Reduced Decoherence in Large Quantum Registers

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Among the biggest obstacles for building larger (and thus more powerful) quantum-information processors is decoherence, the decay of quantum-information by the coupling between the quantum register and its environment. Procedures for reducing decoherence processes will be essential for successful operation of larger quantum processors. We study model quantum registers consisting of up to 4900 qubits and measure their decay as a function of the register size. We demonstrate that appropriate sequences of qubit rotations reduce the coupling between system and environment for all sizes of the quantum register, thus preserving the quantum-information 50 times longer than without decoupling.

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Quantum computers possess enormous potential for solving some computational problems that cannot be solved efficiently on classical computers [1,2]. To realize this potential, well-controlled quantum systems are required that consist of at least several tens to several thousands of qubits. The conditions that must be met to reliably operate such quantum-information processors [3,4] include quantum error correction schemes as well as the capability to perform a large number of quantum operations within the decoherence time of the system. A number of systems have been proposed as potential candidates to realize such scalable quantum computers [5–11]. In most cases, it is possible to estimate the gate operation times that might be realized with them. Much less is known about the decoherence time for the systems. Where estimates and experimental data are available, they typically refer to individual qubits, while very little is known about the dependence of the decoherence time on the size of the quantum register [12]. From the early times of quantum mechanics, it was considered evident that coherence in large quantum systems decays on a very short time scale. This assumption is generally used to explain why superposition states are not observed in macroscopic bodies [13].

In those cases, where decoherence processes are too fast to allow direct application of quantum error correction, it has been proposed to impose an additional time dependence on the quantum system in such a way that it reduces the decoherence processes [14,15]. Techniques of this type have been shown to work in small systems [16,17], but so far it has not been possible to test their viability in large quantum systems.

To address these issues, we have experimentally implemented model quantum registers consisting of several thousand nuclear spin qubits. In a magnetic field, the Zeeman interaction splits the two states of spins $I = \frac{1}{2}$, which represent the logical states of qubits. The qubits in our quantum registers cannot be addressed individually; to execute quantum algorithms in such a system will require a symmetry-breaking interaction somewhere in the lattice.

The decoherence of these superposition states is dominated by the magnetic dipole-dipole couplings between pairs of nuclear spins. These couplings depend on the orientation of the nuclear spins with respect to the external field. This orientation dependence can be used to eliminate them by averaging their effect over at least three mutually orthogonal orientations. We use this possibility to compare the decoherence of large quantum registers with and without actively decoupling it from its environment.

The experiment starts with a spin system in thermal equilibrium, where all the nuclear spins are uncorrelated (Fig. 1). The corresponding density operator of N nuclear spins can be written in the high-temperature approximation

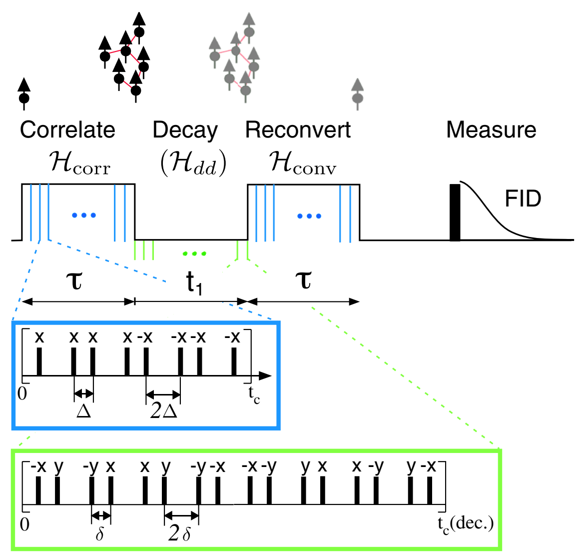

FIG. 1 (color online). Experimental scheme: the Hamiltonian $\mathcal{H}_{\rm corr}$ correlates the spin qubits. The resulting highly correlated states decay during t_1 and after a reconversion back into longitudinal magnetization, we measure the coherence amplitudes as the amplitude of the free induction decay (FID). The effective Hamiltonians driving the evolution are generated by sequences of qubit rotations (radio-frequency pulses), which are indicated in the insets: x indicates a $\pi/2$ rotation around the x axis.

as [18]

$$\rho_{eq} = \frac{1}{2^N}\left(\mathbf{1} + \frac{\hbar\gamma B_0}{k_B T}\sum_{j=1}^{N} I_z^j\right), \tag{1}$$

where γ is the gyromagnetic ratio, B_0 the static magnetic field, k_B is the Boltzman constant, T the temperature, and I_z^j are the z components of the spin operators.

