2, since $[H_0, H_4] = 0$. H_2 describes scattering between fermions moving in opposite directions, and, as shown below, it entangles excitation modes of opposite q.

Using the transformation (1) and (2) and its inverse, along with $b(q,t) = e^{iH_{\rm LM}t/\hbar} b(q) e^{-iH_{\rm LM}t/\hbar} = e^{-iv(q)|q|t} b(q)$ and $b^\dagger(q,t) = e^{iv(q)|q|t} b^\dagger(q)$, the exact evolution of $b_0(q)$ under $H_{\rm LM}$ can be obtained:

$$b_0(q,t) = f(q,t) b_0(q) + g^*(q,t) b_0^\dagger(-q), \quad (5)$$

where $b_0(q,t) = e^{iH_{\rm LM}t/\hbar} b_0(q) e^{-iH_{\rm LM}t/\hbar}$, $f(q,t) = \cos v(q)|q|t - i\sin v(q)|q|t \cosh 2\varphi(q)$, and $g(q,t) = i\sin v(q)|q|t \sinh 2\varphi(q)$. Note that this form obeys the correct boundary condition $b_0(q,0) = b_0(q)$. Entanglement between modes of opposite q vanishes for $\varphi(q) = 0$ [i.e., $g_2(q) = 0$] in agreement with the above discussion.

The evolution of one-particle correlations [i.e., $O(x) = \psi_\alpha(x)$] can be obtained from Eq. (5) and the bosonization formula [17,19,21,22]: $\psi_\alpha(x) = \frac{\eta_\alpha}{(2\pi a)^{1/2}} e^{is_\alpha \phi_\alpha(x)}$, $\eta_R \neq \eta_L$ being two different Pauli matrices that ensure the anticommutation of the left- and right-moving Fermi fields; $s_\alpha = 1$ for $\alpha = R$ and $s_\alpha = -1$ for $\alpha = L$, $\phi_\alpha(x) = s_\alpha \varphi_{0\alpha} + 2\pi x N_\alpha/L + \Phi_\alpha^\dagger(x) + \Phi_\alpha(x)$, where $[N_\alpha, \varphi_{0\beta}] = i\delta_{\alpha,\beta}$, $\Phi_\alpha(x) = \sum_{q>0}(2\pi/qL)^{1/2} e^{-qa/2} e^{is_\alpha qx} b_0(s_\alpha q)$, and $a \to 0^+$. Expressions for the correlations following the quench look formally very similar to those obtained for time-dependent correlations in the ground state of $H_{\rm LM}$ [17]. Calculations are greatly simplified by considering a model where $\sinh 2\varphi(q) = e^{-|qR_0|/2}\sinh 2\varphi$, with $R_0 \ll L$ being of the order of the range of the $g_2(q)$ interaction, and replacing $v(q)$ by its $q = 0$ value. This is the natural extension of the model used to obtain the *universal* (i.e., independent of R_0, up to prefactors) behavior of the ground-state correlations [17,19]. This behavior is dominated by states where excitations are predominately near the Fermi level so that it is accurate to replace in the calculations $\varphi(q)$ and $v(q)$ by their $q = 0$ values [16,17]. In the present case, although $|0\rangle$ is a rather complicated excited state of $H_{\rm LM}$, since $g_2(q)$ and $g_4(q)$ decay very rapidly far from $q = 0$, this approximation is also expected to be accurate. For a system of size L at $T = 0$, the leading term of the one-body correlations is given by the following expression:

$$C_{\psi_R}(x, t>0|L) = G_R^{(0)}(x|L)\left[\frac{R_0}{d(x|L)}\right]^{\gamma^2} \times \left[\frac{d(x-2vt|L)d(x+2vt|L)}{[d(2vt|L)]^2}\right]^{\gamma^2/2}, \quad (6)$$

where $d(x|L) = L|\sin(\pi x/L)|/\pi$ is the *cord* function, $G_R^{(0)}(x|L) = i/[2L\sin\pi(x+ia)/L]$ the noninteracting correlation function, and $\gamma = \sinh 2\varphi$. The above expression is accurate asymptotically, i.e., for $d(x \pm 2vt|L)$,

$d(x|L), d(2vt|L) \gg R_0$. It can be seen that the one-particle correlations are periodic in time: $C_{\psi_R}(x, t + T_0|L) = C_{\psi_R}(x, t|L)$ with $T_0 = L/2v$. This implies that the finite-size LM does not relax, which is a consequence of the (approximately) linear dispersion of the eigenmodes near $q = 0$ along with the absence of any damping mechanisms in the LM (see discussion at the end). However, in the thermodynamic limit, $L \to \infty$ and $d(x|L) \to |x|$. Therefore, Eq. (6) becomes

$$C_{\psi_R}(x, t > 0) = \frac{i}{2\pi(x + ia)} \left|\frac{R_0}{x}\right|^{\gamma^2} \left|\frac{x^2 - (2vt)^2}{(2vt)^2}\right|^{\gamma^2/2}. \tag{7}$$

