Spin Correlations of Strongly Interacting Massive Fermion Pairs as a Test of Bell's Inequality

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We report the results of the first-time test of the local hidden variable theories (Bell-Clauser-Horne-Shimony-Holt) involving strongly interacting pairs of massive spin 1/2 hadrons from the decay of short-lived ($\tau < 10^{-21}$ sec) ^{2}He spin-singlet state, populated in the nuclear reaction $^2\mathrm{H} + {}^1\mathrm{H} \rightarrow {}^2\mathrm{He} + n$. The novel features of this experiment are (a) the use of an "event-ready" detector of nearly 100% efficiency to prepare an unbiased sample and (b) a focal-plane polarimeter of full 2π sr acceptance with a random "post selection" of the reference axes. The spin-correlation function is deduced to be $S_{\mathrm{exp}}(\pi/4) = 2.83 \pm 0.24_{\mathrm{stat}} \pm 0.07_{\mathrm{sys}}$. This result is in agreement with nonlocal quantum mechanical prediction and it violates the Bell-CHSH inequality of $|S| \leq 2$ at a confidence level of 99.3%.

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It is well known that Einstein, Podolsky, and Rosen (EPR) [1] asserted that quantum mechanics is incomplete in terms of local realism, a classical physicist's conception of nature. Bell [2] rendered this argument quantitative and amenable to experimental verification as he showed that any local hidden variable theory will result in an inequality, which can contradict quantum mechanical predictions. Clauser, Horne, Shimony, and Holt (CHSH) [3] further sharpened this discussion as they presented a geometrical scheme where one can easily identify the regions of clear disagreement between local hidden variable theories and quantum mechanics. In a seminal review, Clauser and Shimony [4] analyzed the physics of idealized versus practical experimental arrangements. They identified an important, delicate problem of biased data samples in which the detected events may not be fully representative of the true spin correlations of pairs. To test Bell's inequalities, they noted that the experimental observables of the pairs of decay particles must correspond to strong quantum mechanical correlations and be detected with high efficiency. Filters to eliminate the unwanted emissions and polarization analyzers of good discriminating power are warranted. They asserted that locality prescribes "that the measured value of a quantity on one system is not causally affected by what one chooses to measure on the other system, since the two systems are well separated (e.g., spacelike separated) when the measurements are performed" (see Ref. [4], page 1892).

Here, we report our measurements where a nondepolarizing "event-ready" detector system (see Fig. 1 in [4]) of high efficiency ensured unbiased sampling. Also, the complete solid angle coverage of 2π sr of the polarimeter for each proton (a) allowed us to vary the reference axes at will after the experiment was performed, thus overcoming the observer dependent reality, (b) with no constraint on the choice of reference axes, renders the system an infinite channel polarization analyzer, and (c) overcomes the counterfactuality loophole.

It is noteworthy that the Bell-CHSH formulation concerns itself with experiments where spin orientations have two possible outcomes (up/down or ±) with no reference to interaction mechanism. Numerous experiments measured polarization correlations between entangled photon pairs produced by positronium decay [5], atomic cascade [6–9], and parametric down-conversion of lasers [10–15] to demonstrate that Bell's inequality is violated. References [4,16] summarize these experiments. Recently, experiments using $^9\mathrm{Be}^+$ ions [17] and a hybrid system of an atom and a photon [18] have been performed.

All experiments, with the exception of Lamehi-Rachti and Mittig (LRM) [19], and Polachic *et al.* [20], relied on entangled systems produced by electromagnetic (EM) interactions. Spacelike separation, dictated by the locality condition, means that there is no conceivable communication between the interacting partners after they are separated from the source point. It is indeed a significant achievement that the optics experiments address this loophole by employing very long cables (see, for example, [15]). We take advantage of the fact that the correlations of hadronic pairs and also nuclear spin scatterings of hadrons in the analyzers are of ranges of about 10^{-15} meters, unlike the EM processes which are of infinite range. The present experiment relies on the purity of entangled pairs as determined from energy and momentum correlations of protons in the pair. Though LRM also employed strong interactions, they fixed the directions of the spin-reference axes before the measurement was carried out and the polarizers had small acceptances, which were the sources of contextuality and observer dependent reality. They had to resort to model estimates for the contamination from the triplet state contributions. The experiment of Polachic *et al.* [20] was similar to the present work, but they were unable to draw definitive conclusions due to poor statistics.

