Constraining the Phase of B_s-$\bar{B}_s$ Mixing

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New physics contributions to B_s-$\bar{B}_s$ mixing can be parametrized by the size (r_s^2) and the phase ($2\theta_s$) of the total mixing amplitude relative to the standard model amplitude. The phase has so far been unconstrained. We first use the D0 measurement of the semileptonic CP asymmetry $A_{\rm SL}$ to obtain the first constraint on the semileptonic CP asymmetry in B_s decays, $A_{\rm SL}^s = -0.008 \pm 0.011$. Then we combine recent measurements by the CDF and D0 Collaborations—the mass difference (ΔM_s), the width difference ($\Delta\Gamma_s$), and $A_{\rm SL}^s$—to constrain $2\theta_s$. The errors on $\Delta\Gamma_s$ and $A_{\rm SL}^s$ should still be reduced to have a sensitive probe of the phase, yet the central values are such that the regions around $2\theta_s \sim 3\pi/2$ and, in particular, $2\theta_s \sim \pi/2$, are disfavored.

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Introduction.—Flavor changing $b \to s$ transitions are a particularly sensitive probe of new physics. Among these, B_s-$\bar{B}_s$ mixing occupies a special place. New physics contributions to the mixing amplitude M_{12}^s can be parametrized in the most general way as follows:

$$M_{12}^s = r_s^2 e^{2i\theta_s}(M_{12}^s)^{\rm SM}, \tag{1}$$

where $(M_{12}^s)^{\rm SM}$ is the standard model (SM) contribution to the mixing amplitude. Values of $r_s^2 \neq 1$ and/or $2\theta_s \neq 0$ would signal new physics. Assuming that the new physics can affect any loop processes but is negligible for tree level processes, and that the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix is unitary (i.e., no quarks beyond the known three generations), we can use various experimental measurements to constrain the new physics parameters r_s^2 and $2\theta_s$: (1) the mass difference between the neutral B_s states:

$$\Delta M_s = (\Delta M_s)^{\rm SM} r_s^2. \tag{2}$$

(2) The width difference between the neutral B_s states [1,2]:

$$\Delta\Gamma_s^{CP} = \Delta\Gamma_s \cos 2\theta_s = (\Delta\Gamma_s)^{\rm SM}\cos^2 2\theta_s. \tag{3}$$

(3) The semileptonic asymmetry in B_s decays:

$$A_{\rm SL}^s = -{\rm Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right)^{\rm SM}\frac{\sin 2\theta_s}{r_s^2}. \tag{4}$$

(4) The CP asymmetry in B_s decays into final CP eigenstates such as $\psi\phi$:

$$S_{\psi\phi(CP=+)} = -\sin 2\theta_s. \tag{5}$$

Our convention here is defined by $\Delta M_s \equiv M_{sH} - M_{sL}$ and $\Delta\Gamma_s \equiv \Gamma_{sH} - \Gamma_{sL}$. The observable $\Delta\Gamma_s^{CP}$ is defined by $\Delta\Gamma_s^{CP} \equiv \Gamma_- - \Gamma_+$, where $\Gamma_-(\Gamma_+)$ is deduced from fitting the decay rate into a final CP-odd (-even) state assuming that it is described by a single exponential. This assumption introduces an error of $\mathcal{O}(y_s^2) = 0.01$ [$y_s \equiv \Delta\Gamma_s/(2\Gamma_s)$]. In the expressions for $\Delta\Gamma_s$ and $S_{\psi\phi}$ we neglect terms of $\mathcal{O}(\sin 2\beta_s) = 0.04$ (where $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$), while the approximation for $A_{\rm SL}^s$ is good to $\mathcal{O}((m_c^2/m_b^2)\sin 2\beta_s) = 0.004$.

Until very recently, experiments gave only a lower bound on ΔM_s, a large error on $\Delta\Gamma_s$, and no meaningful information on the CP asymmetries. Under these circumstances, there has been only a lower bound on r_s^2 and no constraint at all on $2\theta_s$.

