Ellipsoidal Universe Can Solve the Cosmic Microwave Background Quadrupole Problem

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The recent 3 yr Wilkinson Microwave Anisotropy Probe data have confirmed the anomaly concerning the low quadrupole amplitude compared to the best-fit Λ−cold dark matter prediction. We show that by allowing the large-scale spatial geometry of our universe to be plane symmetric with eccentricity at decoupling or order 10^{-2}, the quadrupole amplitude can be drastically reduced without affecting higher multipoles of the angular power spectrum of the temperature anisotropy.

The latest results from the Wilkinson Microwave Anisotropy Probe [1] show that the cosmic microwave background (CMB) anisotropy data are in remarkable agreement with the simplest inflation model. At large scale, however, some anomalous features have been reported. The most important discrepancy resides in the low quadrupole moment, which signals an important suppression of power at large scales. Note, however, that the probability of the quadrupole being low is not statistically significant [1]. Nevertheless, if this discrepancy turns out to have a cosmological origin, then it could have far reaching consequences for our understanding of the universe and, in particular, for the standard inflationary picture. Indeed, it has been suggested that the low multipole anomalies in the CMB fluctuations may be a signal of a nontrivial cosmic topology [2−4]. For alternative solutions to the quadrupole problem see Ref. [5], while for other large-scale anomalies in the angular distribution see Ref. [6].

In this Letter we show that the power suppression at large scales can be accounted for if we relax the implicit assumption that the large-scale geometry is spherical. Indeed, if we assume that the large-scale spatial geometry of our universe is plane-symmetric with an eccentricity at decoupling of order 10^{-2}, then we find that the quadrupole amplitude can be drastically reduced with respect to the value of the best-fit Λ−cold dark matter (ΛCDM) standard model without affecting higher multipoles of the angular power spectrum of the temperature anisotropy. This result is generic regardless of the origin of eccentricity.

Let us begin by briefly discussing the standard analysis of the temperature anisotropy [7]. First, the temperature anisotropy is expanded in terms of spherical harmonics:

$$\frac{\Delta T(\theta, \phi)}{\langle T \rangle} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi). \tag{1}$$

After that, one introduces the power spectrum

$$\frac{\Delta T_l}{\langle T \rangle} = \sqrt{\frac{1}{2\pi} \frac{l(l+1)}{2l+1} \sum_m |a_{lm}|^2} \tag{2}$$

that fully determines all the properties of the CMB anisotropy. In particular, the quadrupole anisotropy refers to the multipole $l = 2$:

$$\mathcal{Q} \equiv \frac{\Delta T_2}{\langle T \rangle}, \tag{3}$$

where $\langle T \rangle \simeq 2.73$ K is the actual (average) temperature of the CMB radiation. The quadrupole problem resides in the fact that the observed quadrupole anisotropy is [1]

$$(\Delta T_2)_{\text{obs}}^2 \simeq 211 \ \mu\text{K}^2, \tag{4}$$

while the expected quadrupole anisotropy according the standard inflation is

$$(\Delta T_2)_I^2 \simeq 1252 \ \mu\text{K}^2. \tag{5}$$

If we assume that the large-scale spatial geometry of our universe is plane symmetric with a small eccentricity, then following Ref. [3] we have that the observed CMB anisotropy map is a linear superposition of two independent contributions

$$\Delta T = \Delta T_A + \Delta T_I, \tag{6}$$

where ΔT_A represents the temperature fluctuations due to the anisotropic space-time background, while ΔT_I is the standard isotropic fluctuation caused by the inflation-produced gravitational potential at the last scattering surface. As a consequence, we may write

$$a_{lm} = a_{lm}^A + a_{lm}^I. \tag{7}$$

In this Letter we will focus on the simpler case of plane-symmetric space-time background. The most general plane-symmetric line element [8] is

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2) - b^2(t)dz^2, \tag{8}$$

where we chose the xy plane as the plane of symmetry.

