Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

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Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity η to entropy density s by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s = \hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio η/s. We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

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One of the amazing experimental discoveries of measurements on gold-gold collisions at the RHIC (Relativistic Heavy Ion Collider) at Brookhaven National Laboratory is the surprising amount of collective flow exhibited by the outgoing hadrons. Collective flow is evidenced in both the single-particle transverse momentum distribution [1], commonly referred to as radial flow, and in the asymmetric azimuthal distribution around the beam axis [2], quantified by the functions $v_1(y, p_T)$, $v_2(y, p_T)$, ... in the expansion

$$\frac{d^3N}{dydp_Td\phi} = \frac{1}{2\pi}\frac{d^2N}{dydp_T}[1 + 2v_1(y, p_T)\cos(\phi) + 2v_2(y, p_T)\cos(2\phi) + \cdots], \quad (1)$$

where y is the rapidity and p_T is the transverse momentum. The function $v_2(y = 0, p_T)$, in particular, was expected to be much smaller at RHIC than it is at the lower energies of the SPS (Super Proton Synchrotron) at CERN [3], but in fact it is about twice as large. Various theoretical calculations [4] support the notion that collective flow is mostly generated early in the nucleus-nucleus collision, and is present at the partonic level before partons coalesce or fragment into hadrons. Theoretical calculations including only two-body interactions between partons cannot generate sufficient flow to explain the observations unless partonic cross sections are artificially enhanced by more than an order of magnitude over perturbative QCD predictions [5]. This has emphasized that the quark-gluon matter created in these collisions is strongly interacting, unlike the type of weakly interacting quark-gluon plasma expected to occur at very high temperatures on the basis of asymptotic freedom [6]. On the other hand, lattice QCD calculations yield an equation of state that differs from an ideal gas only by about 10% once the temperature exceeds $1.5T_c$, where $T_c \approx 175$ MeV is the critical or crossover temperature from quarks and gluons to hadrons [7]. Furthermore, perfect fluid dynamics (with zero shear and bulk viscosities) reproduces the measurements of radial flow and v_2 very well up to transverse momenta of order 1.5 GeV/c [8].

An amazing theoretical discovery was made by Kovtun, Son, and Starinets [9], who showed that certain special field theories, special in the sense that they are dual to black branes in higher space-time dimensions, have the ratio $\eta/s = 1/4\pi$ (we use units with $\hbar = k_B = c = 1$) where η is the shear viscosity and s is the entropy density. They conjectured that all substances have this value as a lower limit, and gave as examples helium, nitrogen, and water at pressures of 0.1, 10, and 100 MPa, respectively. Interesting enough, this bound is also obeyed by $\mathcal{N} = 4$ supersymmetric $SU(N_c)$ Yang-Mills theory in the large N_c limit [10].

We are motivated by these discoveries to study what happens in QCD at finite temperature. The relatively good agreement between perfect fluid calculations and experimental data for hadrons of low to medium transverse momentum at RHIC suggests that the viscosity is small, however, it cannot be zero. Indeed, the calculations within anti-de Sitter space/conformal field theory suggest that $\eta \geq s/(4\pi)$. Our conclusion will be that sufficiently precise calculations and measurements should allow for a determination of the ratio η/s as a function of temperature, and that this ratio can pinpoint the location of the phase transition or rapid crossover from hadronic to quark and gluon matter. This is a different method than trying to infer the equation of state of QCD in the form of pressure P as a function of temperature T or energy density ϵ.

The energy-momentum tensor density for a perfect fluid (which does not imply that the matter is noninteracting) is $T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu$. Here $w = P + \epsilon = Ts$ is the local enthalpy density and u^μ is the local flow velocity.

