Strongly Coupled Chameleon Fields: New Horizons in Scalar Field Theory

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We show that, as a result of nonlinear self-interactions, scalar field theories that couple to matter much more strongly than gravity are not only viable but could well be detected by a number of future experiments provided that they are properly designed to do so.

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There is widespread interest in the possibility that, in addition to the matter described by the standard model of particle physics, our Universe may be populated by one or more scalar fields. These are a general feature in high energy physics beyond the standard model and are often related to the presence of extra dimensions. The existence of scalar fields has also been postulated as a means to explain the early and late time acceleration of the Universe. It is almost always the case that such fields interact with matter: either due to a direct Lagrangian coupling or indirectly through a coupling to the Ricci scalar or as the result of quantum loop corrections. If the scalar field self-interactions are negligible, then the experimental bounds on such a field are very strong: requiring it to either couple to matter much more weakly than gravity does or to be very heavy [1]. Recently, a novel scenario was presented by Khoury and Weltman [2] that employed self-interactions of the scalar field to avoid the most restrictive of the current bounds. They dubbed such scalars to be "chameleon fields" due to the way in which the field's mass depends on the density of matter in the local environment. A chameleon field might be very heavy in relatively high density environments, such as the Earth and its atmosphere, but almost massless cosmologically where the density is some 10^{-30} times lower. This feature allows the field to evade local constraints on fifth force effects and is deemed the *chameleon mechanism*.

Chameleon field theories involve nonlinear self-interactions, which makes finding analytical solutions difficult, particularly in highly inhomogeneous environments. Most commentators invariably, therefore, linearize the chameleon theories when studying their behavior in such backgrounds [2,3]. In this Letter, we show that this linearization procedure is often invalid. When properly accounted for, the nonlinearities increase the strength of the chameleon mechanism: further hiding the field from present day constraints, particularly those on possible violations of the weak equivalence principle (WEP). Our results not only reveal interesting behavior at the level of field theory but that today's experimental bounds on the parameters of these theories could be much weaker than previously realized. Furthermore, they imply that experiments which probe possible violations of the WEP should be redesigned if they are to have a chance of detecting chameleon fields.

We consider theories where the chameleon field ϕ has a self-interaction potential given by:

$$V(\phi) = \lambda M^4 (M/\phi)^n,$$

where M has units of mass, n is some integer, and λ is a parameter. We set $c = \hbar = 1$ and define $G = M_{\rm pl}^{-2}$. Theories with $n > 0$ were first consider in this context in Ref. [2], while a ϕ^4 theory was initially noted to have chameleonlike behavior in Ref. [4]. When $n \neq -4$, we can, by rescaling M, set $\lambda = 1$, whereas when $n = -4$, the mass scale M does not appear in V. As argued in Ref. [4], $\lambda = 1/4!$ would be a "natural" value when $n = -4$. If $M \sim (0.1\ \text{mm})^{-1}$, the chameleon may play the role of dark energy [3].

We parametrize the matter coupling of the chameleon by a function $\beta B_{,\phi}(\beta\phi/M_{\rm pl})\rho/M_{\rm pl}$. Astrophysical constraints require that $|\beta\phi/M_{\rm pl}| \lesssim 0.1$ since nucleosynthesis [3]. Preempting this requirement, we simplify our calculations by expanding $B_{,\phi}$ about $\phi = 0$ and scale β so that $B_{,\phi}(0) = 1$. The equation of motion for ϕ then becomes

$$-\Box\phi = V_{,\phi}(\phi) + \beta(\rho + \omega P)/M_{\rm pl}, \qquad (1)$$

where ρ is the energy density of matter, P is its pressure, and ω parametrizes the way in which the chameleon couples to matter. In the simplest models, ϕ couples to the trace of the energy momentum tensor, and so $\omega = -3$. In what follows, we take this to be our fiducial value of ω and note that the results for different $O(1)$ values of ω are very similar [5]. We note that the right-hand side of Eq. (1) vanishes when $\phi = \phi_c(\rho + \omega P)$:

$$\phi_c(\rho + \omega P) = M(\beta(\rho + \omega P)/(\lambda n M_{\rm pl} M^3))^{-1/n+1}.$$

For $\phi_c(\rho + \omega P)$ to be real when $\beta(\rho + \omega P) > 0$, we need either $n \geq 0$ or for n to be negative and even and $n \neq 0, -2$ for the theory to be nonlinear. The mass of small perturbations about $\phi = \phi_c$ is $m_c = \sqrt{V_{,\phi\phi}(\phi_c)} = \sqrt{\lambda n(n+1)} M|M/\phi_c|^{n/2+1}$.

