Lagrangian Measurements of Inertial Particle Accelerations in Grid Generated Wind Tunnel Turbulence

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We describe Lagrangian measurements of water droplets in grid generated wind tunnel turbulence at a Taylor Reynolds number of $R_\lambda = 250$ and an average Stokes number ($\langle \mathrm{St} \rangle$) of approximately 0.1. The inertial particles are tracked by a high speed camera moving along the side of the tunnel at the mean flow speed. The standardized acceleration probability density functions of the particles have spread exponential tails that are narrower than those of a fluid particles ($\mathrm{St} \approx 0$) and there is a decrease in the acceleration variance with increasing Stokes number. A simple vortex model shows that the inertial particles selectively sample the fluid field and are less likely to experience regions of the fluid undergoing the largest accelerations. Recent direct numerical simulations compare favorably with these first measurements of Lagrangian statistics of inertial particles in highly turbulent flows.

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The bulk of our empirical knowledge of fluid turbulence stems from measurements of the fluid motion as it passes a fixed probe. These ''Eulerian'' measurements have provided detailed knowledge of the turbulence velocity spectrum and its probability density function (PDF) and have shown that turbulence exhibits internal intermittency: temporal and spatial variation in the energy dissipation rate at the small scales [1]. Intermittency affects mixing rates of passive scalars such as temperature or humidity inhomogeneities [2] as well as the trajectories of small particles in the fluid field [3]. While Eulerian measurements provide insight into the velocity field, $\underline{u}(\underline{x}, t)$ they do not directly yield information on the acceleration $\underline{a}(\underline{x}, t)$, which is the sum of the temporal and spatial variation of the velocity: $\underline{a} = \partial \underline{u}/\partial t + (\underline{u} \cdot \Delta)\underline{u}$. By following the motion of particles in fluid, be they particles having the same density as the surroundings (fluid particles) or particles whose density is greater than that of the fluid (inertial particles), $\underline{a}$ can be directly measured, and thus the forces on an advected particle can be determined. Measurements of particle trajectories in this so-called Lagrangian framework are more difficult than their Eulerian counterpart because of the extremely rapid variations of $\underline{a}$ at the smallest scales [3], but recently there have been major technical advances [3–6] in the measurement of fluid particles with concomitant developments in simulations [7–11]. The experiments and simulations show that intermittency is most strongly manifest in the particle acceleration statistics [3,4,6]. Their PDF's show extremely broad (stretched) exponential tails, indicating rare events that occur at the small scales [1,3,7]. How particles with significant inertia, such as water drops in clouds (where the particle-fluid density ratio is order 1000), or fuel drops in combustors, respond to the intense intermittency is less well understood, and is the subject of this Letter.

Inertial particles are expected to have trajectories different from those of fluid particles in the same flow. For example, they are ejected from regions of high vorticity due to centrifugal forces, and accumulate in regions of high strain [12–17]. Numerical simulations of inertial particles show dramatic increases in the particle collision rate as a result of clustering [8–10]. Recent evidence suggests this increase may occur for water droplets in clouds [18–20], an effect presently neglected in most cloud models that leads to an overestimate in the time required for rain initiation [18,21]. Particle accelerations play a crucial role in clouds by enhancing the collision rate of droplets of differing sizes [13]. Experiments are needed to better understand how droplet accelerations differ from fluid accelerations and how they depend upon the particle and turbulence characteristics, including the particle distribution.

In this Letter we present the first Lagrangian measurements of the acceleration statistics of inertial particles. Grid generated wind tunnel turbulence is seeded with water droplets. The particle loading is low so that turbulence modulation and particle-particle interactions can be neglected. Their size is less than the smallest scale of the turbulence, the Kolmogorov scale (η), and thus they do not affect the fluid motion. Gravitational forces are determined to be insignificant compared to the forces due to the fluid motion. We show that the tails of the normalized acceleration PDF become systematically less stretched due to selective sampling of the fluid by the inertial particles as the inertial effects increase, and we compare our results to recent numerical estimates.

Our experiments were conducted in a large (1 m $\times$ 0.9 m $\times$ 20 m) open circuit wind tunnel with an active grid (triangular agitator wings attached to the rotating grid bars, randomly flipping) at the beginning of the test section (Fig. 1, [22]) to produce turbulence in the range $100 \leq R_\lambda \leq 1000$. The water spray consists of an array of four nozzles symmetrically placed downstream of the grid. The particle drop size distribution was measured using a

