Universal Quantum Computation in Decoherence-Free Subspace with Neutral Atoms

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We show how realistic cavity-assisted interaction between neutral atoms and coherent optical pulses, and measurement techniques, combined with optical transportation of atoms, allow for a universal set of quantum gates acting on decoherence-free subspace in a deterministic way. The logical qubits are immunized to the dominant source of decoherece—dephasing, while the influences of additional errors are shown by numerical simulations. We analyze the performance and stability of all required operations and emphasize that all techniques are feasible with current experimental technology.

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Introduction.—Manipulation of atoms in microscopic traps is one of the major highlights of the extraordinary progress experienced by atomic, molecular, and optical (AMO) physics over the past few years, and has led to important successes in the implementation of quantum information processing [1]. Hence, several implementations of neutral atoms quantum computing, exploiting various trapping methods and entangling interactions, have been proposed [2–6]. Nevertheless, the experimental requirements with these approaches turn out to be very challenging, such as a large number of atoms, each of which is strongly coupled with cavity mode, individually addressing, and localization to the Lamb-Dicke limit.

A quantum memory stores information in superposition states, but interactions between the quantum memory and its environments destroy the stored information, so-called decoherence. Decoherence-free subspaces (DFSs) have been proposed [7] to protect fragile quantum information against the detrimental effects of decoherence. There have been a lot of theoretical researches for achieving fault tolerant universal quantum computation in DFSs [8]. Also significant experimental efforts have been made for realization of such a decoherent-free quantum memory in different physical systems [9–11].

In this Letter, we present a scheme to realize a universal set of quantum gates in a deterministic way, acting on neutral atoms through cavity-assisted interaction of coherent optical pulses in DFS, which from the beginning immunizes our logical qubits against the dominant source of decoherence—collective dephasing. Our idea is at least twofold. First, we implement computation using specific physical mechanisms that allow for gates in the encoded space without any overhead associated with encoded gates. Second, in our construction the system never leaves the DFS during the entire execution of gates, so that fault tolerance is natural and, in stark contrast to the usual situation in quantum error correction, necessitates no extra resources during the computation.

Neutral atoms in our scheme are stored in transverse optical lattices and translated into and outside of the cavity [12] for gate operations to obviate the requirement for individual addressing, each of which has three relevant levels as shown in Fig. 1. Atomic states $|0\rangle$ and $|1\rangle$ are two stable ground states. The atomic transition from $|0\rangle$ to excited level $|e\rangle$ is resonantly coupled to a cavity mode a_c. The state $|1\rangle$ is decoupled due to a large hyperfine splitting. The coherent time of the superposition of the internal atomic states τ_{co} is with a magnitude of milliseconds [13]. There are two dominant sources of decoherence: (i) photon loss during gate operations and (ii) dephasing during the storage and transmission of the atoms in the optical lattices. We will later show that in realistic setting gate errors due to photon loss are characterized by the detailed numerical simulation which demonstrates the practicality of this scheme within the reach of the current experimental technology. Furthermore, unlike single-photon detection , since a homodyne detection of the coherent state directly measures the relative phase of the signal state, the photon losses only decrease the signal-to-noise ratio but do not lead to a failure in the measurements. Also, we describe a specific encoding that allows suppression of the error of type (ii) by considering a DFS by the

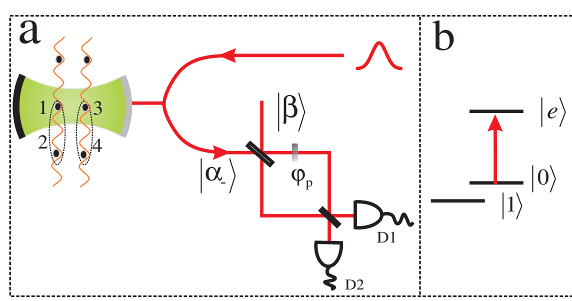

FIG. 1 (color online). (a) Schematic setup for implementation of the logical CZ gate on two logical atomic qubits in DFS through the cavity-assisted interaction. In order to verify the projection, the scattering coherent optical pulses leaking out are detected by the homodyne detectors after reflection. Here $|\beta\rangle$ is the state of local oscillator. (b) The relevant level structure of the atom and the coupling configuration.