Here, the scale factors a and b are functions of the cosmic time t only. The most general energy-momentum tensor consistent with the planar symmetry is of the form

$$T^\mu{}_\nu = \text{diag}(\rho, -p_\parallel, -p_\parallel, -p_\perp), \qquad (9)$$

so that the Einstein equations read

$$\left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} = 8\pi G\rho, \qquad \frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} = -8\pi G p_\parallel,$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi G p_\perp, \qquad (10)$$

where a dot indicates the derivative with respect to the cosmic time. The total energy-momentum tensor $T^\mu{}_\nu$ can be made up of two different components: an anisotropic contribution, $(T_A)^\mu{}_\nu = \text{diag}(\rho^A, -p_\parallel^A, -p_\parallel^A, -p_\perp^A)$, which induces the planar symmetry, as, for example, a domain wall, a cosmic string, or a uniform magnetic field, and an isotropic contribution, $(T_I)^\mu{}_\nu = \text{diag}(\rho^I, -p^I, -p^I, -p^I)$, such as vacuum energy, radiation, matter, or cosmological constant. Exact solutions of the Einstein equations for different kinds of plane-symmetric plus isotropic components can be found in Ref. [9].

In the following we will focus on the universe in the matter-dominated era $p^I = 0$ with a plane-symmetric component given by a uniform magnetic field [10,11]. (Different physical models giving rise to a plane-symmetric metric are discussed in [12].) We will work in the limit of small eccentricity,

$$e = \sqrt{1 - (b/a)^2}, \qquad (11)$$

and we normalize the scale factors such that $a(t_0) = b(t_0) = 1$ at the present time t_0. Moreover, we will suppose that, due to the high conductivity of the primordial plasma, the magnetic field is frozen into the plasma, so that it evolves as $B \propto a^{-2}$ [10]. The energy-momentum tensor for a uniform magnetic field can be written as $(T_B)^\mu{}_\nu = \rho_B \text{diag}(1, -1, -1, 1)$, where $\rho_B = B^2/(8\pi)$ is the magnetic energy density. Thus, from the Einstein equations, we find that the eccentricity evolves according to

$$\frac{d(e\dot{e})}{dt} + 3H(e\dot{e}) = 16\pi G\rho_B, \qquad (12)$$

where $H = \dot{a}/a$. In the matter-dominated era and at the lowest order in e, we have $a \propto t^{2/3}$ and then $H = 2/(3t)$. The solution of Eq. (12), with the condition $e(t_0) = 0$, is $e^2 = 8\Omega_B^{(0)}(1 - 3a^{-1} + 2a^{-3/2})$, where $\Omega_B^{(0)} = \rho_B(t_0)/\rho_{\rm cr}^{(0)}$, and $\rho_{\rm cr}^{(0)} = 3H_0^2/8\pi G$ is the actual critical energy density. At the decoupling, $t = t_{\rm dec}$, we have $e_{\rm dec}^2 \simeq 16\Omega_B^{(0)} z_{\rm dec}^{3/2}$, where $e_{\rm dec} = e(t_{\rm dec})$ and $z_{\rm dec} \simeq 1088$ is the redshift at decoupling [13]. As a consequence, we get

$$e_{\rm dec} \simeq 10^{-2} h^{-1} \frac{B_0}{10^{-8}\,{\rm G}}, \qquad (13)$$

where $B_0 = B(t_0)$ and $h \simeq 0.72$ [13] is the little-h constant.

We are interested in the distortion of the CMB radiation in a universe with planar symmetry described by the metric (8). As before, we will work in the small-eccentricity approximation. From the null geodesic equation, we get that a photon emitted at the last scattering surface having energy $E_{\rm dec}$ reaches the observer with an energy equal to $E_0(\hat{n}) = \langle E_0\rangle(1 - e_{\rm dec}^2 n_3^2/2)$, where $\langle E_0\rangle \equiv E_{\rm dec}/(1 + z_{\rm dec})$, and $\hat{n} = (n_1, n_2, n_3)$ are the direction cosines of the null geodesic in the symmetric (Robertson-Walker) metric.

It is worth mentioning that the above result applies to the case of a magnetic field directed along the z axis. We may, however, easily generalize this result to the case of a magnetic field directed along an arbitrary direction in a coordinate system (x_g, y_g, z_g) in which the $x_g y_g$ plane is, indeed, the galactic plane. To this end, we perform a rotation $\mathcal{R} = \mathcal{R}_x(\vartheta)\mathcal{R}_z(\varphi + \pi/2)$ of the coordinate system (x, y, z), where $\mathcal{R}_z(\varphi + \pi/2)$ and $\mathcal{R}_x(\vartheta)$ are rotations of angles $\varphi + \pi/2$ and ϑ about the z and x axis, respectively. In the new coordinate system the magnetic field is directed along the direction defined by the polar angles (ϑ, φ). Therefore, the temperature anisotropy in this new reference system is