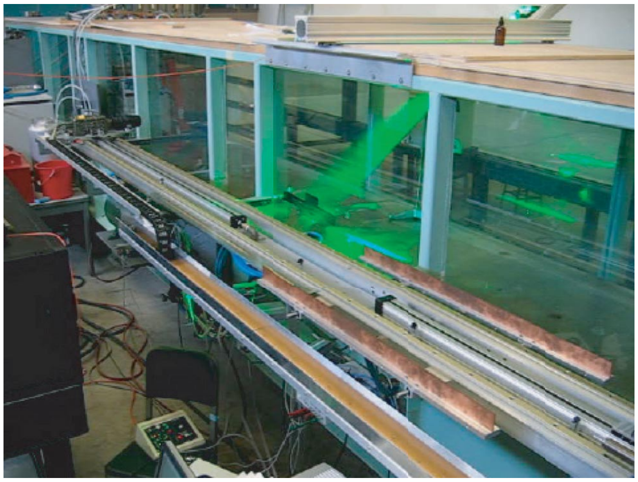

FIG. 1 (color). The wind tunnel showing the camera (far left, at the beginning of its trajectory), the sled, and the laser sheet. The active grid and spray system are at the tunnel entrance (just above the camera lens). The copper strips (right foreground) are the magnetic braking system for the camera sled.

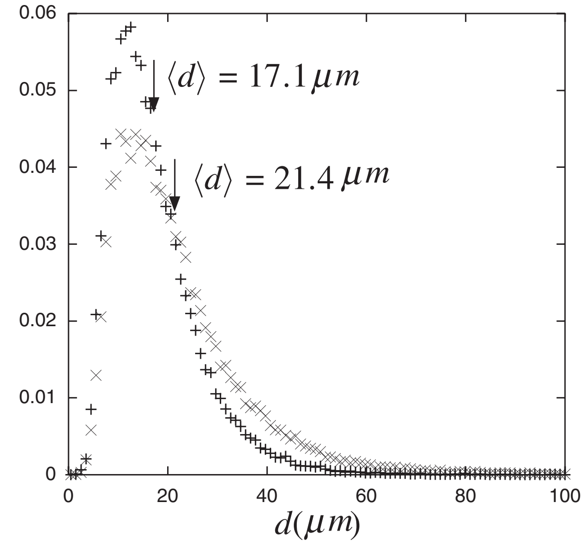

FIG. 2. Particle size distributions at the measurement station. The vertical arrows are the mean diameters calculated from the distributions. The resulting mean Stokes numbers (see text) for the two cases are 0.09 ± 0.03 (×) and 0.15 ± 0.04 (+).

phase doppler particle sizer (Fig. 2). The particle mass loading was approximately 10^{-4} kg water/kg dry air. A high speed camera (Phantom v7.1) attached to a precision linear motion pneumatic driven sled was accelerated to the mean flow speed and 2D particle tracks were measured at region 30 mesh lengths ($M = 11.4$ cm) downstream from the grid and 20.3 cm from the tunnel wall [23]. The camera frame rate was 8000 fps with a resolution of 512 pixels × 512 pixels. The laser light sheet (Nd-YAG, 20 W, pulse width 120 ns at a repetition rate of 40 kHz) was projected from the top of the tunnel such that the camera received light forward scattered at an angle of 30°. The width of the sheet was approximately 2 mm (Fig. 1). The sampling area was 1.9×1.9 cm^2, and the intersample time was $(1/100)$ τ_η, where τ_η is the Kolmogorov time scale $[(\nu/\epsilon)^{1/2}$, where ϵ is the turbulence kinetic energy dissipation rate [24], and ν is the kinematic viscosity], and the spatial resolution was $(1/12)\ \eta$. The camera tracked the particles over a distance of 40 cm (0.2 s) as they moved across the light sheet. Approximately 15 000 data points were taken per sled run, and 400 runs were completed to provide 6×10^6 data points per set. Data analysis followed the approach developed by the Bodenschatz group [4].

The particle inertial effects are described by the Stokes number $\mathrm{St} \equiv \tau_p/\tau_\eta$, where τ_p is the time scale defined as $(1/18)[\rho_p/\rho_f]d_2/\nu$, where ρ_p, ρ_f, d, and ν are the particle density (998 kg/m^3), fluid density (1.23 kg/m^3), particle diameter, and the fluid viscosity (1.5×10^{-5} m^2/s). Thus St is the ratio of the particle inertial response time to the time scale of the smallest eddies. It is these eddies that have the most intense accelerations [1] and thus they will have the strongest effect on the motion of the inertial particles. The apparatus is designed for $0 \leq \mathrm{St} \leq 10$. Here we provide measurements for $\mathrm{St} \sim 0.1$ and $R_\lambda =$ 250. Theory [12] and numerical experiments [9,11,14] show that significant departures from fluid particle behavior, including the preferential concentration of inertial particles, occur in the range $0.1 < \mathrm{St} < 1$.