It is interesting to analyze the above expression in the limit $2vt \ll |x|$, where $C_{\psi_R}(R_0 \ll 2vt \ll |x|) \approx \frac{iZ(t)}{2\pi(x+ia)}$, $Z(t) = (R_0/2vt)^{\gamma^2}$ being a time-dependent renormalization constant of the Fermi quasiparticles. Thus for short times the system behaves as a Fermi liquid, with a singularity at the Fermi points given by $Z(t)$, which decreases with time. On the other hand, for $2vt \gg |x|$ the correlation takes a non-Fermi liquid form:

$$C_{\psi_R}(R_0 \ll |x| \ll 2vt) \approx \frac{i}{2\pi(x + ia)} \left|\frac{R_0}{x}\right|^{\gamma^2}. \tag{8}$$

In particular, in the limit $t \to +\infty$ one-particle correlations relax to the power law on the right-hand side of Eq. (8). Notice that, although $C_{\psi_R}(x, t \to \infty)$ exhibits a power-law behavior, the latter is governed by an exponent that is different from the one that governs asymptotic ground-state correlations [17,19], $\gamma_0^2 = 2\sinh^2\varphi < \gamma^2 = \sinh^2 2\varphi$ for $\varphi \neq 0$. The origin of this new exponent will be discussed below.

The different behavior of $C_{\psi_R}(x, t)$ for short and long times can be understood in terms of a "light-cone" effect [11]: The initial state $|0\rangle$ has higher energy than the ground state of $H_{\rm LM}$ (see discussion further below). Therefore, it contains long wavelength phonons that propagate (approximately) from time = 0 to time = t along light cones where the role of speed of light is played by v. These excitations determine which points retain the same type of correlations found in $|0\rangle$ and which points acquire new correlations. The latter phenomenon and the overall structure of (7) bear some resemblance to results reported in Ref. [11]. Nevertheless, I have so far failed to extend the methods of [11] to the quench in the LM. There are two main differences: First, the initial state in the present case is critical, and therefore it does not have any characteristic (gap) energy scale as the initial states considered in [11]. Second, and more importantly, the critical exponent found above is different from the bulk or boundary exponents of the field operator $\psi_R(x)$.

One may think that the relaxation behavior exhibited by $C_{\psi_R}(x, t)$ in the thermodynamic limit is because the field operator, $\psi_R(x)$, is a nonlinear function of $b_0(q)$ and $b_0^\dagger(q)$. However, the (density) operator $J_R(x) = \partial_x \phi_R(x)/2\pi$ also exhibits relaxation. Setting $O(x) = J_R(x)$ in (4), the following is obtained using Eq. (5):

$$C_{J_R}(x, t|L) = -\frac{1}{4\pi^2}\left\{\frac{1 + \gamma^2}{[d(x|L)]^2} - \frac{\gamma^2}{2[d(x - 2vt|L)]^2} - \frac{\gamma^2}{2[d(x + 2vt|L)]^2}\right\}. \tag{9}$$

For finite L the density correlation function is again periodic in time. However, for $L \to \infty$, it shows relaxation: $C_{J_R}(x, t \to \infty|L) \to -(1 + \gamma^2)/(4\pi^2 x^2)$. This form again deviates from the ground-state behavior, where the prefactor of $-1/(4\pi^2 x^2)$ is $\cosh 2\varphi - \sinh 2\varphi = e^{-2\varphi}$ [19,21,22].

It is interesting to find that the above results in the $t \to \infty$ limit can be analytically obtained from the generalized Gibbs distribution introduced in Ref. [12], which is described by the following density matrix:

$$\rho_{gG} = \frac{1}{Z_{gG}} e^{\sum \lambda(q) I(q)}, \tag{10}$$

where $Z_{gG} = \mathrm{Tr} e^{\sum \lambda(q) I(q)}$ and $[H, I(q)] = [I(q), I(q')] = 0$, that is, the set of all independent integrals of motion. Since $[H_{\rm LM}, n(q)] = 0$, where $n(q) = b^\dagger(q) b(q)$, the phonon occupancy operators seem as the most natural choice for $I(q)$. The Lagrange multipliers $\lambda(q)$ are obtained from the condition [12]

$$\langle n(q)\rangle_{t=0} = \langle 0|n(q)|0\rangle = \langle n(q)\rangle_{gG} = \mathrm{Tr}[\rho_{gG} n(q)], \tag{11}$$

where $T = 0$ was assumed. Using (1) and (2), $\langle 0|n(q)|0\rangle = \sinh^2\varphi(q)$, which is a nonthermal distribution. However, $\lambda(q)$ does not need to be obtained explicitly, as it suffices to realize that ρ_{gG} has the same form as the distribution in the canonical ensemble with $H/T = -\sum_q \lambda(q) n(q)$. One can also regard ρ_{gG} as a canonical distribution with a q-dependent temperature, $T(q) = -\hbar v(q)|q|/\lambda(q)$. Using this fact, I find that

$$C_{\psi_R}^{gG}(x) = \mathrm{Tr}\rho_{gG}\psi_R^\dagger(x)\psi_R(0) = \lim_{t\to+\infty} C_{\psi_R}(x, t), \tag{12}$$

$$C_{J_R}^{gG}(x) = \mathrm{Tr}\rho_{gG} J_R(x) J_R(0) = \lim_{t\to+\infty} C_{J_R}(x, t). \tag{13}$$