$$\frac{\Delta T_A}{\langle T\rangle} \equiv \frac{\langle E_0\rangle - E_0(n_A)}{\langle E_0\rangle} = \frac{1}{2}e_{\rm dec}^2 n_A^2, \qquad (14)$$

where $n_A \equiv (\mathcal{R}\hat{n})_3$ is equal to

$$n_A(\theta, \phi) = \cos\theta\cos\vartheta - \sin\theta\sin\vartheta\cos(\phi - \varphi). \qquad (15)$$

Alternatively, when the ellipticity is small, Eq. (8) may be written in a more standard form:

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j, \qquad (16)$$

where h_{ij} is a metric perturbation which takes on the form $h_{ij} = -e^2\delta_{i3}\delta_{j3}$. The null geodesic equation in a perturbed Friedman-Robertson-Walker metric gives the temperature anisotropy (the Sachs-Wolfe effect):

$$\frac{\Delta T}{\langle T\rangle} = -\frac{1}{2}\int_{t_0}^{t_{\rm dec}} dt\, \frac{\partial h_{ij}}{\partial t} n^i n^j, \qquad (17)$$

where n^i are the directional cosines. Using $e(t_0) = 0$, from Eqs. (16) and (17) one gets $\Delta T/\langle T\rangle = (1/2)e_{\rm dec}^2 n_3^2$, which indeed agrees with the above result.

Before proceeding further, we note that the result (14) is subject to a simple geometrical interpretation. Indeed, let the surface of last scattering be an ellipsoid with semiaxes $a_{\rm dec}$ and $b_{\rm dec}$, and eccentricity $e_{\rm dec} = \sqrt{1 - (b_{\rm dec}/a_{\rm dec})^2}$ (see Fig. 1). Then, a photon starting from the last scattering surface at the emitter point E with spherical coordinates (r, θ, ϕ) reaches the observer O with an energy proportional to r, $E_0(r) = E_{\rm dec} r/a_{\rm dec}$. Therefore, taking into account the equation of an ellipsoid whose b axis is directed along the direction defined by the polar angles ϑ, φ,
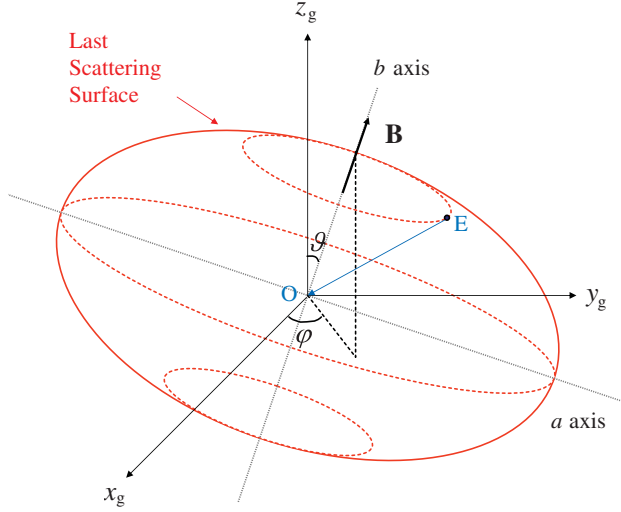
FIG. 1 (color). Geometrical interpretation of the CMB anisotropy in an "ellipsoidal" universe. A photon emitted at point E of the ellipsoidal surface of last scattering reaches the observer O with an energy proportional to the distance between emitter and observer. This causes a quadrupole anisotropy in the CMB radiation.

$$\frac{a_{\text{dec}}^2}{r^2} = 1 + \frac{e_{\text{dec}}^2}{1 - e_{\text{dec}}^2} n_A^2(\theta, \phi), \tag{18}$$

we recover, for small eccentricity, the previous results.