Figure 3 shows the Eulerian velocity spectrum at the measurement location measured using hot-wire anemometers. These hot-wire measurements enable accurate determination of dissipation rates and associated flow quantities (Table I). A well-developed scaling range (inertial subrange) is observed. There is some large scale anisotropy (Table I). However, detailed studies [22] indicate isotropy in the inertial and dissipation ranges for this flow. The Eulerian fluid velocity PDFs are close to Gaussian [22].

Figure 4 shows the normalized acceleration PDFs for $\langle \mathrm{St} \rangle \sim 0.09 \pm 0.03$ and 0.15 ± 0.04. They exhibit a stretched exponential form but are narrower than that of a fluid particle. (By contrast, the PDFs of the inertial particle velocities (not shown) were found to be close to Gaussian. This is expected since they are determined by the large scales.) The mean Stokes number, defined as $\langle \mathrm{St} \rangle \equiv (1/18\nu\tau_\eta)[\rho_p/\rho_f]\langle d^2 \rangle$, was determined from the second moment of the drop size distribution. [Note that for this flow $(1/18\nu\tau_\eta)[\rho_p/\rho_f]$ is a constant.] There is a small decrease in the width of the tails at the higher Stokes number. The un-normalized PDFs (not shown) indicate that the variance also decreases for the higher Stokes number case by a factor of 0.8. Also plotted is the fluid particle acceleration PDF from the work of the Bodenschatz group [6]. The inertial particle PDFs of normalized accelerations are substantially below the fluid PDFs for the normalized accelerations beyond ± 4.0 (see also Fig. 5). We note that the passive particles data were measured at $R_\lambda = 690$. The Voth *et al.* [4] experiment

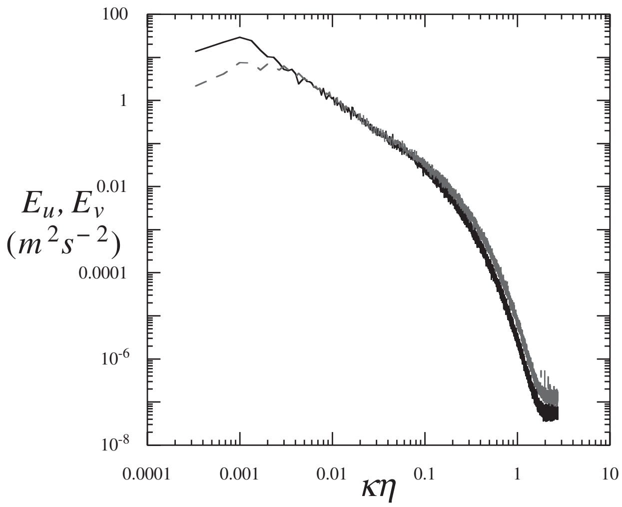

FIG. 3. Eulerian wave number velocity spectra, determined from hot-wire measurements, of the fluctuating longitudinal velocity, $E_u(\kappa\eta)$ in solid black, and transverse velocity, $E_v(\kappa\eta)$ in dashed gray.

(Fig. 19) shows a weak dependence on R_λ over the range 200–690 but the tails are always much broader than those of the inertial particles reported here.

In order to explain the changes in the inertial particle accelerations, two simplified simulations were performed. In the first, we subjected inertial particles to a fluid velocity obtained from a stochastic model of the Lagrangian fluid velocity [25–27]. The particle velocity was obtained by solving the equation of motion for the particle assuming a Stokes drag force. For the Sawford model we observed a systematic decrease in the variance of particle accelerations with increasing particle Stokes number, in accord with our experimental measurement; however, the shape of the PDF for the inertial particle remained unchanged (i.e., there was no reduction in the tails of the distribution with increasing inertial effects). The second simulation consisted of a two-dimensional array of potential-flow vortices. The strength of the vortices was set randomly and was smoothly varied in time. A number of trajectories for the inertial particle were computed based on Stokes drag and the fluid velocity field. In this case, the inertial particle acceleration variance decreased with increasing St *and* the PDF of the normalized particle inertial acceleration showed narrower stretched exponential tails compared to those of a fluid particle. We conclude from these studies that the change in the variance of the acceleration for inertial particles results from linear damping of the fluid acceleration by inertia, whereas the change in the shape of the PDF is a result of biased sampling of the underlying fluid flow due to inertia, although these two effects are not entirely decoupled. This result is consistent with recent numerical simulations that show a strong correlation between regions of a turbulent flow where particles accumulate and zero-acceleration points of the fluid [28].

In Fig. 5, we compare the present measurements to recent numerics of Bec *et al.* [11]. Our results are in good agreement: both the numerics and experiment show that the inertial particle PDF's fall below the fluid particle PDF at $\sim 4\langle a^2\rangle^{1/2}$. We note that the numerics are for monodispersed particles, while ours are polydispersed particles (Fig. 2) [29].