Thus, at least for these simple correlation functions, it seems that the generalized Gibbs distribution describes the stationary state of the LM after an interaction quench. The reason why the critical exponent γ^2 turns out to be different from the known equilibrium exponents can be thus explained in two different ways: Mathematically, it is seen that in order to obtain the evolution of the operator $b_0(q)$ [Eq. (5)] one has to do and undo the Bogoliubov transformation (1) and (2). However, these transformations do not cancel each other exactly (except at $t = 0$) because of the phase factors $e^{\pm i v(q)|q|t}$ introduced by the time evolution operator. In contrast, in the equilibrium problem, since the expectation value is taken over the ground state of $H_{\rm LM}$, the Bogoliubov transformation is performed only once. Physically, in view of the results (12) and (13), the difference in exponent can be regarded a consequence of the nonequilibrium distribution of phonons $\langle 0|n(q)|0\rangle = \sinh^2\varphi(q)$, which is a constant of motion.

Let us finally consider where the above predictions could be experimentally relevant. To date, there are no exact realizations of the LM in nature. However, one can exploit the fact that the LM describes the low-energy properties of Tomonaga-Luttinger liquids [19,21,22], of which several physical realizations in cold atomic gases are available [14,23,24]. Let us therefore consider a single-species Fermi gas confined to one dimension in a strongly anisotropic trap [24]. In a single-species cold Fermi gas, p-wave interactions are naturally negligible. One possibility to realize a sudden change of the interaction strength is to use a p-wave Feshbach resonance [24], which enhances the strength of this interaction. Alternatively, one can use a 1D dipolar Fermi gas, where long-distance interactions are described by the potential:

$$V_{\rm dip}(x,\theta) = \frac{1}{4\pi\epsilon_0}\frac{D^2(1-3\cos\theta)}{[x^2+R_0^2]^{3/2}}, \qquad (14)$$

D being the dipolar momentum of the atoms and θ is the angle subtended by the direction of the atomic motion and an electric field (or magnetic, for magnetic dipoles) that polarizes the gas. In the above expression R_0 is of the order of the transverse size of the cloud. The Fourier transform of (14) is $V_{\rm dip} = \lambda(\theta)|qR_0|K_1(|qR_0|) \propto g_2(q) = g_4(q)$, where $\lambda(\theta) = D^2(1-3\cos\theta)/2\pi\epsilon_0 R_0^2$ and $K_1(x)$ is the modified Bessel function of the first kind. A suddenly switched on $V_{\rm dip}(q,\theta)$ can be realized by deviating the electric field that polarizes the gas from the "magic" angle $\theta_m = \cos^{-1}(\frac{1}{3})$, for which (14) vanishes [i.e., $\lambda(\theta_m) = 0$].

However, the full Hamiltonian for a TLL contains an infinite series of terms that spoil the integrability of the LM [18,19]. Roughly speaking, these stem from the nonlinearity of the fermion dispersion and the fact that interactions couple right- and left-moving modes in a way that is highly nonlinear in terms of the boson fields $\phi_\alpha(x)$ (umklapp scattering) [18]. In a TLL all these deviations are irrelevant in the renormalization-group sense, which means that their effect on low-energy states is small. Nevertheless, after a sudden change of the interaction in the systems described above, high-energy excitations will be created that are not described by the LM. Exciting many fermions to levels very far from the Fermi level where the LM description is not accurate can be avoided by turning on the interaction to a value much smaller than the Fermi energy. On the other hand, low-energy excitations will survive for longer times and, since they dominate the long-time dynamics, the behavior of the correlations will be described by the above results. Thus, if the quench was conducted at zero temperature, since the atomic systems are finite, an approximately periodic behavior of correlations can be expected. However, Fermi gases are usually hard to cool down, and a situation where temperature is larger than level spacing (i.e., $T \gg \hbar\pi v_F/L$) is perfectly realistic. In this situation, one should consider correlations at finite T, neglecting finite-size effects. The latter can be obtained from Eq. (6) upon replacing $L\sin(\pi x/L)/\pi$ by $(\hbar v_F/\pi T)\times \sinh(\pi T x/\hbar v_F)$, etc. Thus relaxation takes place because temperature induces a finite correlation length in the initial state and therefore correlations decay exponentially. One-body correlations can be accessed through the momentum distribution, which can be measured in a time of flight experiment. Thus the steady state momentum distribution following a suddenly switched-on interaction should differ from the equilibrium distribution at the same temperature. A more detailed analysis will be given elsewhere [25].

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