It is easy to see from Eq. (14) that only the quadrupole terms ($l = 2$) are different from zero:

$$a_{20}^A = \frac{\sqrt{\pi}}{6\sqrt{5}}[1 + 3\cos(2\vartheta)]e_{\text{dec}}^2,$$

$$a_{21}^A = -(a_{2,-1}^A)^* = -\sqrt{\frac{\pi}{30}}e^{-i\varphi}\sin(2\vartheta)e_{\text{dec}}^2, \tag{19}$$

$$a_{22}^A = (a_{2,-2}^A)^* = \sqrt{\frac{\pi}{30}}e^{-2i\varphi}\sin^2\vartheta e_{\text{dec}}^2.$$

Consequently, the quadrupole anisotropy is

$$\mathcal{Q}_A = \frac{2}{5\sqrt{3}}e_{\text{dec}}^2. \tag{20}$$

Since the temperature anisotropy is a real function, we have $a_{l,-m} = (-1)^m(a_{l,-m})^*$. Observing that $a_{l,-m}^A = (-1)^m(a_{l,-m}^A)^*$ [see Eq. (19)], we get $a_{l,-m}^I = (-1)^m \times (a_{l,-m}^I)^*$. Moreover, because the standard inflation-produced temperature fluctuations are statistically isotropic, we will make the reasonable assumption that the a_{2m}^I coefficients are equals up to a phase factor. Therefore, we can write

$$a_{20}^I = \sqrt{\frac{\pi}{3}}e^{i\phi_1}\mathcal{Q}_I, \qquad a_{21}^I = -(a_{2,-1}^I)^* = \sqrt{\frac{\pi}{3}}e^{i\phi_2}\mathcal{Q}_I,$$

$$a_{22}^I = (a_{2,-2}^I)^* = \sqrt{\frac{\pi}{3}}e^{i\phi_3}\mathcal{Q}_I, \tag{21}$$

where $0 \le \phi_i \le 2\pi$ are arbitrary phases. Taking into ac-

count Eqs. (19) and (21) and Eqs. (1), (2), and (7), we get for the total quadrupole

$$\mathcal{Q}^2 = \mathcal{Q}_A^2 + \mathcal{Q}_I^2 - 2f\mathcal{Q}_A\mathcal{Q}_I, \tag{22}$$

where

$$f(\vartheta, \varphi; \phi_1, \phi_2, \phi_3) = \frac{1}{4\sqrt{5}}\{2\sqrt{6}[-\sin\vartheta\cos(2\varphi + \phi_3)$$
$$+ 2\cos\vartheta\cos(\varphi + \phi_2)]$$
$$- [1 + 3\cos(2\vartheta)]\cos\phi_1\}. \tag{23}$$

Looking at Eq. (22) we see that, if the space-time background is not isotropic, the quadrupole anisotropy can become smaller than the one expected in the standard picture of the ΛCDM (isotropic) cosmological model of temperature fluctuations. We may fix the direction of the magnetic field and the eccentricity by minimizing the total quadrupole anisotropy. Let $\bar{e}_{\text{dec}}$, $\bar{\vartheta}$, and $\bar{\varphi}$ be the values which minimize $\mathcal{Q}^2$, and $\bar{f}$ the expression $f(\bar{\vartheta}, \bar{\varphi}; \phi_1, \phi_2, \phi_3)$, then we get

$$\mathcal{Q}^2 = (1 - \bar{f}^2)\mathcal{Q}_I^2, \qquad \bar{e}_{\text{dec}}^2 = \frac{5\sqrt{3}}{2}\bar{f}\mathcal{Q}_I. \tag{24}$$

It is straightforward to show that $\bar{f}$ is a strictly positive function, such that, for every $\bar{\vartheta}$, $\bar{\varphi}$, ϕ_1, ϕ_2, and ϕ_3, it results in

$$\frac{1}{\sqrt{5}} \le \bar{f} \le \frac{\sqrt{39 + 6\sqrt{6}} + \sqrt{6} - 1}{4\sqrt{5}}. \tag{25}$$

Taking $(\Delta T_2)_I^2 = 1252 \ \mu\text{K}^2$, we have

$$46.2 \ \mu\text{K}^2 \lesssim (\Delta T_2)^2 \lesssim 1001.6 \ \mu\text{K}^2, \tag{26}$$

$$0.50 \times 10^{-2} \lesssim \bar{e}_{\text{dec}} \lesssim 0.74 \times 10^{-2}. \tag{27}$$

From the above equations we see that the inflation-produced quadrupole anisotropy embedded in a plane-symmetric universe with eccentricity of order 10^{-2} can be brought into agreement with observations (see Fig. 2). In particular, from Eq. (13) it follows that the eccentricity $\bar{e}_{\text{dec}} \simeq (0.5\text{–}0.7) \times 10^{-2}$ is produced by cosmic magnetic fields $B_0 \simeq (4\text{–}5) \times 10^{-9}$ G (these values for B_0 are compatible with the constraints on cosmic magnetic fields derived in Ref. [14]).