states $|0_L\rangle = |01\rangle$ and $|1_L\rangle = |10\rangle$, which from the beginning immunizes our logical qubits against the dominant source of decoherence—dephasing provided by stray fields, random variation of the atom-cavity coupling rate, and the instability of the optical lattice. We denote the logical Bell states by $|\Phi_L^\pm\rangle = (|0_L 0_L\rangle \pm |1_L 1_L\rangle)/\sqrt{2} = (|0101\rangle \pm |1010\rangle)/\sqrt{2}$ and $|\Psi_L^\pm\rangle = (|0_L 1_L\rangle \pm |1_L 0_L\rangle)/\sqrt{2} = (|0110\rangle \pm |1001\rangle)/\sqrt{2}$, which take the full advantage of these properties, suppressing phase noise. The logical qubit decoheres only insofar as the dephasing fails to be collective.

Dynamical decoupling pulses and their application.—We briefly review the decoupling technique [14] as it pertains to our problem. Assume a phase noise term $\varepsilon(t)$ acts on the internal states of atoms, characterized by a power spectrum $S(\omega)$ of integrated power $(\tau_{\rm co})^2$ with a high frequency cutoff at $\omega_c \ll 1/\tau_{\rm co}$. The action of $\varepsilon(t)$ can be represented by a stochastic evolution operator $U_x(t) = e^{-i\int_0^t \varepsilon(t')dt'\sigma_x^L}$, where σ_x^L is a Pauli operator for the encoded subspace, which can be implemented simply by swapping the two qubits. The pulse sequence $[\Delta t, U_x, \Delta t, U_x]$ gives a reduced power spectrum $S_{\rm DFS}(\omega) \propto S(\omega)\sin^4(\Delta t\omega/2)/(\Delta t\omega)^2$, where Δt is free evolution time (cycle time). For frequencies below $1/\Delta t$, the bath-induced error rate is reduced by a factor proportional to $(\Delta t\omega)^2$.

The DFS also reduces phase errors during transport of atoms with a separation time τ_T. Replacing $\varepsilon(t)$ with $\varepsilon(x, t)$, we set $\langle\varepsilon(x, t)\varepsilon(x', t')\rangle = N(|x - x'|) \times \int_{-\infty}^{\infty} S(\omega)e^{i\omega(t-t')}d\omega$ for transport into or outside of the cavity, where $N(x) = e^{-x^2/d^2}$, $d = n\lambda/2$ is the distance between two atoms with n integer and λ is the wavelength of the counterpropagating laser used to form a 1D optical lattice. The resulting spectral function is $S_{\tau_T}(\omega) = \int_{-\infty}^{\infty} S(\omega - \nu)\sin^2[(\omega - \nu)\tau_T/2]\frac{e^{-(\tau_T/4)^2\nu^2/2}}{\sqrt{2\pi(4/\tau_T)^2}}d\nu$, which has a suppression of noise with frequencies $\ll 1/\tau_T$ by $(\tau_T\omega)^2/8$.