The present experiment was performed with the 270 MeV deuteron beam in conjunction with a liquid hydrogen target at the RIKEN Accelerator Research Facility. Entangled proton pairs were produced in the decay of ^{2}He, formed in the ^{1}H(d, ^{2}He)n reaction. The ^{2}He denotes a pair of protons with small relative kinetic energies for which the quantum mechanical description is of an entangled spin-singlet state $((|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)/\sqrt{2})$ by the strong final state interaction.

The schematic arrangement of the experimental setup is shown in Fig. 1. It consists of three components in the following sequence: (a) the magnetic spectrometer SMART [21] for the momentum selection of protons; (b) the focal-plane detector system comprising of multiwire drift chamber (MWDC1) and scintillator hodoscope (HOD1) constitute the "event-ready" detector; (c) the polarimeter (EPOL [22]) of a spin-analyzer target (5 cm thick graphite slab), two sets of plastic scintillation counter hodoscopes (HOD2 and HOD3), and two multiwire drift chambers (MWDC2 and MWDC3). The "event-ready" detector permits the selection of proton pairs produced by the ^{1}H(d, ^{2}He)n reaction. The overall detection efficiency for protons was determined to be >96%. Event triggers of resolving time faster than 1 ns (FWHM) were generated by light output signals from the scintillation counters and the event-by-event information was stored on data storage system for later analysis.

The data analyses were comprised of the following stages. The geometrical information of the proton tracks before the graphite slab was ion-optically transformed into the scattering angles and momenta of the two protons at the primary hydrogen target, which yielded the missing energy of the residual nucleus of the ^{1}H(d, ^{2}He)n reaction and the relative kinetic energy E_{rel} between two protons. Figure 2 shows the excitation energy of the proton pairs calculated from the missing mass spectrum. The sharp Gaussian peak at $E_x = 0$ MeV are the ^{2}He events. The peak width of 1.2 MeV FWHM is mainly due to the experimental resolution. Figure 2 also shows the relative kinetic energy (E_{rel}) of protons for $|E_x| < 1$ MeV. A lot of spectroscopic evidence (for example, see [23,24]) shows that such proton pairs with low relative kinetic energies are *s* wave ($L = 0$) of high purity. Thus for the final event selection, we require that the $E_{\mathrm{rel}} < 1$ MeV to be compared to about 7 MeV for the LRM measurement [19]. The contribution of the accidental coincidences to the data amounted to $8.19 \pm 0.26\%$ and it was corrected for.

The polarization analysis of protons was carried out by a focal-plane polarimeter EPOL [22]. The analyzer system is

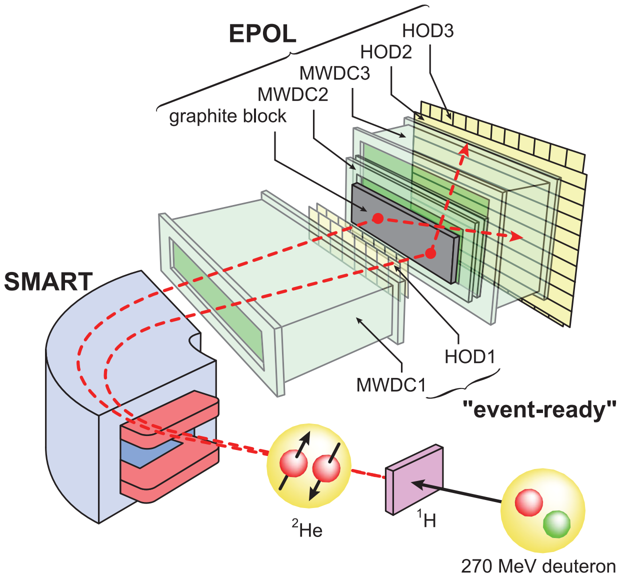

FIG. 1 (color). A schematic diagram of the experimental layout. Deuterons interact with protons in liquid hydrogen (^{1}H) target. The proton pairs (^{2}He) are momentum selected by SMART spectrometer, which are subsequently tracked by the "event-ready" detector (MWDC1 and HOD1). The spin analysis of the protons was then achieved by the EPOL polarimeter, where graphite block is the analyzer and the MWDC2, MWDC3, HOD2, and HOD3 are correlation detectors.