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Corrections to this expression are proportional to first derivatives of the local quantities whose coefficients are the shear viscosity η and bulk viscosity ζ. (Thermal conductivity is neither relevant nor defined when all net conserved charges, such as electric charge and baryon number, are zero). Explicit expressions may be found in textbooks [11]. Perfect fluid dynamics applies when the viscosities are small, or when the gradients are small, or both. The dispersion relations for the transverse and longitudinal (pressure) parts of the momentum density are

$$\omega + iD_t k^2 = 0, \qquad \omega^2 - v^2 k^2 + iD_l \omega k^2 = 0, \qquad (2)$$

where $D_t = \eta/w$ and $D_l = (\frac{4}{3}\eta + \zeta)/w$ are diffusion constants with the dimension of length and v is the speed of sound. Since $w = Ts$, and since usually the bulk viscosity is small compared to the shear viscosity, the dimensionless ratio of (shear) viscosity to entropy (disorder) η/s is a good way to characterize the intrinsic ability of a substance to relax towards equilibrium independent of the actual physical conditions (gradients of pressure, energy density, etc.). It is also a good way to compare very different substances.

In Figs. 1–3 we plot the ratio η/s versus temperature at three fixed pressures, one of them being the critical pressure (meaning that the curve passes through the critical point) and the other ones being larger and smaller, for helium, nitrogen, and water. The ratio was constructed with data obtained from the National Institute of Standards and Technology (NIST) [12]. (Care must be taken to absolutely normalize the entropy to zero at zero temperature; we did that using data from CODATA [13]). The important observation [14,15] is that η/s has a minimum at the critical point where there is a cusp. At pressures below the critical pressure there is a discontinuity in η/s, and at pressures above it there is a broad smooth minimum.

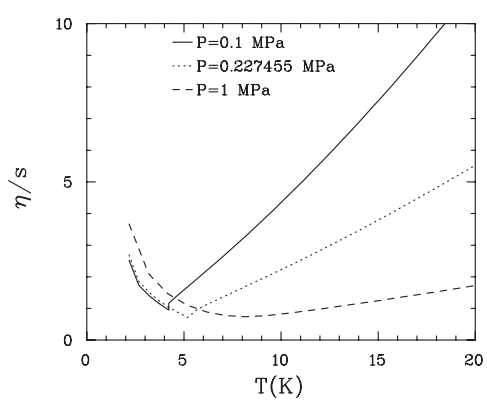

FIG. 1. The ratio η/s as a function of T for helium with s normalized such that $s(T = 0) = 0$. The curves correspond to fixed pressures, one of them being the critical pressure, and the others being greater (1 MPa) and the other smaller (0.1 MPa). Below the critical pressure there is a jump in the ratio, and above the critical pressure there is only a broad minimum. They were constructed using data from NIST and CODATA.

The simplest way to understand the general behavior was presented by Enskog, as explained in [16]. Shear viscosity represents the ability to transport momentum. In classical transport theory of gases $\eta/s \sim T l_{\rm free} \bar{v}$, where $l_{\rm free}$ is the mean free path and $\bar{v}$ is the mean speed. For a dilute gas the mean free path is large, $l_{\rm free} \sim 1/n\sigma$, with n the particle number density and σ the cross section. Hence it is easy for a particle to carry momentum over great distances, leading to a large viscosity. (This is the usual paradox, that a nearly ideal classical gas has a divergent viscosity). In a liquid there are strong correlations between neighboring atoms or molecules. A liquid is homogeneous on a mesoscopic scale, but on a microscopic scale it is a mixture of clusters and voids. The action of pushing on one atom is translated to the next one and so on until a whole row of atoms moves to fill a void, thereby transporting momentum over a relatively large distance and producing a large viscosity. Reducing the temperature at fixed pressure reduces the density of voids, thereby increasing the viscosity. The viscosity, normalized to the entropy, is observed to be the smallest at or near the critical temperature, corresponding to the most difficult condition to transport momentum. This is an empirical observation. The mean free path must lie somewhere between the dilute gas limit, $1/n\sigma$, and the close-packing limit, $1/n^{1/3}$. For a massless gas of N bosonic degrees of freedom, with entropy density $s = N(4\pi^2/90)T^3$, the close-packed limit gives $\eta/s \approx 2/N^{1/3}$.

How does this relate to hadrons and quark-gluon plasma? In the low energy chiral limit for pions the cross section is proportional to $\hat{s}/f_\pi^4$, where $\hat{s}$ is the usual Mandelstam variable for invariant mass-squared and f_π is the pion decay constant. The thermally averaged cross section is $\langle\sigma\rangle \propto T^2/f_\pi^4$, which leads to $\eta/s \propto (f_\pi/T)^4$.