One would expect, in the absence of any chameleon mechanism, the force mediated by ϕ to be β^2 as strong as gravity. As a result of quantum corrections, β will generally differ slightly for different particle species, which would standardly lead to a composition-dependent force that would, in turn, violate WEP. Solar system bounds on WEP violation require $\beta \lesssim 10^{-5}$ in nonchameleon theories [1]. Chameleon theories have been shown to be compatible with $\beta \sim O(1)$ [2]. In this Letter, however, we will go much further and report how, as a result of nonlinear effects, it is possible for a chameleon field to couple to matter much more strongly than gravity does (i.e., $\beta \gg 1$) and yet for it to have remained thus undetected. We define $M_\phi = M_{\rm pl}/\beta$, which is roughly the energy at which chameleon particles would be produced in particle colliders. It would be pleasant in the light of the hierarchy problem if $M_{\rm pl}/\beta \ll M_{\rm pl}$, say, of the grand unified theory scale, or, if we hoped to find traces of it at the LHC, maybe even the TeV scale. We show below that both of these scenarios are allowed for.

Crucial to our ability to constrain chameleon theories is a full understanding of how they behave as field theories. It transpires that when $\beta \gg 1$, the nonlinear nature of the potential $V(\phi)$ becomes very important. Even in the, supposedly, simple case of the field produced by a single large body, there might not exist any self-consistent linearization of Eq. (1) that is valid everywhere [5]. Nonlinear effects are also non-negligible when calculating the force produced by one body upon another. When linearized theory fails, the solution to the two-body problem cannot be found simply by superimposing two copies of the field produced a single body.

Nonlinear effects also play a role in determining the effective large-scale or macroscopic theory associated with the chameleon. Equation (1) defines the microscopic, or particle-level, field theory for ϕ, whereas in most cases we are interested in the large-scale or coarse-grained behavior of ϕ. In macroscopic bodies, the density is actually strongly peaked near the nuclei of the individual atoms from which it is formed, and these are separated from each other by distances much greater than their radii. Rather than explicitly considering the microscopic structure of a body, it is standard practice to define an "averaged" field theory that is valid over scales comparable to the body's size. If our field theory were linear, then the averaged equations would be the same as the microscopic ones, e.g., as in Newtonian gravity. But it is important to note that this is very much a property of *linear theories* and is not, in general, true of nonlinear ones. Nonlinear effects must, therefore, be taken into account. We do this by combining matched asymptotic expansions with exact analytical solution of the full nonlinear equations under certain reasonable assumptions. We confirm our results by numerically integrating the field equations.

First, we define the concept of a *thin shell*. A body is said to have a thin shell if the coarse-grained value of ϕ (as defined on scales that are large compared to the sizes of the constituent particles of the body) is approximately constant everywhere inside the body, except in a thin shell near the surface of the body where large changes $[O(1)]$ in its value occur. The existence of a thin shell is related to the presence of nonlinear behavior. Deep inside a body with a thin shell, ϕ is constant, and so we might expect $\phi = \phi_c(\rho)$, where ρ is the density of the body (we assume $P \ll \rho$). The effective chameleon mass $m_{\rm eff}$ in the body would then be given by $m_{\rm eff} = m_c(\rho)$. The effect of the nonlinearities on the averaging, however, is to limit the averaged value of m_ϕ to be smaller than some critical value $m_{\rm crit}$ [5]. $m_{\rm crit}$ is a macroscopic quantity, but it depends only on the microscopic properties of the body and the index n. It is *independent* of β, M, and λ [5]. We have modeled the body as being composed of particles of radius R_p separated by an average distance d_p. The macroscopic mass of the chameleon in the body is then $m_{\rm eff} = \min(m_c(\rho), m_{\rm crit})$, where

$$m_{\rm crit} \approx \sqrt{3|n+1|}\, d_p^{-1}(R_p/d_p)^{q(n)/2}, \qquad n \neq -4,$$

where $q(n) = \min(1, (n+4)/(n+1))$ and $m_{\rm crit} \approx 1.4/d_p$ when $n = -4$. Whenever $m_{\rm eff} = m_{\rm crit}$, it is because the individual particles that make up the body have themselves developed thin shells. This critical behavior emerges from the requirement that nonlinear effects are negligible outside of the particle from $r = R_p$ to $r = d_p$: This implies a maximal value of $m_{\rm eff}$, i.e., $m_{\rm crit}$, that depends only on R_p, d_p, and n. The n dependence arises because n determines precisely when linear theory breaks down.