The spins interact with each other by the magnetic dipole-dipole coupling, which can be described by the Hamiltonian

$$\mathcal{H}^{\text{dd}} = \sum_{j,k} d^{jk}\left\{2I_z^j I_z^k - \frac{I_+^j I_-^k + I_-^j I_+^k}{2}\right\}. \tag{2}$$

d^{jk} are the coupling constants and $I_\pm := I_x \pm iI_y$ are the raising and lowering spin operators. This operator commutes with the total Hamiltonian of the system; the equilibrium density operator therefore does not evolve under the dipole-dipole interaction. The dipole-dipole coupling is the dominant interaction of the spins, apart from the Zeeman interaction.

To create the highly correlated spin clusters that serve as our model quantum registers, we need an interaction between the spins that does not commute with the density operator. This can be achieved by modifying the dipole-dipole coupling, using multiple pulse sequences to create an average Hamiltonian [19,20]. Specifically, we used a pulse sequence that is shown in the inset of Fig. 1 [21,22], which changes the Hamiltonian of Eq. (2) into

$$\mathcal{H}_{\text{corr}} = -\tfrac{1}{2}\sum_{j,k} d^{jk}[I_+^j I_+^k + I_-^j I_-^k]. \tag{3}$$

This form of the dipole-dipole coupling drives simultaneous flips of two spins. It does not commute with the density operator and creates correlations between initially uncorrelated spins. Under its influence, the equilibrium density operator evolves into a state that includes correlations between all spins that are coupled by the network of dipole-dipole interaction. These are the same correlations that carry the information of a quantum register. Because of the $\sim 1/R^3$ dependence of the dipole-dipole interaction of two spins separated by a distance R, the correlations first develop in small clusters of spins that are close to each other and therefore strongly coupled. For longer times, these clusters become larger and involve also weakly coupled spins.

To measure the information stored in the quantum register, we used a method introduced by Yen and Pines [23]. It converts the unobservable multispin coherences back into measurable single spin magnetization. This can be achieved by reverting the time evolution of the preparation period by generating a Hamiltonian $\mathcal{H}_{\text{conv}} = -\mathcal{H}_{\text{corr}}$ acting for the same duration τ (see Fig. 1). This reconversion step transfers the order remaining in the system into Zeeman polarization, which can be read out by applying a single $\pi/2$ rotation and measuring the amplitude of the resulting free induction decay (FID). The reconverting Hamiltonian $\mathcal{H}_{\text{conv}}$ is also generated as an average Hamiltonian through the same multiple pulse sequence that was used for the correlation step, but with an overall 90° phase shift of the eight pulses.

The average size of the clusters, and therefore the size of the model quantum register, can be measured by a technique developed by Baum *et al.* [22]. The basic idea of this approach is to measure the behavior of the density operator under rotations around an axis parallel to the magnetic field. In general, such a rotation changes the state as

$$\rho(\phi) = \sum_M \rho_M e^{iM\phi}, \tag{4}$$

where ϕ is the angle by which the state has been rotated and ρ_M are components of irreducible tensor operators. In a cluster of K spins, the number of the components ρ_M forms a binomial distribution whose width is of the order $\sqrt{K}$, and which vanishes for $M > K$.

Figure 2 shows how the average size $\bar{K}$ of the model quantum registers increases with the time τ during which the coupling Hamiltonian $\mathcal{H}_{\text{corr}}$ [Eq. (3)] is applied. The experimental data points are represented by crosses, and for two of the crosses, the observed distribution of multiple quantum coherence amplitudes $||\rho_M||$ is shown. The experimental results shown here (and below) were obtained at room temperature on a home-built solid-state NMR spectrometer operating at a ^{1}H frequency of 300 MHz. The proton spins of a powdered adamantane sample served as the spin system. The correlation pulse sequence is shown in Fig. 1; the experimental pulse durations and