Recently, three important experimental developments took place in this context: (1) the CDF Collaboration measured ΔM_s [3]:

$$\Delta M_s = 17.33^{+0.42}_{-0.21} \pm 0.07\ {\rm ps}^{-1}. \tag{6}$$

(The D0 Collaboration provided a milder two-sided bound [4].) (2) The D0 Collaboration measured [5] $\Delta\Gamma_s^{CP} = -0.15 \pm 0.10^{+0.03}_{-0.04}\ {\rm ps}^{-1}$. Averaging this result with the earlier measurements by CDF [6] and ALEPH [7], we obtain

$$\Delta\Gamma_s^{CP} = -0.22 \pm 0.08\ {\rm ps}^{-1}. \tag{7}$$

(3) The D0 Collaboration searched for the semileptonic CP asymmetry [8,9]:

$$A_{\rm SL} = -0.0026 \pm 0.0024 \pm 0.0017. \tag{8}$$

As obvious from Eq. (2), the main implication for new physics of the new result for ΔM_s, Eq. (6), is a range for r_s^2 which can be further translated into constraints on parameters of specific models [10–16]. Here, we would like to focus instead on the phase of the mixing amplitude $2\theta_s$. In order that a measurement of $\Delta\Gamma_s^{CP}$ can be used to constrain $\cos^2 2\theta_s$, the experimental error should be at or below the level of $(\Delta\Gamma_s)^{\rm SM}$. The new D0 measurement of $\Delta\Gamma_s^{CP}$ is the first to reach the required level. There are three necessary conditions in order that a measurement of $A_{\rm SL}$ can be used to constrain $2\theta_s$: (1) the experimental error on $A_{\rm SL}$ should be at or below the level of $|\Gamma_{12}^s/M_{12}^s|^{\rm SM}$; (2) an upper bound on r_s^2 should be available; (3) an independent upper bound on $A_{\rm SL}^d$ (the semileptonic asymmetry in B_d decays) should be available. Both the D0 measurement of A_{SL} and the CDF measurement of ΔM_s are thus crucial for our purposes because they satisfy, for the first time, the first and second condition, respectively.

Relating A_{SL} to A_{SL}^s.—The semileptonic asymmetry measured at the TeVatron,

$$A_{SL} \equiv \frac{\Gamma(b\bar{b} \to \mu^+\mu^+X) - \Gamma(b\bar{b} \to \mu^-\mu^-X)}{\Gamma(b\bar{b} \to \mu^+\mu^+X) + \Gamma(b\bar{b} \to \mu^-\mu^-X)} = \frac{\Gamma_{RS}^+\Gamma_{WS}^+ - \Gamma_{RS}^-\Gamma_{WS}^-}{\Gamma_{RS}^+\Gamma_{WS}^+ + \Gamma_{RS}^-\Gamma_{WS}^-}, \tag{9}$$

sums over all B-hadron decays. Given that the quark subprocesses are $b \to \mu^- X$ and $\bar{b} \to \mu^+ X$, the right-sign (RS) and wrong-sign (WS) rates can be decomposed as follows:

$$\begin{aligned}
\Gamma_{RS}^- &= f_d T(\bar{B}_d \to \bar{B}_d)\bar{\Gamma}_{SL}^d + f_s T(\bar{B}_s \to \bar{B}_s)\bar{\Gamma}_{SL}^s + f_u\bar{\Gamma}_{SL}^u,\\
\Gamma_{RS}^+ &= f_d T(B_d \to B_d)\Gamma_{SL}^d + f_s T(B_s \to B_s)\Gamma_{SL}^s + f_u\Gamma_{SL}^u,\\
\Gamma_{WS}^- &= f_d T(B_d \to \bar{B}_d)\bar{\Gamma}_{SL}^d + f_s T(B_s \to \bar{B}_s)\bar{\Gamma}_{SL}^s,\\
\Gamma_{WS}^+ &= f_d T(\bar{B}_d \to B_d)\Gamma_{SL}^d + f_s T(\bar{B}_s \to B_s)\Gamma_{SL}^s.
\end{aligned} \tag{10}$$

Here, f_q is the production fraction of B_q (we assume that there is no production asymmetry, $f_q = \bar{f}_q$), T is the time integrated probability, and Γ_{SL}^q ($\bar{\Gamma}_{SL}^q$) is the semileptonic decay rate of B_q-($\bar{B}_q$) mesons. (One should think of the $q = u$ terms as representing all b hadrons that do not mix, that is, the charged B mesons and the Λ_b baryons.)

Within our assumptions, there is no direct CP violation in semileptonic decays, that is, $\Gamma_{SL}^q = \bar{\Gamma}_{SL}^q$. The time integrated probabilities fulfill $T(B_{d,s} \to B_{d,s}) = T(\bar{B}_{d,s} \to \bar{B}_{d,s})$. Consequently, we have $\Gamma_{RS}^- = \Gamma_{RS}^+$. This leads to a considerable simplification of Eq. (9):

$$A_{SL} = \frac{\Gamma_{WS}^+ - \Gamma_{WS}^-}{\Gamma_{WS}^+ + \Gamma_{WS}^-}. \tag{11}$$

Thus, the semileptonic asymmetry depends only on the wrong-sign rates. In particular, it is independent of the $B^\pm$ (and similarly of the Λ_b) decay rates.

