Marzlin and Sanders Reply: Here we address Comments by Ma *et al.* [1] (MZWW) and by Duki *et al.* [2] (DMN) on our analysis [3] (MS) of problems concerning incautious applications of the adiabatic theorem. These Comments primarily concern our argument of inconsistency, although they differ between each other in conclusions, and MZWW also discuss our counterexample to the sufficiency of the adiabatic condition. The main objections are that MZWW claim our "proof of inconsistency" is due to the mathematical error of integrating an approximate differential equation beyond its duration of validity, and DMN provide an alternative explanation of the inconsistency, which is actually similar to our own argument.

MZWW agree that our proof of inconsistency is spurious but argue against our explanation of its origin. To understand and refute the MZWW criticism, consider an example for which the adiabatic theorem is valid so the approximate solution to the Schrödinger equation using the adiabatic theorem is close to the exact solution for a large propagation time T_A. The adiabatic approximation works on the large time scale T_A because perturbations due to coupling to other instantaneous energy eigenstates in the Schrödinger equation [Eq. (1) of Ref. [3]] oscillate rapidly so their cumulative effect cancels. In contrast application (5) of Ref. [3], which exploits this approximation, yields an inconsistency for large time. Eqs. (7–9) of Ref. [3] reveal that in this particular application the rapidly oscillating phase factors are removed so that the adiabatic approximation cannot be applied on the time scale T_A. This inapplicability is the perfunctory use we cautioned against in MS [3]. In the absence of rapidly oscillating phase factors, deviations from the exact solution obviously grow on the "perturbation" time scale $T_p \sim 1/|\langle E_n|\dot{E}_m\rangle| \ll T_A$.

MZWW claim our explanation is unrelated to the adiabatic theorem because, in general, a function that only approximately fulfills a differential equation could quickly deviate from the exact solution, but casting the problem in a general context oversimplifies the nature of the inconsistency for adiabatic evolution. The MZWW line of reasoning basically states that deviations generally grow on time scale T_p, but this is equivalent to the explanation of the inconsistency we provided in MS and fails to elucidate why the approximation breaks down on time scale T_A. Furthermore, their argument erroneously disqualifies the adiabatic theorem itself: it only guarantees the adiabatic approximation on a time scale T_p instead of T_A because it ignores the role of rapid oscillations.

MZWW also proffer two naïve arguments: (i) that our proof of inconsistency and concomitant warning are trivial because it is wrong to commit any mathematical error; and (ii) that the onus is on us to expose a flaw in a theorem. Our response is: (i) a warning, and accompanying proof of inconsistency to amplify this warning, are appropriate if the error is subtle, which it is in this case; and (ii) a counterexample suffices to negate a theorem, and then the burden of discovering flaws and tightening the theorem falls to its defenders. Our early concerns about limitations concerning the adiabatic theorem are among a growing movement to deal with the need for tightening this theorem [4,5].

The DMN analysis is refreshingly lucid and draws similar conclusions to our own. DMN's explanation of the difference between their Eqs. (1) and (2) is similar to our observation that rapidly oscillating phase factors in Eq. (1) of Ref. [3] cancel out in Eq. (9) Ref. [3]. Furthermore, their conclusion that the inconsistency is not a deficiency of the adiabatic approximation is in perfect agreement with our statement that it is a perfunctory use of, not a problem with, the adiabatic theorem that generates the inconsistency. However, their conclusion that the existing form of the adiabatic theorem is complete only recognizes that the proof of inconsistency does not challenge the adiabatic theorem. On the other hand our counterexample, which has not been considered by DMN, does pose a challenge (as evidenced by more recent work on refining the theorem [4]) and is only ruled out by the adiabatic condition introduced by Ambainis and Regev [5].

Our work is supported by NSERC, iCORE, and CIAR.

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Received 2 May 2006; published 18 September 2006
DOI: 10.1103/PhysRevLett.97.128903
PACS numbers: 03.65.Ca, 03.65.Ta

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