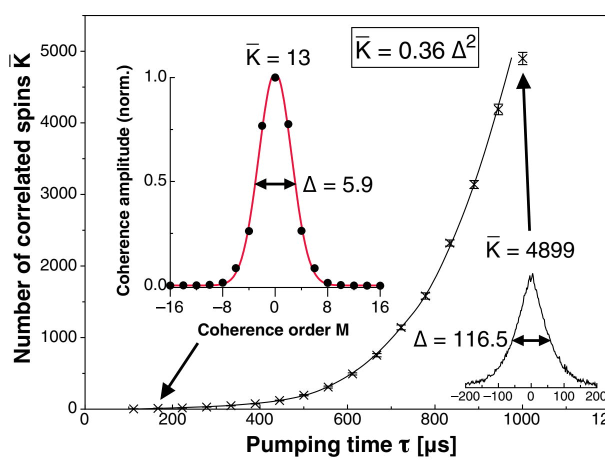

FIG. 2 (color online). Average size $\bar{K}$ of the model quantum register as a function of the excitation time τ. The crosses represent measured values whereas the solid line is a guide to the eye. The large inset shows the distribution of the coherence amplitudes for a pumping time of $\tau = 167$ μs together with the result of the Gaussian fit to determine the cluster size $\bar{K}$. The small inset shows the corresponding distribution for the longest pumping time of $\tau = 1$ ms.

delays were $t_p = 2.448\ \mu$s and $\Delta = 4.632\ \mu$s, resulting in a cycle time $t_c = 55.584\ \mu$s.

For each of these different sizes of model quantum registers, we measured the decay under free evolution and when an rf pulse sequence was applied that decoupled the quantum register from its environment. In the case of free evolution, the dominant interaction of the quantum register with the environment is the magnetic dipole-dipole coupling of Eq. (2).

Figure 3 shows the decay of six model quantum registers of different size as a function of the decay time t_1. The observed decays (shown here on a logarithmic time scale) clearly demonstrate that larger clusters of correlated spins decay on a significantly shorter time scale than small clusters or individual, uncorrelated qubits.

A number of theoretical proposals exist for reducing decoherence by applying suitable single qubit rotations to the system in such a way that the coupling to the environment is effectively averaged away [14,15]. In our system, the dominant interaction is the dipole-dipole coupling, which can be averaged to zero by suitable sequences of radio-frequency pulses that were developed in the context of high-resolution NMR in solids [19,20] to eliminate unwanted line broadening. Up to now, this approach had only been tested in systems of uncorrelated spins.

In our experiment, we applied such multiple pulse sequences to the system when it was in the highly correlated state, whose survival probability we measured. We tested several sequences for their performance in slowing down the decay of the quantum information and found qualitatively similar behavior. A typical result is shown in the right-hand part of Fig. 3: for all sizes of quantum registers, the decoupling sequence increased the lifetime of the quantum coherence by more than an order of magnitude. The data shown here were obtained with the 16 pulse sequence MREV-16 [17] for decoupling during the decay period (t_1) (see Fig. 1), using delays $\delta = 4.628\ \mu$s, resulting in a cycle time of $t_c = 111.072\ \mu$s.

For a more quantitative evaluation of the decoherence process and its dependence on the system size, we calculated decoherence rates for each case. We determined the decoherence rates as $r = 1/t_{1/e}$, where $t_{1/e}$ is the time when the coherence has decayed to $1/e$ times the initial value.

In Fig. 4, we plot this decoherence rate against the average number of qubits in the quantum register. The experimental data points are fitted to a power law dependence. In the case of free evolution, we find an exponent of 0.481 ± 0.004. The decoupling sequence reduced the decoherence rate by a factor of about 50, independent of the size of the quantum register: the observed exponent is slightly smaller (0.433 ± 0.005). This clearly shows that the decoupling procedure works well for all sizes of quantum registers.

The overall scaling of the decoherence rates is relatively benign: while the larger clusters decay more rapidly than smaller ones, the increase is significantly slower than the linear dependence that would be expected if the decoherence of the individual qubits was uncorrelated. This clearly demonstrates that correlations between different contributions to the decoherence process are important and must be taken into account when designing and implementing large-scale quantum computers. Theoretical work indicates that it is possible to quantify such correlations [24],