To a very good approximation we expect $\Gamma_{SL}^d = \Gamma_{SL}^s$ [this SU(3)-flavor equality is violated only by terms of $\mathcal{O}(m_s\Lambda_{QCD}/m_b^2)$] which leads to

$$A_{SL} = \frac{f_d T_d^- + f_s T_s^-}{f_d T_d^+ + f_s T_s^+}, \tag{12}$$

where

$$T_q^\pm = T(\bar{B}_q \to B_q) \pm T(B_q \to \bar{B}_q). \tag{13}$$

The relevant time integrated transition probabilities are as follows [17]:

$$\begin{aligned}
T(B_q \to \bar{B}_q) &= \left(\frac{1-\delta_q}{1+\delta_q}\right)\frac{Z_q}{2\Gamma_q},\\
T(\bar{B}_q \to B_q) &= \left(\frac{1+\delta_q}{1-\delta_q}\right)\frac{Z_q}{2\Gamma_q},
\end{aligned} \tag{14}$$

where [$y_q = \Delta\Gamma_q/(2\Gamma_q)$, $x_q = \Delta M_q/\Gamma_q$]

$$Z_q \equiv \frac{1}{1-y_q^2} - \frac{1}{1+x_q^2}. \tag{15}$$

The quantity δ_q characterizes CP violation in mixing [$\delta_q \equiv (1-|q/p|_q^2)/(1+|q/p|_q^2)$]. Given that it is small, one can write to leading order $\delta_q = A_{SL}^q/2$, $T_q^- = A_{SL}^q Z_q/\Gamma_q$, and $T_q^+ = Z_q/\Gamma_q$. Taking again the SU(3) limit, $\Gamma_d = \Gamma_s$ (the equality is violated at high order in $1/m_b$; experimentally [18] $\tau_s/\tau_d \sim 0.96 \pm 0.04$), we obtain [19]

$$A_{SL} = \frac{f_d Z_d A_{SL}^d + f_s Z_s A_{SL}^s}{f_d Z_d + f_s Z_s}. \tag{16}$$

Given the experimental ranges [21] $|y_d| = 0.004 \pm 0.019$ and $|y_s| = 0.16 \pm 0.06$ we can safely neglect y_d^2 and y_s^2. (Within our framework, we expect [22,23] $y_s^2 \sim 0.01$). Using the experimental values [18] $f_d = 0.4$, $f_s = 0.1$, $x_d = 0.78$, and $x_s = 25.3$, we obtain

$$A_{SL} \simeq 0.6A_{SL}^d + 0.4A_{SL}^s. \tag{17}$$

There are two sets of measurements that, in combination, allow us to extract a range for A_{SL}^s. First, we have the D0 measurement of A_{SL} [Eq. (8)], which we can average together with previous measurements by the LEP experiments OPAL [24] and ALEPH [25] (we neglect here the small difference between LEP and the TeVatron regarding the measured values of $f_{d,s}$). We find

$$A_{SL} = -0.0027 \pm 0.0029. \tag{18}$$

Second, we have measurements of A_{SL}^d at the $\Upsilon(4S)$ energy by *BABAR* [26], Belle [27], and CLEO [28]. We find

$$A_{SL}^d = +0.0011 \pm 0.0055. \tag{19}$$

Thus, we obtain

$$A_{SL}^s = -0.008 \pm 0.011. \tag{20}$$

Constraining $2\theta_s$.—Our constraints on $2\theta_s$ involve Eqs. (3) and (4). As concerns $(\Gamma_{12}/M_{12})^{SM}$, we use [22] (see also [23] for a different calculation with similar results)

$$\mathrm{Re}\left(\frac{\Gamma_{12}}{M_{12}}\right)^{SM} = -0.0040 \pm 0.0016. \tag{21}$$

As concerns $(\Delta M_s)^{SM}$, we use [11]

$$(\Delta M_s)^{SM} = \frac{G_F^2}{6\pi^2}\eta_B m_{B_s}\hat{B}_{B_s}F_{B_s}^2 S(x_t)|V_{tb}V_{ts}|^2 = 17.8 \pm 4.8\ \mathrm{ps}^{-1}. \tag{22}$$

It is important to note that the range for $|V_{ts}V_{tb}|$ is derived using tree level processes and CKM unitarity. The combination of (21) and (22) gives

$$(\Delta\Gamma_s)^{SM} = -0.07 \pm 0.03\ \mathrm{ps}^{-1}. \tag{23}$$

We can now fit the new physics parameters r_s^2 and $2\theta_s$ to the experimental values of Eqs. (6), (7), and (20), via Eqs. (2)–(4). To do so, we use the SM estimates of Eqs. (21)–(23).