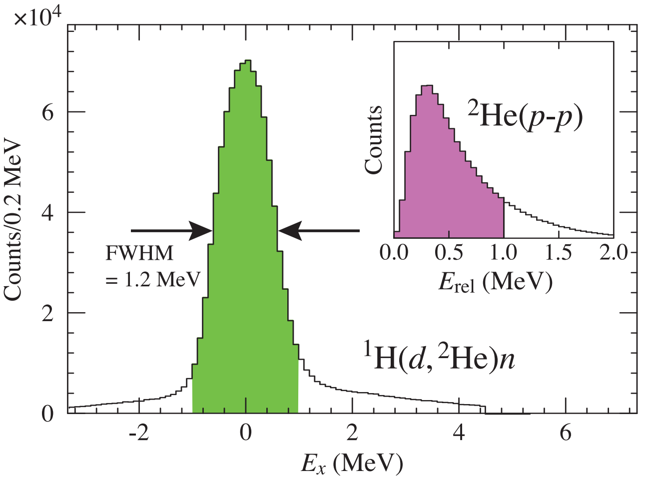

FIG. 2 (color). The missing energy of two protons (E_{rel}) in the ^{1}H(d, ^{2}He) reaction. Events in the green shaded area ($|E_x(\mathrm{MeV})| < 1$ MeV) are selected for further analyses. The inset shows the relative kinetic energies (E_{rel}) of the events in green shaded area. Proton pairs $E_{\mathrm{rel}} < 1$ MeV (pink histogram) are finally selected for correlation analyses. Suppression of the events at high measured E_{rel} is mainly due to the finite momentum and angular acceptances of SMART.

also of detection efficiency >96%. The effective analyzing powers were measured in a separate experiment by injecting a polarized proton beam into EPOL and measuring the directional asymmetries due to nuclear elastic scattering at the graphite slab. The analyzing power A is deduced from the definition $A \times P = [N_L - N_R]/[N_L + N_R]$, where P is the degree of polarization of protons, and N_L and N_R are the number of protons scattered to the left and the right, respectively. For proton kinetic energies of our interest ($T_p = 128$–140 MeV), it was measured to vary between $A = 0.16$–0.23, with overall uncertainties of 1%–2%. The protons transmitted by the analyzer without scattering were also recorded. It should be emphasized that the entire history of each proton (for more than 90% of them, to be precise) entering the polarimeter has been recorded, but only the events in which both protons undergo scattering in the analyzer target are used for further analyses. Thus, the selected subensemble of ^{2}He events constitutes the unbiased sample of proton pairs.

To explain the principle of the spin-correlation measurement, we shall consider a schematic representation shown in Fig. 3. It consists of a pair of polarimeters, each made up of an analyzer target and a pair of detectors L and R. Thus each polarimeter is sensitive to the spin component of the incident protons along the direction of $\vec{n}^{(i)}$ ($i = 1, 2$). We define the spin-correlation function, $C(\vec{n}^{(1)}, \vec{n}^{(2)})$, as the expectation value of the product of the signs of two proton spins with respect to the axes $\vec{n}^{(1)}$ and $\vec{n}^{(2)}$. In this setup, the polarimeter 1 is oriented at an angle Φ with respect to the polarimeter 2. If the incident protons are in the singlet state, then the quantum mechanical prediction is given by $C_{\mathrm{QM}}(\vec{n}^{(1)}, \vec{n}^{(2)}) \equiv \langle \vec{\sigma}^{(1)} \cdot \vec{n}^{(1)} \sigma^{(2)} \cdot \vec{n}^{(2)} \rangle = -\cos\Phi$. Experimentally, $C(\vec{n}^{(1)}, \vec{n}^{(2)})$ ($\equiv C(\Phi)$) is given by [19]