Basic tools.—For the logical gate operations, we should introduce some basic tools—physical controlled-Z (CZ) gate operation and projective measurements. To perform a collective CZ gate on two atoms [5], we reflect a weak coherent light pulse with the so-called odd coherent state from the cavity, which is resonant with the bare cavity mode and is given in this form as $|\alpha_-\rangle = N_-(|\alpha\rangle - |-\alpha\rangle)$, where N_- is a normalization constant and $|\alpha\rangle$ is a coherent state. Recently, this novel state of light has been generated and characterized by a nonpositive Wigner function experimentally [15]. For the case that both atoms are in the state $|1\rangle$, the coherent light performed in the limit with $T \gg 1/\kappa$ (here T is the pulse duration and κ is the cavity decay rate) is resonantly reflected by the bare cavity mode with a flipped global phase. For the three other cases, the effective frequency of the dressed cavity mode will be shifted due to the atom-cavity coupling, which is described by the Hamiltonian $H = \hbar\sum_{i=1,2} g_i(|e\rangle_i\langle 0|a + |0\rangle_i\langle e|a^\dagger)$. If the coupling rates satisfy $g_i \gg (1/T, \kappa, \gamma)$, where γ is the rate of spontaneous decay of the excited state, then the frequency shift will have a magnitude comparable with g_i, so that the incident single-photon pulse will be reflected by an off-resonant cavity. Hence, both the shape and global phase will remain unchanged for the reflected pulse. The net effect of these two subprocesses is that the reflection of a single-photon pulse from the cavity actually performs a CZ operation $U_{\rm CZ} = \exp(i\pi|11\rangle\langle 11|)$ on the two atoms while leaving the photon state unchanged.

If the input optical pulse is prepared in a weak coherent state $|\alpha\rangle$, which is reflected following the above analysis from atom-cavity system, then the projection is obtained after the homodyne detection of the states of the coherent light as the form

$$P_1 = |11\rangle\langle 11|; \qquad P_2 = I - P_1. \tag{1}$$

Now we show that by making a little change to the realistic setting one obtains another projection. First, the weak coherent optical pulse enters the cavity with only atom 1 inside. After the interaction between atom and cavity mode, an operation $\exp(i\pi|1, \alpha\rangle\langle 1, \alpha|)$ is applied on atom and the optical pulse. Atom 2 now is moved into the cavity while 1 outside, and the pulse is reflected successively to enter the cavity again, so that the same operation is applied on atom 2 and the pulse. After detection, we obtain

$$P_3 = |00\rangle\langle 00| + |11\rangle\langle 11|; \qquad P_4 = I - P_3. \tag{2}$$

Logical single-qubit operations.—The (physical) single-qubit rotation $R_z(\alpha) = \exp(-i\alpha\sigma_z)$, which can be implemented by rf pulses or the Raman transition applied on atom 1, has already provided arbitrary logical z rotation, $U_z(\alpha)$, i.e., $U_z(\alpha)|0_L\rangle = e^{-i\alpha}|0_L\rangle$ and $U_z(\alpha)|1_L\rangle = e^{i\alpha}|1_L\rangle$.

Then we show another important logical single-qubit gate—Hadamard gate. Consider a system A including atoms 1 and 2, on which we want to apply a Hadamard operation and obtain the outcome state on an ancilla system B including atoms 3 and 4 prepared in the state $|+_L\rangle$ initially, where $|\pm_L\rangle = (|0_L\rangle \pm |1_L\rangle)/\sqrt{2}$. We perform a physical CZ gate on atoms 1 and 3, and measure system A in logical x basis $\{|+_L\rangle, |-_L\rangle\}$. If the outcome $|-_L\rangle$ is obtained, we apply $\sigma_x \otimes \sigma_x$ on system B, or else we do nothing. Then $H_L = (|0_L\rangle\langle 0_L| + |0_L\rangle\langle 1_L| + |1_L\rangle\langle 0_L| - |1_L\rangle\langle 1_L|)/\sqrt{2}$ is obtained.

Hence, an arbitrary logical single-qubit rotation can be implemented with a sequence of Hadamard operations and z rotations $U = U_z(\alpha)H_L U_z(\beta)H_L U_z(\varsigma)$.

Logical single-qubit measurements.—We can realize logical single-qubit Z measurement of the observable σ_z^L by the sequence of operations: first, one applies $\sigma_x \otimes I$ and then the measurement $\{P_1, P_2\}$, followed by $\sigma_x \otimes \sigma_x$, $\{P_1, P_2\}$ again, finally, $I \otimes \sigma_x$. The measurement outcome

(π_1, π_2) with π_i being the outcome associated with P_i, corresponds—in the logical subspace—to $P^L_{z,+} = |0_L\rangle \times \langle 0_L|$; while one obtains $P^L_{z,-} = |1_L\rangle\langle 1_L|$ for the outcome (π_2, π_1). Measurements of arbitrary single-qubit observables can be realized by applying the corresponding basis change.