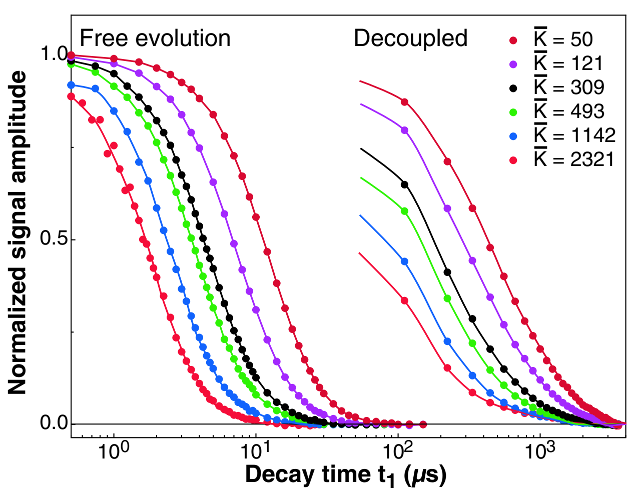

FIG. 3 (color online). Effect of decoupling for six quantum register sizes $\bar{K}$. The six left curves show the decay for the case of free evolution and the others show the decay under decoupling. The lines are meant to guide the eye. The lines at the beginning of the decay under decoupling indicate that the decay starts at $t_1 = 0$, and the normalization of the signal amplitude is in both cases relative to the value at $t_1 = 0$.

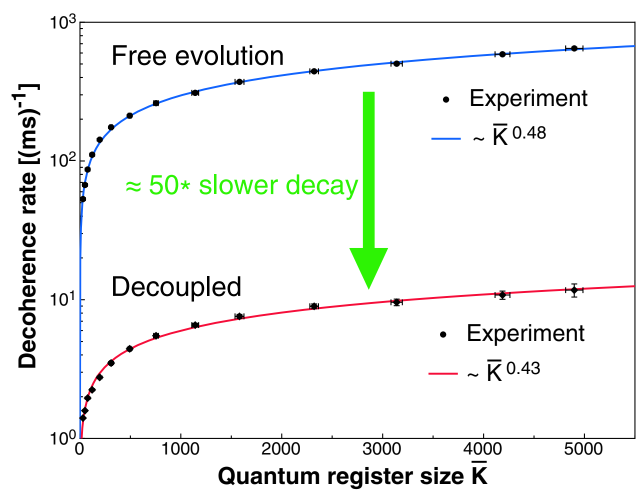

FIG. 4 (color online). Average decoherence rates for the two cases of free evolution and decoupling of the dipolar coupled spins during t_1. The decoupling reduces the decoherence rates by a factor of 50. The comparison of the decoherence rates, as well as the power function fits (blue and red line), indicate that the decoupling works slightly better for larger quantum registers than for smaller ones.

as well as to use them in order to increase the overall decoherence time of large quantum registers by encoding the qubits in clusters appropriate to the geometry of the physical system used for quantum computing [25].

In conclusion, we have measured the decoherence rates of large quantum registers as a function of the number of qubits and demonstrated that suitable sequences of single qubit rotations can significantly reduce the environment-induced decoherence, independent of the size of the register. In our model system, the quantum register as well as the environment consist of nuclear spins. The distinction between system and environment is achieved by the detection process, which is selective for the same spin clusters that have been created by the initial pumping process, while all the roughly 10^{20} spins of the sample contribute to the environment.

While it is possible to describe the coupling between the quantum register and the environment as well as the interactions within the heat bath in terms of a Hamiltonian, it is well known (see, e.g., [26,27]) that in the limit of many degrees of freedom, the resulting dynamics appears as an irreversible relaxation process.

The pulse sequence that we used to decouple the quantum register from its environment was designed to average to zero interactions that transform as second rank irreducible tensors under rotations. Magnetic dipole-dipole couplings, which are the dominant interaction for spins $1/2$ in solids [18], transform as second rank tensors, and our experimental results show that this decoupling sequence removes most of the interactions that cause decoherence. The nature of the remaining decoherence terms remains unsure at this stage [28].

The experimental results presented here were obtained in a system in the high-temperature limit, while many proposed implementations of quantum-information processors will be operated in the low-temperature regime or in pure states. Nevertheless, we expect our conclusions to remain valid for arbitrarily high polarization, since relaxation rates do not depend on the population of the different states. Furthermore, the rates that we measure here, as well as the observed decoherence rates that describe the decay of quantum-information during the execution of a quantum algorithm, are averaged over a very large number of relaxation rates that describe the dephasing of individual elements in the density operator.

If our findings for the scaling of the decoherence rates and the efficiency of decoupling for highly correlated states can be verified in other systems, it raises the prospects for successful future implementations of large-scale quantum computers. Experiments are currently under way to test the universality of these results in systems that allow independent manipulation of quantum register and environment. Initial results confirm the findings reported here.

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