In summary, we have provided the first results of acceleration statistics for inertial particles in moderately high Reynolds number wind tunnel turbulence for St ~ 0.1. Our results show that the tails of the acceleration PDF decrease in width compared to those of fluid particles, and this is consistent with simple modeling which indicates selective sampling of the fluid field by the inertial particles. Finally, preliminary measurements of inertial particles in the same tunnel [31] exhibit pronounced spatial clustering at the small dissipation scale revealing another manifestation of

TABLE I. Flow Parameters.

Mean velocity, U (ms^{-1})	1.89
$R_\lambda \equiv \langle u^2\rangle^{1/2}\lambda/\nu$	250
rms longitudinal velocity, $\langle u^2\rangle^{1/2}$ (ms^{-1})	0.28
rms transverse velocity, $\langle v^2\rangle^{1/2}$ (ms^{-1})	0.22
Taylor scale, $\lambda \equiv [U^2\langle u^2\rangle/\langle(\partial u/\partial t)^2\rangle]^{1/2}$ (m)	1.35×10^{-2}
Dissipation rate, ε (m^2 s^{-3})	0.096
Integral length scale, $l \equiv \langle u^2\rangle^{3/2}/\varepsilon$ (m)	0.22
Kolmogorov length scale, $\eta \equiv (\nu^3/\varepsilon)^{1/4}$ (m)	4.33×10^{-4}
Kolmogorov time scale, $\tau_\eta \equiv (\nu/\varepsilon)^{1/2}$ (s)	1.25×10^{-2}
Stokes Number, St	0.09 ± 0.03; 0.15 ± 0.04
Normalized Reynolds stress, $\frac{\langle uv\rangle}{\langle u^2\rangle^{1/2}\langle v^2\rangle^{1/2}}$	0.004

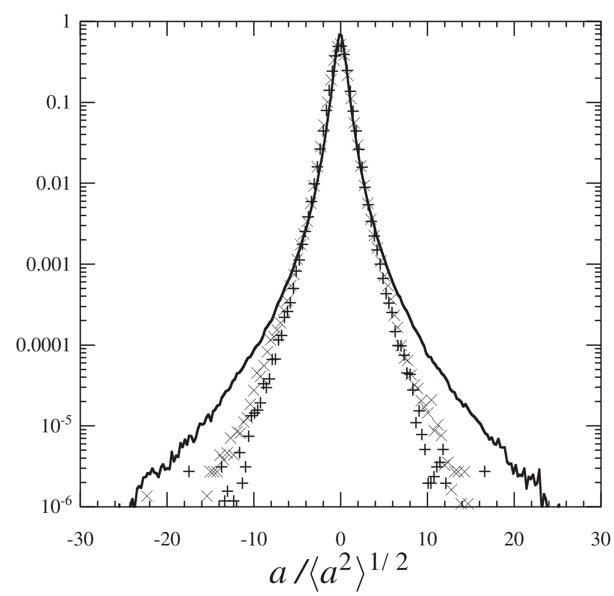

FIG. 4. The normalized PDF of the longitudinal component of the acceleration of the inertial particles ($\langle$St$\rangle = 0.09 \pm 0.03$, $\times$; $\langle$St$\rangle = 0.15 \pm 0.04$, +) compared with those of particles (St ≈ 0) measured by Mordant *et al.* [6] (solid line). The PDF's are of the accleration normalized by the rms, $\langle a^2\rangle^{1/2}$.

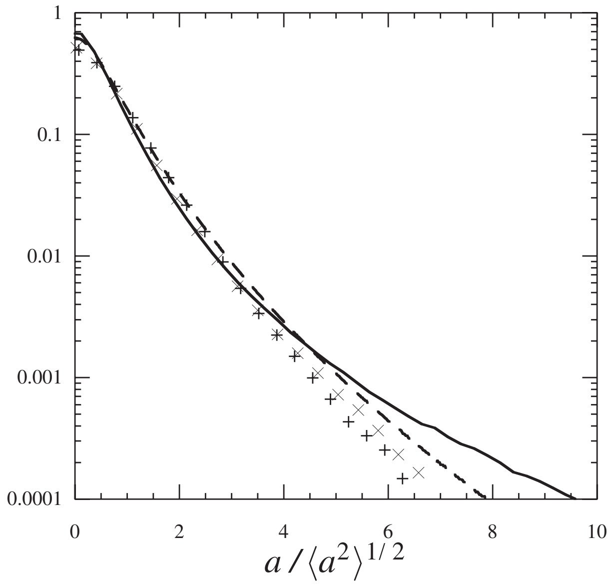

FIG. 5. Same normalized inertial particle acceleration PDFs as Fig. 4 (× and + symbols) compared with recent numerical simulations [11] at $R_\lambda = 185$, St = 0.16 (dashed line). Also shown is the Mordant *et al.* [6] acceleration PDF for fluid particles (solid line).

the intermittent structure of turbulence and its effect on inertial particles.

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