Logical Bell-state measurement (BSM).—Performing the measurement $\{P_3, P_4\}$ on atoms 1 and 3 belonging to two logical qubits, respectively, allows one to distinguish the subspace spanned by $\{|\Phi^+_L\rangle, |\Phi^-_L\rangle\}$ and $\{|\Psi^+_L\rangle, |\Psi^-_L\rangle\}$. The measurement outcomes π_3 and π_4 correspond to $P_{\{|\Phi^+_L\rangle,|\Phi^-_L\rangle\}} = |\Phi^+_L\rangle\langle\Phi^+_L| + |\Phi^-_L\rangle\langle\Phi^-_L|$ and $P_{\{|\Psi^+_L\rangle,|\Psi^-_L\rangle\}} = |\Psi^+_L\rangle\langle\Psi^+_L| + |\Psi^-_L\rangle\langle\Psi^-_L|$, respectively. More generally, one can obtain nondestructive projections onto subspaces spanned by two arbitrary Bell states using additional logical single-qubit unitary operations which allow one to permute Bell states. For instance, the application $H_L \otimes H_L$ consequently before and after the measurement $P_{\{|\Phi^+_L\rangle,|\Phi^-_L\rangle\}}$ corresponds to $P_{\{|\Phi^+_L\rangle,|\Psi^+_L\rangle\}}$. Obviously, using these nondestructive projections, we can achieve a full logical BSM.

Two-qubit gate.—A logical CZ gate described by $U^L_{\mathrm{CZ}} = \mathrm{diag}\,(1, 1, 1, -1)$ in the logical basis can be realized shown in Fig. 1(a) via atoms in a cavity by performing a physical CZ operation on atoms 1 and 3 belonging to two logical qubits, respectively.

Now we analyze the fidelity of the logical CZ gate under the influence of some practical sources of noise. For the initial state of the system $|\Psi_{\mathrm{in}}\rangle = \sum_{m,n=0,1}\epsilon_{mn}|m_L\rangle|n_L\rangle|\varphi\rangle_{\mathrm{in}}$, $|\varphi_{\mathrm{in}}\rangle \propto \{\exp[\alpha\int_0^T f_{\mathrm{in}}(t)a^\dagger_{\mathrm{in}}(t)dt] - \exp[-\alpha\int_0^T f_{\mathrm{in}}(t)a^\dagger_{\mathrm{in}}(t)dt]\}|\mathrm{vac}\rangle$ is the state of the input coherent optical pulse with a normalized shape function $f_{\mathrm{in}}(t)$, where $|\mathrm{vac}\rangle$ denotes the vacuum state and $a^\dagger_{\mathrm{in}}(t)$ is the one-dimensional optical field operator with the commutation relation $[a_{\mathrm{in}}(t), a^\dagger_{\mathrm{in}}(t')] = \delta(t - t')$. The cavity mode a_c is driven by the input field $a_{\mathrm{in}}(t)$ through the Langevin equation $\dot{a}_c = -i[a_c, H] - (\kappa/2)a_c - \sqrt{\kappa}a_{\mathrm{in}}(t)$. The output field $a_{\mathrm{out}}(t)$ of the cavity is connected with the input through the input-output relation $a_{\mathrm{out}}(t) = a_{\mathrm{in}}(t) + \sqrt{\kappa}a_c$. The output state of the whole system can be written as $|\Psi_{\mathrm{out}}\rangle = \sum_{m,n=0,1} e^{i\theta_{mn}}\epsilon'_{mn}|m_L\rangle|n_L\rangle|\varphi_{\mathrm{out}}\rangle_{mn}$, where the output state of the coherent light $|\varphi_{\mathrm{out}}\rangle_{mn}$ corresponds to the atomic component $|m_L\rangle|n_L\rangle$ with a shape $f^{\mathrm{out}}_{mn}(t)$ and amplitude α'_{mn}. In general, the amplitude α'_{mn} (for m, $n \neq 1$) is different from α because of the effect of the atomic spontaneous emission loss—the fundamental source of photon loss in cavity can be quantified by the photon loss parameter $\eta = 1 - |\alpha'|^2/|\alpha|^2 \propto \kappa\gamma/g_o^2$ through the numerical simulations. Ideally, the output state $|\Psi^{\mathrm{id}}_{\mathrm{out}}\rangle$ would have the unchanged amplitude α and shape functions $f^{\mathrm{out}}_{11}(t) = -f_{\mathrm{in}}(t)$ and $f^{\mathrm{out}}_{mn}(t) = f_{\mathrm{in}}(t)$ (for m, $n \neq 1$). Then the fidelity can be defined as $F \equiv |\langle\Psi^{\mathrm{id}}_{\mathrm{out}}|\Psi_{\mathrm{out}}\rangle|^2$, which decreases with the mean photon number $\langle n\rangle = |\alpha|^2$.