β-independent critical behavior is also seen in the ϕ force between two bodies. The onset of this critical behavior is linked to the emergence of a thin shell. A body of radius R and density ρ_c in a background of density $\rho_b \ll \rho_c$ has a thin shell if:

$$m_{\rm eff} R \gtrsim \sqrt{3|n+1|}\,|1 - (\rho_c/\rho_b)^{1/n+1}|^{1/2}, \qquad n \neq -4. \tag{2}$$

The existence of a thin shell is essentially due to nonlinearities being strong near the surface of a body but weak in other regions. When $n = -4$, a thin shell occurs for $m_{\rm eff} R \gtrsim 4$, whereas linearized theory fails to be accurate for $m_{\rm eff} R \gtrsim 1.4$. When $n > 0$, $(\rho_c/\rho_b)^{1/n+1} \gg 1$ and so the thin shell condition, Eq. (2), depends greatly upon on the background density. The same is *not* true when $n \leq -4$ since here $(\rho_c/\rho_b)^{(1/n+1)} \ll 1$. Therefore, $n > 0$ theories can behave differently in space-based experiments than they do in laboratory ones, because the thin-shell condition is more restrictive in the low-density background of space than it is in the lab [2]. In contrast, theories with $n \leq -4$ will exhibit no big difference in their behavior in space-based tests to that seen on Earth.

The existence of a thin shell in the test masses used in experimental searches for deviations from general relativity is vital if we are to evade their bounds. Whereas the force between two non-thin-shelled bodies with separation r is $\beta^2(1+m_b r)e^{-m_b r}$ times the gravitational force between them (m_b is the chameleon mass in the region between the bodies), the force between two bodies, of masses M_1 and M_2 with thin shells is found to be independent of the coupling β [5]. When $d \gg R_1, R_2$, where R_1 and R_2 are the respective radii of the two bodies, this force is found to be α_{12} times the strength of gravity, where for $n \neq -4$:

$$\alpha_{12} = \frac{S(n, m_b) M_{\rm pl}^2 (1 + m_b r) e^{-m_b r}}{M_1 M_2} (M^2 R_1 R_2)^{q(n)},$$

where $S(n, m_b)$ is $(3/|n|)^{2/|n+2|}$ for $n < -4$, whereas for $n > 0$ it equals $(n(n+1)M^2/m_b^2)^{2/(n+2)}$. When $n = -4$,

$$\alpha_{12} = \frac{M_{\rm pl}^2 (1 + m_b r) e^{-m_b r}}{8\lambda M_1 M_2 \sqrt{\ln(r/R_1)\ln(r/R_2)}}.$$

For $d \lesssim R_1, R_2$, a different value for α_{12} applies and is given below. This β independence was first noted in Ref. [6], in the context of ϕ^4 theory. However, the authors were mostly concerned with a region of parameter space $\beta < 1$, $\lambda \ll 1$; in our analysis, we go further: considering a wider range of theories and also the possibility that $\beta \gg 1$.

We can understand the β independence as follows: Just outside a thin-shelled body, the potential term in Eq. (1) is large and negative [$\sim O(-\beta\rho/M_{\rm pl})$], and it causes ϕ to decay very quickly. At some point, ϕ will reach a critical value $\phi_{\rm crit}$ that is small enough so that nonlinearities are no longer important. Since this all occurs outside the body, $\phi_{\rm crit}$ can depend only on the size of the body, the choice of potential (M, λ, n), and the mass of ϕ in the background, m_b. This is precisely what was found above.

This β independence is of great importance if one wishes to design an experiment to detect the chameleon through WEP violations. Since the ϕ force is independent of the coupling β for bodies with thin shells, any microscopic composition dependence in β will be hidden on macroscopic length scales. The only "composition" dependence in α_{12} is through the masses of the bodies and their dimensions (R_1 and R_2). The strength of WEP violations is quantified by the Eötvos parameter η. If we measure the differential accelerations of two test masses M_1 and M_2 of radii R_1 and R_2 towards a third body, mass M_3 and radius R_3, then $\eta = \alpha_{13} - \alpha_{23}$. Taking the third body to be the Sun or the Moon, experimental searches for WEP violations have up to date found that $\eta \lesssim 10^{-13}$ [7]. In most of these searches, although the composition of the test masses is different, they are made to have the *same* mass ($M_1 = M_2$) and the *same* size ($R_1 = R_2$). Therefore, if the test masses have thin shells, we have $\eta = 0$, and *no* WEP violation will be detected. The only implicit dependence of this result on β is that the *larger* the coupling is, the more likely it is that the test masses will satisfy the thin-shell conditions. The first important consideration for future experiments is that, if one wishes to detect a chameleon field through WEP violations, one must ensure either that test masses do not satisfy the thin-shell conditions or that they are of different masses and/or dimensions.