It is easy to understand the constraint on r_s^2 by simply using Eq. (2):

$$r_s^2 = \frac{(\Delta M_s)^{\text{expt}}}{(\Delta M_s)^{\text{SM}}} = 0.97 \pm 0.26. \tag{24}$$

To get a feeling for the situation concerning $2\theta_s$, we first use Eqs. (3) and (4) separately. The $\Delta\Gamma_s$ measurement gives

$$\cos^2 2\theta_s = \frac{(\Delta\Gamma_s)^{CP}}{(\Delta\Gamma_s)^{\text{SM}}} = 3.1 \pm 1.7. \tag{25}$$

This range disfavors (at the 1.8σ level) small $\cos^2 2\theta_s$ values, that is $2\theta_s \sim \pi/2, 3\pi/2$. The A_{SL}^s measurement gives

$$\sin 2\theta_s = -\frac{A_{\text{SL}}^s}{\text{Re}\,(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}} \frac{(\Delta M_s)^{\text{expt}}}{(\Delta M_s)^{\text{SM}}} = -1.9 \pm 2.8. \tag{26}$$

This range disfavors large positive $\sin 2\theta_s$ values, that is $2\theta_s \sim \pi/2$. The combination of the two sources of constraints should therefore disfavor the regions around $2\theta_s \sim \pi/2, 3\pi/2$, with stronger significance for the first. In Fig. 1 we present the constraints in the r_s^2-$2\theta_s$ plane. Note that Eqs. (25) and (26) do not take into account the correlations between the contributions to the various observables, since they are meant to emphasize the impact of each measurement separately. The correlations are, however, fully taken into account in Fig. 1.

We learn that the constraints on $2\theta_s$ are still rather weak. In principle, the error on A_{SL}^s is still a factor of 3 larger than what is needed to have sensitivity to $\sin 2\theta_s$. However, since the central value for $\sin 2\theta_s$ happens—presumably due to statistical fluctuations—to lie below the physical region, large positive values of $\sin 2\theta_s$ are disfavored (at the 1σ level). The error on $\Delta\Gamma_s^{CP}$ is closer to what is needed to be sensitive to $2\theta_s$ and, indeed, the resulting constraint is more significant.

We also consider a subclass of our framework, where new physics contributions are significant only in $b \to s$ transitions. This modifies the analysis in three ways: (1) we can now extract a narrower range for $(\Delta M_s)^{\text{SM}}$ by using, in addition to the direct calculation of Eq. (22), an indirect calculation [29,30] that makes use of experimental measurements of $b \to d$ (and $s \to d$) processes and, in particular, identify $\Delta M_d^{\text{expt}} = \Delta M_d^{\text{SM}}$: $(\Delta M_s)^{\text{SM}} = 21.7^{+5.9}_{-4.2}\ \text{ps}^{-1}$ [31]. The direct calculation of Eq. (22) and the indirect one quoted here are essentially independent of each other. Therefore, we average over these two results and get

$$(\Delta M_s)^{\text{SM}} = 19.7 \pm 3.5\ \text{ps}^{-1}. \tag{27}$$

(2) We can set $A_{\text{SL}}^d = 0$ and then

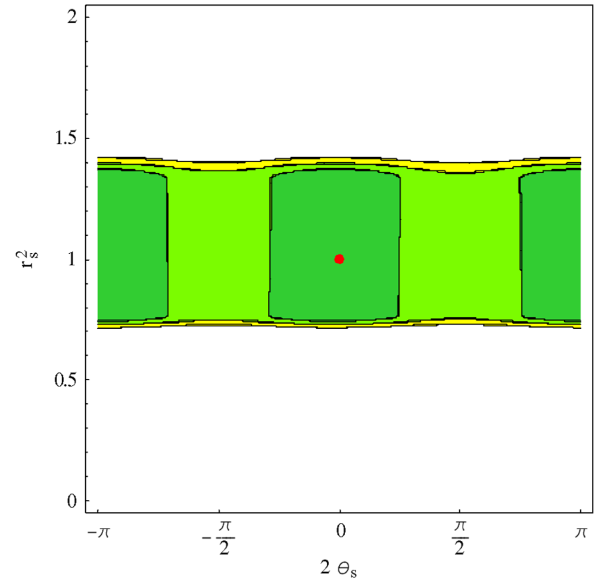

FIG. 1 (color online). The constraints in the r_s^2-$2\theta_s$ plane allowing for new physics in all loop processes. The dark green, light green, and yellow regions correspond to probability higher than 0.32, 0.046, and 0.0027, respectively. The SM point, $2\theta_s = 0$, $r_s^2 = 1$, is marked with red.