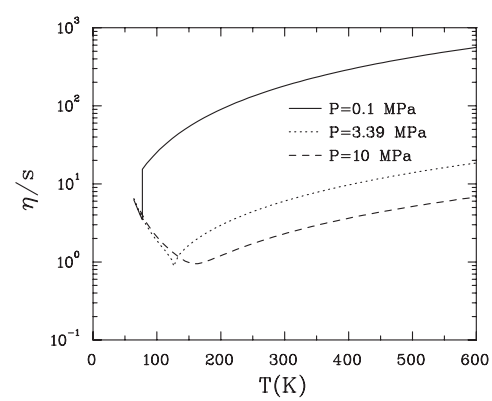

FIG. 2. The ratio η/s as a function of T for nitrogen with s normalized such that $s(T = 0) = 0$. The curves correspond to fixed pressures, one of them being the critical pressure, and the others being greater (10 MPa) and the other smaller (0.1 MPa). Below the critical pressure there is a jump in the ratio, and above the critical pressure there is only a broad minimum. They were constructed using data from NIST and CODATA. The curves are plotted on logarithmic scale to make the behavior around the critical point more visible.

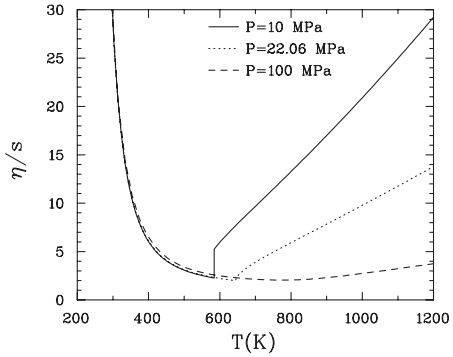

FIG. 3. The ratio η/s as a function of T for water with s normalized such that $s(T=0)=0$. The curves correspond to fixed pressures, one of them being the critical pressure, and the others being greater (100 MPa) and the other smaller (10 MPa). Below the critical pressure there is a jump in the ratio, and above the critical pressure there is only a broad minimum. They were constructed using data from NIST and CODATA.

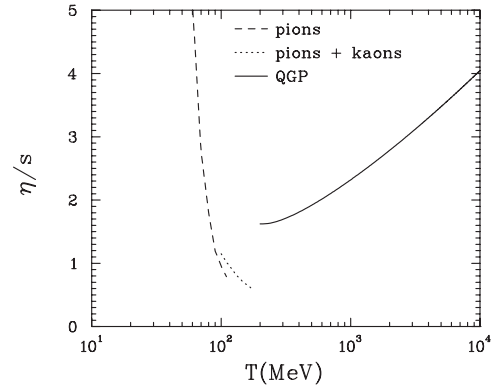

FIG. 4. The ratio η/s for the low temperature hadronic phase and for the high temperature quark-gluon phase. Neither calculation is very reliable in the vicinity of the critical or rapid crossover temperature.

Explicit calculation gives [17]

$$\frac{\eta}{s} = \frac{15}{16\pi}\frac{f_\pi^4}{T^4}. \tag{3}$$

Thus the ratio η/s diverges as $T \to 0$. At the other extreme lies quark-gluon plasma. The parton cross section behaves as $\sigma \propto g^4/\hat{s}$. A first estimate yields $\eta/s \propto 1/g^4$. Asymptotic freedom at one loop order gives $g^2 \propto 1/\ln(T/\Lambda_T)$, where Λ_T is proportional to the scale parameter $\Lambda_{\rm QCD}$ of QCD. Therefore η/s is an increasing function of T in the quark-gluon phase. As a consequence, η/s must have a minimum. Based on atomic and molecular data, this minimum should lie at the critical temperature if there is one, otherwise at or near the rapid crossover temperature.