We shall assume that such an experiment has been conducted, using two spherical test bodies both with a mass of 10 g, where one is made entirely of copper and the other of aluminum. The strongest bounds on chameleon fields would then come from measuring the differential acceleration of these bodies towards the Moon. We indicate in Fig. 1 the restrictions that finding $\eta \lesssim 10^{-13}$ in such an experiment would place on these chameleon theories. The Moon is a better choice of attractor than the Earth or the Sun for such experiments, since α_{13} is proportional to $M_{\rm pl}^2/M_1 M_3$ and so the smaller mass of the test bodies M_1 and the attractor M_3, the larger η will be compared to gravity. The corollary of this result is that if we are unable to detect ϕ in lab-based, microgravity experiments where both M_1 and $M_2 \sim O(10\text{ g})$ (such as the Eöt-Wash experiment), then the ϕ force between larger (say, human-sized) objects would also be undetectably small. For this reason, measurements of the differential acceleration of the Earth and Moon towards the Sun, e.g., lunar laser ranging, are not competitive with lab-based experiments.

Future, space-based tests of WEP promise to be able to detect η up to a precision of 10^{-18}; we indicate in Fig. 1 the

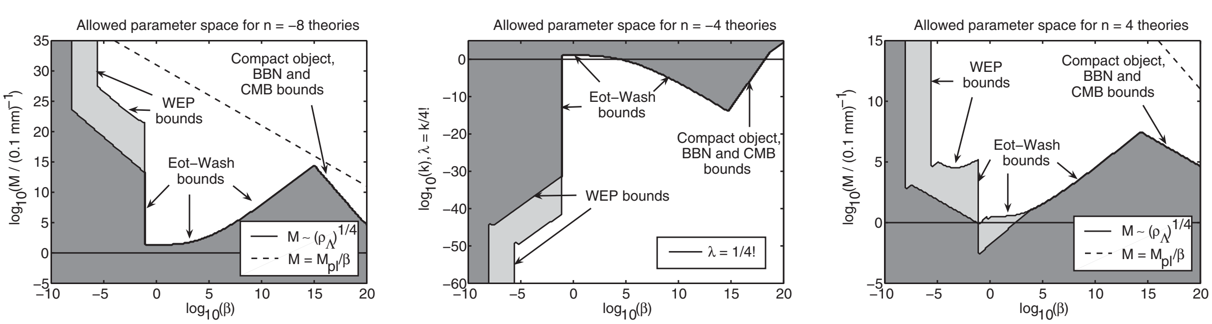

FIG. 1. The whole of the shaded area shows the allowed parameter space with all current bounds. For some values of M and λ, we need the BeCu sheet to have a thin shell; this results in a region near $\beta \sim O(1)$ being ruled out. Future space-based tests could detect the more lightly shaded regions. The solid horizontal lines indicate the case where the chameleon field behaves like dark energy. Plots for theories with $n < -4$ or $n > 0$ are similar to cases $n = -8$ and $n = 4$, respectively.

regions of parameter space that such experiments would be able to detect. The ϕ-mediated force will also produce effective corrections to the $1/r^2$ behavior of gravity. The best bounds on such corrections come from the Eöt-Wash experiment performed by Hoyle *et al.* [8], which employs a torsion balance to measure the torque induced on a pendulum by a rotating attractor at a separation d. For $d \gtrsim 0.1$ mm, they find that $\alpha_{12} \lesssim 10^{-2}$ [8]. For a chameleon theory to satisfy this bound, we need both the attractor and pendulum to have thin shells. In this scenario, d is small compared to the size of test masses ($d < R_1, R_2$) and so the previous formula for α_{12} does not apply. When the mass of the chameleon inside the attractor and pendulum m_ϕ obeys $m_\phi d \gg 1$ (as is the case for $\beta \gtrsim 1$), we find that the ϕ force is α times the strength of gravity, where α_{12} is