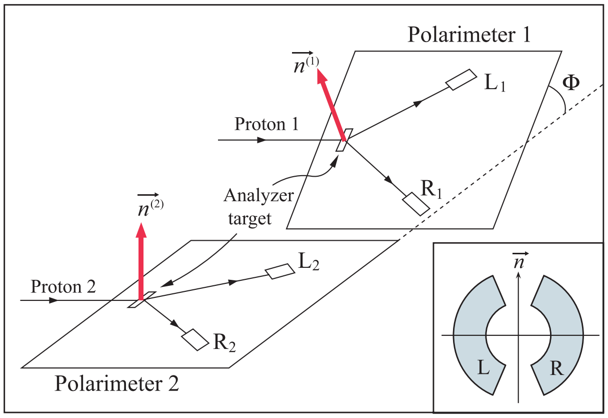

FIG. 3 (color). A conceptual view of the spin-correlation measurements by EPOL (Fig. 1). Correlated proton pairs are scattered by a pair of spin-analyzer targets (graphite block). In each polarimeter, scattered protons are detected with detector L or R. $\vec{n}^{(i)}$ ($i = 1, 2$) denotes the normal of the scattering plane of each polarimeter. The role of the left-right detectors for both protons is served by the MWDC2, MWDC3 and HOD2, HOD3 assembly shown in Fig. 1. The inset shows virtual sectors of angular ranges $8° \le \theta \le 20°$ and $\Delta\phi = 134°$, used in the polarization analyses. The angles are with respect to reference axes $\vec{n}^{(i)}$ ($i = 1, 2$).

$$C_{\mathrm{exp}}(\Phi) = \frac{1}{A^{(1)}A^{(2)}} \frac{N_{L_1L_2} + N_{R_1R_2} - N_{L_1R_2} - N_{R_1L_2}}{N_{L_1L_2} + N_{R_1R_2} + N_{L_1R_2} + N_{R_1L_2}}, \tag{1}$$

where $N_{L_1R_2}$ is the number of coincidences between detector L_1 and R_2 and so forth.

Bell-CHSH [3] states that local realistic theories satisfy the following inequality for arbitrary directions $\vec{a}$, $\vec{b}$, $\vec{a}'$, and $\vec{b}'$:

$$|C(\vec{a}, \vec{b}) - C(\vec{a}, \vec{b}')| + |C(\vec{a}', \vec{b}) + C(\vec{a}', \vec{b}')| \le 2. \tag{2}$$

This inequality is maximally violated by quantum mechanics as

$$S_{\mathrm{QM}}(\pi/4) = |C_{\mathrm{QM}}(\pi/4) - C_{\mathrm{QM}}(3\pi/4)| + |C_{\mathrm{QM}}(-\pi/4) + C_{\mathrm{QM}}(\pi/4)| = 2\sqrt{2}. \tag{3}$$

In our analyses of EPOL data to derive the $C_{\mathrm{exp}}(\Phi)$, two pairs of virtual sectors (L_1, R_1) and (L_2, R_2) were defined on the scattering angle plane for protons 1 and 2, respectively. The spin-reference axis $\vec{n}^{(i)}$ corresponds to the symmetry axis for the pair of sectors (L_i, R_i). These four sectors play the roles of the four detectors shown in Fig. 3. Once the sectors are properly defined, $C_{\mathrm{exp}}(\Phi)$ can be calculated from Eq. (1).

The full 2π sr acceptance of the polarimeter allows us to make a "post selection" of reference axes in an arbitrary manner. We *randomly rotate* $\vec{n}^{(1)}$ and $\vec{n}^{(2)}$ while maintaining the event-by-event relationship and relative angle Φ as a constant. The advantage of this method is that it dilutes the spin correlations of rotationally noninvariant unentangled pairs. If the two spin-up-down protons are due to a mixed ensemble of singlet state and other configurations, the correlations are weaker. As an example, for an unentangled $|\uparrow\downarrow\rangle$ configuration, $|C_{\mathrm{QM}}(\Phi)| \le 1/2$. Thus, a measured value of $C_{\mathrm{exp}}(\Phi) = -\cos\Phi$, which is same as $C_{\mathrm{QM}}(\Phi)$, for $\vec{n}^{(1)}$ and $\vec{n}^{(2)}$ randomly rotated, is strong evidence that the incident two protons are in the entangled state. We evaluated the $C_{\mathrm{exp}}(\Phi)$ for various values of Φ between $-\pi \le \Phi \le \pi$ from the same data set, by software selection during the offline analyses. To compare the results with Bell's inequality, we calculated a correlation function of the CHSH type: $S_{\mathrm{exp}}(\Phi) = 2C_{\mathrm{exp}}(\Phi) + C_{\mathrm{exp}}(-\Phi) - C_{\mathrm{exp}}(3\Phi)$. Bell's inequality in Eq. (2) yields $|S(\Phi)| \le 2$.