The most accurate and detailed calculation of the viscosity in the low temperature hadron phase was performed in [17]. The two-body interactions used went beyond the chiral approximation, and included intermediate resonances such as the ρ meson. The results are displayed in Fig. 4, both two flavors (no kaons) and three flavors (with kaons). The qualitative behavior is the same as in Eq. (3). The most accurate and detailed calculation of the viscosity in the high temperature quark-gluon phase was performed in [18]. They used perturbative QCD to calculate the full leading-order expression, including summation of the Coulomb logarithms. For three flavors of massless quarks the result is

$$\frac{\eta}{s} = \frac{5.12}{g^4 \ln(2.42/g)}. \tag{4}$$

We used this together with the two-loop renormalization group expression for the running coupling

$$\frac{1}{g^2(T)} = \frac{9}{8\pi^2}\ln\left(\frac{T}{\Lambda_T}\right) + \frac{4}{9\pi^2}\ln\left(2\ln\left(\frac{T}{\Lambda_T}\right)\right) \tag{5}$$

with $\Lambda_T = 30$ MeV, which approximately corresponds to using an energy scale of $2\pi T$ and $\Lambda_{\overline{MS}} = 200$ MeV. The result is also plotted in Fig. 4. These results imply a minimum in the neighborhood of the expected value of $T_c \approx 190$ MeV. Whether there is a discontinuity or a smooth crossover cannot be decided since both calculations are unreliable near T_c.

It is interesting to ask what happens in the large N_c limit with $g^2 N_c$ held fixed [19]. In this limit, meson masses do not change very much but baryon masses scale proportional to N_c; therefore, baryons may be neglected in comparison to mesons due to the Boltzmann factor. Since the meson spectrum is essentially unchanged with increasing N_c, so is the Hagedorn temperature. The critical temperature to go from hadrons to quarks and gluons is very close to the Hagedorn temperature, so that T_c is not expected to change very much either. In the large N_c limit the meson-meson cross section scales as $1/N_c^2$. According to our earlier discussion on the classical theory of gases, this implies that the ratio η/s in the hadronic phase scales as N_c^2. This general result is obeyed by (3) since it is known that f_π^2 scales as N_c. The large N_c limit of the viscosity in the quark and gluon phase may be inferred from the calculations of [18] to be

$$\left(\frac{\eta}{s}\right)_{\rm QGP} = \left(\frac{1+3.974r}{1+1.75r}\right)\frac{69.2}{(g^2N_c)^2\ln(26/[g^2N_c(1+0.5r)])} \tag{6}$$

where $r = N_f/N_c$. Thus the ratio η/s has a finite large N_c limit in the quark and gluon phase. Therefore, we conclude that η/s has a discontinuity proportional to N_c^2 if $N_c \to \infty$. This jump is in the opposite direction to that in Fig. 4.

So far the only quantitative results for viscosity in lattice gauge theory have been reported by Nakamura and Sakai [20] for pure SU(3) without quarks. This bold effort obtained $\eta/s \approx 1/2$ in the temperature range $1.6 < T/T_c < 2.2$, albeit with uncertainties of order 100%. Gelman,

Shuryak, and Zahed [21] have modeled the dynamics of long wavelength modes of QCD at temperatures from T_c to $1.5T_c$ as a classical, nonrelativistic gas of massive quasiparticles with color charges. They obtained a ratio of $\eta/s \approx 0.34$ in this temperature range.

It ought be possible to extract numerical values of the viscosity in heavy ion collisions via scaling violations to perfect fluid flow predictions. One should perform a systematic beam energy and projectile or target mass scan from SPS energies to the top RHIC energy, and then on to the LHC. Flow data, in the form of the functions $v_1, v_2, \ldots$ should be obtained and compared with the results of calculations based on relativistic viscous fluid dynamics. This program is analogous to what was accomplished at lower energies of 30 to 1000 MeV per nucleon beam energies in the lab frame. At those energies, scaling violations to perfect fluid dynamics were indeed observed [22]. It was possible to infer the compressibility of nuclear matter and the momentum dependence of the nuclear optical potential via the transverse momentum distribution relative to the reaction plane [23] and via the balance between attractive and repulsive scattering [24]. There is much to be learned about QCD at high energy densities.

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