Obviously, the arbitrary phases ϕ_i are not directly measurable. Therefore, we can treat them as stochastic variables. As a consequence, the quadrupole $\mathcal{Q}$ [Eq. (22)] can be considered as a distribution function which depends on the parameters e_{dec}, ϑ, and φ and the stochastic variables ϕ_i. To determine the distribution of the quadrupole, we have performed numerical simulations keeping the variables ϕ_i random in the interval $[0, 2\pi]$. In Fig. 2 we show the result of our numerical simulations with $N = 250$ runs. In each run, according to our previous discussion, e_{dec}, ϑ, and φ are determined by minimizing the total quadrupole. We find for the arithmetic averages
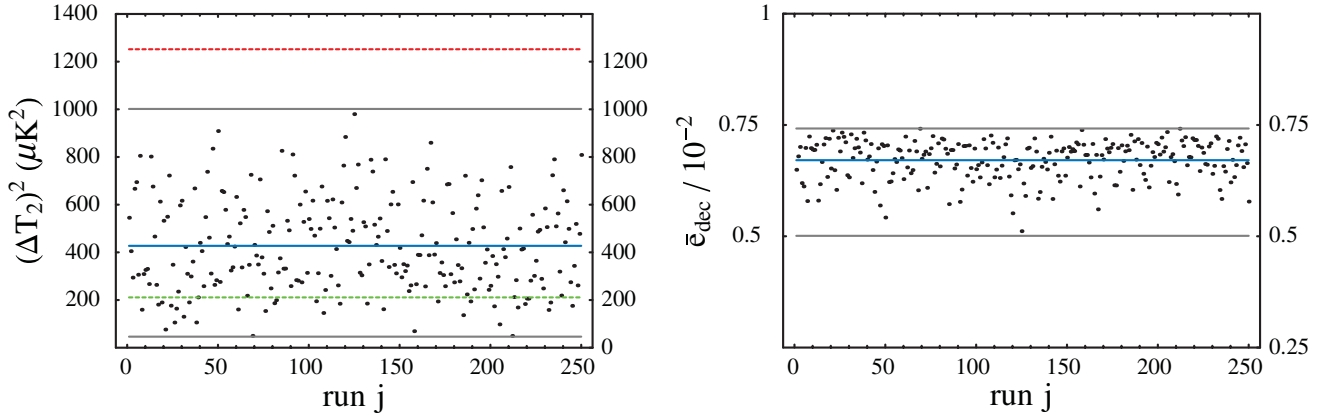
FIG. 2 (color).   Results from our numerical simulation with $N = 250$ runs. Continuous gray lines in the left and right panels refer to Eqs. (26) and (27), respectively. Left panel: The blue continuous line is the mean value Eq. (28), the green dashed line is Eq. (4), and the red dashed line is Eq. (5). Right panel: The blue continuous line is the mean value Eq. (29).

$$[(\Delta T_2)^2]_{\text{mean}} \simeq 427.3 \ \mu K^2, \qquad (28)$$

$$(\bar{e}_{\text{dec}})_{\text{mean}} \simeq 0.67 \times 10^{-2}. \qquad (29)$$

Equation (28) shows that, even if we take care only of the intrinsic uncertainty measured by the cosmic variance,

$$\sigma_{\text{cosmic}} \equiv \sqrt{2/5} [(\Delta T_2)^2]_{\text{mean}} \simeq 270.3 \ \mu K^2, \qquad (30)$$

then our mean value is in agreement with the observed quadrupole anisotropy Eq. (4).

In conclusion, we have shown that allowing a small eccentricity at decoupling $e_{\text{dec}} \sim 10^{-2}$ in the large-scale spatial geometry of our universe results in a drastic reduction in the quadrupole anisotropy without affecting higher multipoles of the angular power spectrum of the temperature anisotropy. Moreover, we have seen that such a small eccentricity could be generated by a uniform cosmic magnetic field whose actual strength, $B_0 \sim 10^{-9}$ G, is of the correct order of magnitude to account for the observed magnetic fields in galaxies and galaxy clusters.

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