We investigate the fidelity under typical experimental configurations and it is shown in Fig. 2(a) as a function of the mean photon number of the input state for the realistic parameters $(g_o, \kappa, \gamma)/2\pi = (27, 2.4, 2.6)$ MHz [12]. We obtain a high fidelity up to 0.99 for these parameters and the coherent input pulse with a remarkable amplitude $\alpha = 1.26$. Furthermore, F is insensitive to the variation of the coupling rate caused by fluctuations in atomic position, and δF describing change of the fidelity is about 10^{-2} for g varying to $g/2$.

The above scheme can also be extended to perform logical controlled-NOT (CNOT) gate—in principle, between two logical qubits represented by remote atoms trapped in different cavities at arbitrary distance since the (physical) CZ gate can be implemented between two atoms belonging to different logical qubits in separated cavities [5,6].

The entangled state $|\Phi^+_L\rangle_{AB}$ is used to generate the logical four-qubit state that corresponds to the CNOT gate. We use notation A, A', B, B' to refer to different atoms, where A and A' referring to atoms trapped in cavity 1 belong to one party, while B referring to atoms trapped in cavity 1 and B' in cavity 2, belong to another separated party. We prepare two ancilla logical qubits A' and B' in the states $|+_L\rangle_{A'}$ and $|0_L\rangle_{B'}$; i.e., the initial state is $|\zeta\rangle = |+_L\rangle_{A'}|\Phi^+_L\rangle_{AB}|0_L\rangle_{B'}$. The following sequence of operations with indicated measurement outcomes generates the desired state: $P^{AA'}_{\{|\Phi^+_L\rangle,|\Phi^-_L\rangle\}}P^{BB'}_{\{|\Phi^+_L\rangle,|\Psi^+_L\rangle\}}|\zeta\rangle = (|0_L0_L\rangle_{AA'}|\Phi^+_L\rangle_{BB'} + |1_L1_L\rangle_{AA'}|\Psi^+_L\rangle_{BB'})/\sqrt{2} \equiv |\Xi\rangle$.

Given two additional logical qubits in an arbitrary state $\rho_{A''B''}$, where A'' and B'' refer atoms trapped in cavity 2 and 1, respectively, one can use the state $|\Xi\rangle$ together with logical BSM, to implement a logical CNOT operation on $\rho_{A''B''}$ and obtain the outcome state on system $A'B'$ following the procedure shown in [16]. This is achieved by measuring systems $A''A$ and $B''B$ in the logical Bell basis $|\psi_{i_1,i_2}\rangle = I \otimes \sigma^L_{i_1,i_2}|\Phi^+_L\rangle$, where $\sigma^L_{i_1,i_2}$ is one of Pauli operators. If the outcome for $A''A$ is $|\psi_{i_1,i_2}\rangle$, we apply $\sigma^L_{i_1,i_2}$ on A' and proceed analogously with $B''B$. One can readily see that the resulting operation on $A'B'$ after the procedure will be U_{CNOT} or U^+_{CNOT} with the same probability. Since