$$5\times 10^{-4}\left(\frac{M}{(0.1\,\mathrm{mm})^{-1}}\right)^{2(n+4)/n+2}\left(\frac{\lambda^{1/n}\sqrt{2}B(\frac{1}{2},\frac{1}{2}+\frac{1}{n})}{|n|d/0.1\,\mathrm{mm}}\right)^{2n/n+2},$$

where $B(p, q)$ is the beta function. We note that α, as before, is *independent* of β. The Eöt-Wash bound is strongest for $n = -4$ where it appears to rule out a "natural" value for λ of $1/4!$: $0.56\lambda^{-1} \lesssim 1$. However, this is not the whole story. In this experiment, a uniform 10 μm thick BeCu membrane is placed between the pendulum and attractor to shield electromagnetic forces. For $O(1)$ values of β and $\lambda \sim 1/4!$ or $M \sim (0.1\ \mathrm{mm})^{-1}$, this sheet does not have a thin shell and makes little difference to the analysis. For slightly larger values of β, however ($\beta \gtrsim 10^4$ and $\lambda = 1/4!$ for $n = -4$), it will develop a thin shell. Taking the mass of the chameleon inside the sheet to be m_s, the effect of this membrane is then to attenuate α_{12} by a factor of $\exp(-m_s d_s)$, where d_s is the thickness of the sheet. The Eöt-Wash bound is then easily satisfied even for $\lambda \sim 1/4!$. The larger β becomes, the larger m_s is and the less restrictive this bound becomes. Experiments such as this must, therefore, be redesigned if they are to be able to detect chameleon theories with $\beta \gg 1$.

The prospect that couplings with $\beta \gg 1$ could be allowed is exciting. But to be taken seriously, we must also consider bounds coming from astrophysical constraints, such as the stability and mass-radius relationship of white dwarfs and neutron stars as well as bounds coming from big bang nucleosynthesis (BBN) and the cosmic microwave radiation temperature anisotropies [3,5]. These bounds can be summarized as requiring $|\beta\phi/M_{\mathrm{pl}}| \lesssim 0.1$ over the whole Universe since the BBN epoch [3,5]. This condition is enough to ensure that there has been no more than a 10% change in particle masses since BBN and in the redshift of the surface of last scattering. While we satisfy the same physical constraints as Amendola for nonchameleon, coupled quintessence [9], the chameleon mechanism ensures a significantly less restrictive bound on β than was found there. Astrophysical constraints place only a weak upper bound on β, which is strongest for $n = -4$, e.g., if $\lambda = 1/4!$, we need $M_{\mathrm{pl}}/\beta \gtrsim 10$ GeV. However, realistically, we probably require $M_{\mathrm{pl}}/\beta \gtrsim 200$ GeV for it not to have been seen so far in particle colliders.

In summary, we have considered a wide spectrum of scalar field theories with a chameleon mechanism, and, for the first time, the nonlinear structure of these theories has been properly taken into account. We have found a surprising result that the chameleon force between two bodies with thin shells is *independent* of their coupling to the field ϕ and that, as a result, the bounds on the coupling β can be exponentially relaxed. We have also noted that some laboratory experiments should be redesigned to detect the chameleon. For "natural" values of $M \sim (0.1\ \mathrm{mm})^{-1}$ or $\lambda \sim 1/4!$, the strongest upper bounds on β probably come from particle colliders, and 200 GeV $\lesssim M_{\mathrm{pl}}/\beta \lesssim 10^{15}$ GeV is allowed for all n. If $M_{\mathrm{pl}}/\beta \sim 1$ TeV, we might even hope to see chameleon production at the LHC; although without a renormalizable quantum theory of the chameleon, it is hard to say for sure if this will happen. Planned space-based tests such as STEP, MICROSCOPE, and SEE [10] promise improved precision, and, when $n > 0$, there is also still the possibility that WEP violations in space can be stronger than the level already ruled out by laboratory-based experiments. As noted in Refs. [2,3], the chameleon field is a good candidate for dark energy if $M \sim (\rho_\Lambda)^{1/4} \approx (0.1\ \mathrm{mm})^{-1}$; this result is unchanged for $\beta \gg 1$.

In conclusion, scalar field theories that couple to matter much more strongly than gravity are not only viable but could well be detected by a number of future experiments provided that they are properly designed to do so. This result opens up an altogether new window which might lead to a completely different view of the role played by scalar fields in particle physics and cosmology.

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