$$A_{\text{SL}}^s \simeq 2.5 A_{\text{SL}} = -0.007 \pm 0.007. \tag{28}$$

(3) We can now use (27) to obtain a more precise estimate of $(\Delta\Gamma_s)^{\text{SM}}$:

$$(\Delta\Gamma_s)^{\text{SM}} = -0.08 \pm 0.03\ \text{ps}^{-1}. \tag{29}$$

Now we get

$$r_s^2 = 0.88 \pm 0.16, \qquad \cos^2 2\theta_s = 2.8 \pm 1.6, \qquad \sin 2\theta_s = -1.4 \pm 1.6. \tag{30}$$

The situation is then quite similar to the first scenario. We show the constraints in the r_s^2-$2\theta_s$ plane in Fig. 2. As can be seen in the figure, $2\theta_s = \pi/2$ is disfavored at the 2σ level.

Conclusions.—The measurement of A_{SL} by D0 probes CP violation in B_s-$\bar{B}_s$ mixing, $A_{\text{SL}}^s = -0.008 \pm 0.011$. In combination with the measurement of ΔM_s by CDF, and the measurements of $\Delta\Gamma_s^{CP}$ by D0 and CDF, the CP violating phase of the mixing amplitude is constrained for the first time. The constraints are still weak. Since experiments favor large values of $\Delta\Gamma_s$ compared to the SM value, small values of $\cos^2 2\theta_s$ (i.e., $2\theta_s \sim \pi/2, 3\pi/2$) are disfavored. Furthermore, since experiments favor a negative A_{SL}^s [see Eqs. (20) and (28)] and $\text{Re}\,(\Gamma_{12}^s/M_{12}^s)^{\text{SM}}$ is negative, large positive values of $\sin 2\theta_s$ (i.e., $2\theta_s \sim \pi/2$) are disfavored even more strongly. This means that, within our framework, values of $S_{\psi\phi(CP=+)}$ close to -1 are disfavored.

Of course, the phase $2\theta_s$ will be strongly constrained once $S_{\psi\phi}$ is measured. Then the combination of the four measurements—ΔM_s, $\Delta\Gamma_s$, A_{SL}^s, and $S_{\psi\phi}$—will provide a test of the assumption that new physics affects only loop

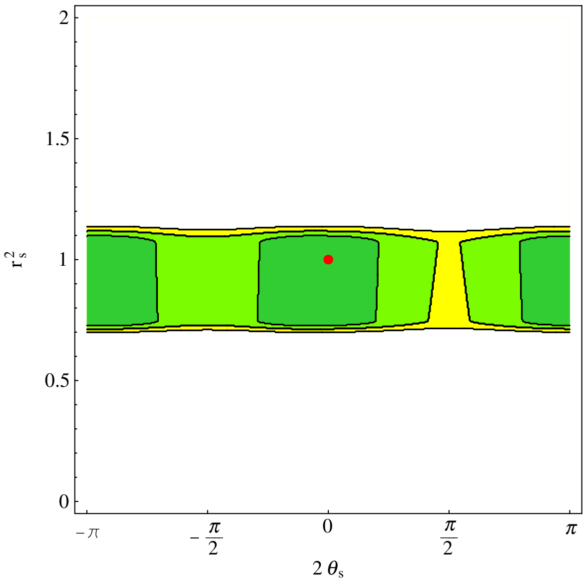

FIG. 2 (color online). The constraints in the r_s^2-$2\theta_s$ plane allowing for new physics in $b \to s$ loop processes only. The dark green, light green, and yellow regions correspond to probability higher than 0.32, 0.046, and 0.0027, respectively. The SM point, $2\theta_s = 0$, $r_s^2 = 1$, is marked with red.

processes [10,11,32]. The strength of this test will, however, be limited by theoretical uncertainties, particularly by the calculation of $\Gamma_{12}^{\rm SM}$.

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