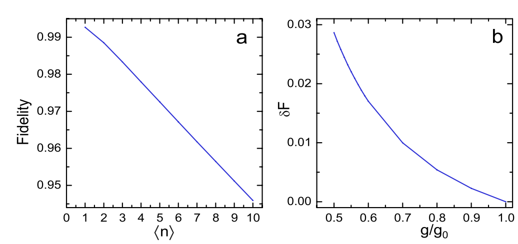

FIG. 2 (color online). (a) The fidelity of the logical CZ gate vs the mean photon number of the coherent optical pulse with the pulse duration $T = 200/\kappa$, and (b) it changes with g/g_o. We have assumed a Gaussian shape for the input pulse with $f(t) \propto \exp[-(t - T/2)^2/(T/5)^2]$. Here we choose the realistic parameters $(g_o, \kappa, \gamma)/2\pi = (27, 2.4, 2.6)$ MHz.

$U_{\mathrm{CNOT}} = U^{+}_{\mathrm{CNOT}}$, we obtain a deterministic implementation of logical CNOT gate, and then atoms A', B' are in the state $U_{\mathrm{CNOT}}\rho_{A''B''}U^{+}_{\mathrm{CNOT}}$.

Leakage error detection.—A method is presented to detect leakage errors, in which the state within the logical subspace $\{|0_L\rangle, |1_L\rangle\}$ is not altered. Consider a system A in some pure state $|\varphi\rangle = |\chi\rangle + |\chi^{\perp}\rangle$, where $|\chi\rangle$ is a state belonging to the logical subspace spanned by $\{|0_L\rangle, |1_L\rangle\}$, while $|\chi^{\perp}\rangle$ belongs to the orthogonal subspace $\{|2_L\rangle = |00\rangle, |3_L\rangle = |11\rangle\}$ and corresponds to leakage error. An ancilla system B is prepared in $|+_L\rangle$, and then the measurement $\{P_3, P_4\}$ is performed on atoms 2 and 4 and then on 1 and 4. If the same outcomes in two measurements are obtained, i.e., (π_3, π_3) and (π_4, π_4), that means the initial system was outside of the logical subspace. In these cases we conclude that a leakage error occurred. For the different outcomes (π_3, π_4) or (π_4, π_3), we perform a logical CNOT operation on systems A and B, then the state of system A is given by $|\chi\rangle$. Hence, this procedure always provides a conclusive leakage error detection.

Feasibility of the proposal.—No particularly demanding assumptions have been made for experimental parameters. The relevant cavity QED parameters for our system are assumed as $g_o^2/\kappa\gamma = 51 \gg 1$, placing our system well into the strongly coupled regime. The cavity consists of two 1-mm-diam mirrors with 10 cm radii of curvature separated by 75 μm [12] assuming the wavelength of the cavity mode is $\sim$780 nm (the rubidium $D2$ line). The distance between two atoms d in an optical lattice has a magnitude of 10 μm, which is larger than the waist $\sim$5 μm to leave only one atom inside the cavity and its neighbor atoms outside for the logical gate operations. The evolution of the states of two atoms is accomplished in the duration of the single-photon pulse $T \sim 200/\kappa = 13$ μs. The maximum velocity of the atoms in the transverse optical lattices is 30 cm/s and the maximum acceleration imparted is 1.5 g. Moving the proper atoms into and outside of the cavity is accomplished in the time τ_T of 100 μs. The gate preformation and transport of atoms can be accomplished within the coherent time (dephasing) of atoms with a magnitude of milliseconds [13,17]. Hence, our scheme fits well the status of current experimental technology.

Summary.—We have proposed a scheme for deterministic quantum gates acting on neutral atoms in DFS which from the beginning immunizes our logical qubits against the dominant source of decoherence—dephasing. The efficiency of this scheme is characterized through exact numerical simulations that incorporate various sources of experiment noise and these results demonstrate the practicality by way of current experimental technology. Some processes proposed here such as full BSM and unitary operations based on teleportation may also find applications in quantum communication and metrology.

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