Figure 4 shows the results of $S_{\mathrm{exp}}(\Phi)$ plotted for $\Delta\Phi = 1°$ intervals. For any arbitrary choice of Φ they are very close to quantum mechanical prediction $S_{\mathrm{QM}}(\Phi) = 3\cos\Phi - \cos3\Phi$. The error band (blue shaded area) includes the statistical and systematic errors. The systematic error was mainly caused by the uncertainty in determining

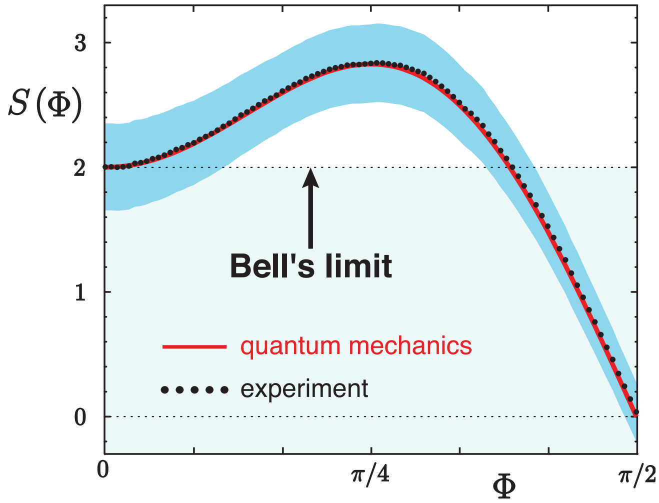

FIG. 4 (color). Plot of the spin-correlation function $S(\Phi)$ vs Φ. Solid circles are the experimental result $S_{\text{exp}}(\Phi)$ derived from the same data set. Each error shown as blue shaded area is correlated. See text for details.

the effective analyzing powers of EPOL. Since adjacent data points use overlapping data, they are not statistically independent. Hence, for a quantitative test against theory one single choice of Φ should be used.

It is very impressive that the experimental data follow the quantum mechanical prediction faithfully for the entire angular region and they exceed Bell's limit over a wide angular range. We obtained $S_{\text{exp}}(\pi/4) = 2.83 \pm 0.24_{\text{stat}} \pm 0.07_{\text{sys}}$, which violates the inequality at a confidence level of 99.3%. It is to be noted that the entanglement of the spin-singlet state was retained even when the two protons traversed through large amounts of material media (50 cm thick argon + ethane gas in wire chambers, 1 cm thick plastic scintillators, and up to 5 cm thick graphite slab). It is indeed remarkable that the strongly interacting pairs maintain the correlations for distances greater than 10^{13}–10^{14} times their coherence length, which was estimated to be 10^{-14} m (the size of a wave packet of two protons at production).

Can some hidden variable theories reproduce quantum mechanical-like correlations in our arrangement? There are two concerns: (a) locality and (b) small analyzing powers. First, the finite distance of 5–40 cm between the two protons, though much larger than the short range strong interactions, does not rule out communications as they pass through the polarimeter. However, the two interacting protons of an event do not know the experimenter's choice of the reference axes since we have "post selection". If the protons make their own choice to conspire to mimic a theory, they need to communicate (1) spin orientation with respect to an axis of their choice, (2) scattering angle (ϕ_1 or ϕ_2), and (3) energy-dependent analyzing powers, which render such a scenario unlikely, if not impossible.

The second concern is the small analyzing power, which might dilute the correlations. Though it is reassuring that the experimental correlations deduced from Eq. (1) are in agreement with quantum mechanics for all choices of reference axes, we are not able to rule out some remote possibility that analyzing powers and hidden variables conspire to yield similar agreement.

In conclusion, we have shown, from an experiment of high statistics involving strongly interacting hadron pairs with nuclear spin interactions and employing "event-ready detector" and "post selection of reference axes", that the result $S_{\text{exp}}(\pi/4) = 2.83 \pm 0.24_{\text{stat}} \pm 0.07_{\text{sys}}$ violates Bell's CHSH inequality at 99.3% confidence level.

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